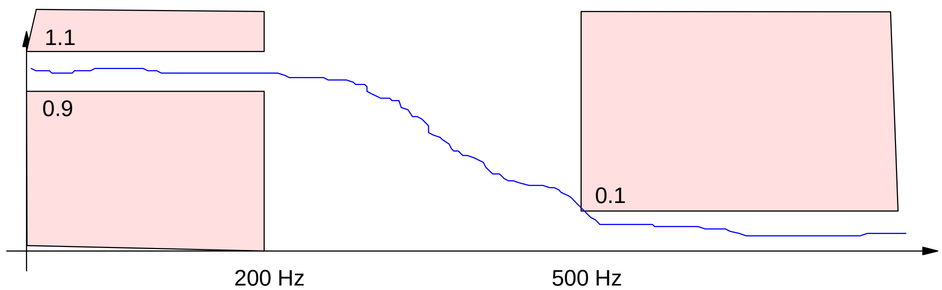
# ECE 321 - Homework #3

Filter Design, Analog Computers. Due Monday, November 20th, 2017

Problem 1-3) Design a low-pass filter to meet the following requirements:

- Input: +/- 10V, capable of 20mA
- Output: +/- 10V capable of 20mA
- Relationship:
  - 1.1 < Gain < 0.9 f < 200 Hz
  - Gain < 0.1 f > 500 Hz




1) Give a filter, G(s), which meets these requirements. Plot the gain vs. frequency for your G(s) in Matlab.

First, determine the number of poles you need:

$$\left(\frac{200Hz}{500Hz}\right)^n = 0.1$$

$$n = \frac{\ln(0.1)}{\ln(0.4)} = 2.51$$

Let n = 3

Second, pick a type of filter. Let's use a Chebychev filter. From Bison Academy, a 3rd-order Chebychev filter with a corner at 1 rad/sec is

$$G(s) = \left(\frac{k}{(s+0.85)\left(s+1.21\angle\pm 69.5^0\right)}\right)$$

Scale this so the corner is at 200Hz (400 pi )

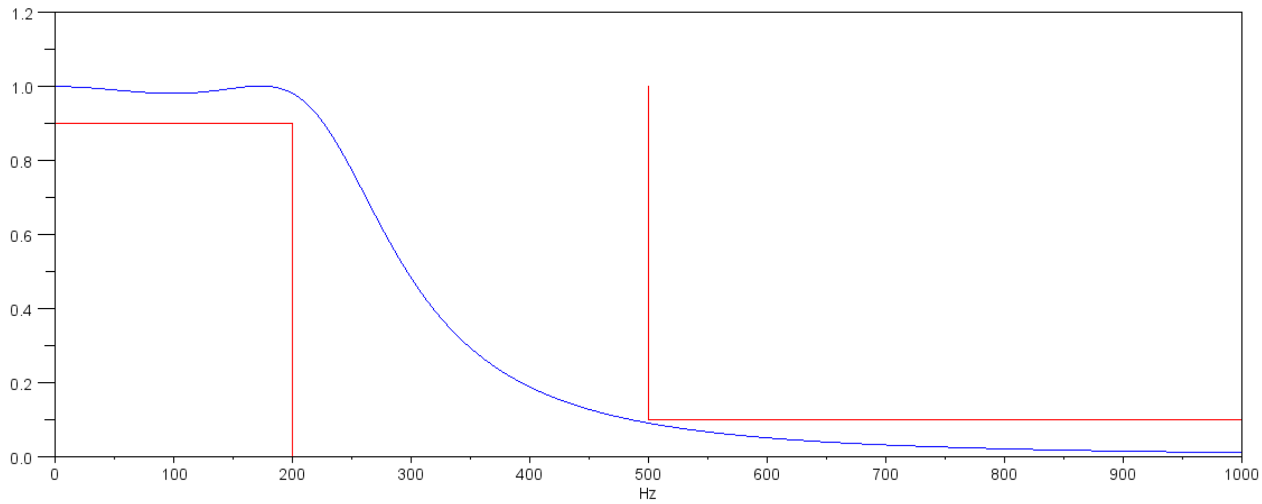
$$G(s) = \left(\frac{k}{(s+1068)\left(s+1520\angle\pm 69.5^0\right)}\right)$$

Plot the gain vs. frequency in Matlab

```
-->f = [0:1000]';
-->w = 2*pi*f;
-->s = j*w;
-->p1 = 1068;
-->p2 = 1520*exp(j*69.5*pi/180);
-->p3 = conj(p2);
-->G = 1 ./ ( (s+p1) .* (s+p2) .* (s+p3) );
-->k = 1/abs(G(1))

    2.468D+09
```

```
-->G = k ./ ( (s+p1) .* (s+p2) .* (s+p3) );
-->plot(f,abs(G))
-->plot([0,200,200],[0.9,0.9,0],'r')
-->plot([500,500,1000],[1,0.1,0.1],'r')
-->xlabel('Hz');
```




Pretty lucky: this meets the requirements.

answer:

$$G(s) = \left( \frac{2.468 \cdot 10^9}{(s+1068)\left(s+1520\angle \pm 69.5^0\right)} \right)$$

2) Design a circuit to implement this circuit

Let R1 = 10k

$$\frac{1}{R_1 C_1} = 1068$$

$$C_1 = 94nF$$

Let R2 = 100k

$$\frac{1}{R_2 C_2} = 1520$$

$$C_2 = 6.58nF$$

For the gain:

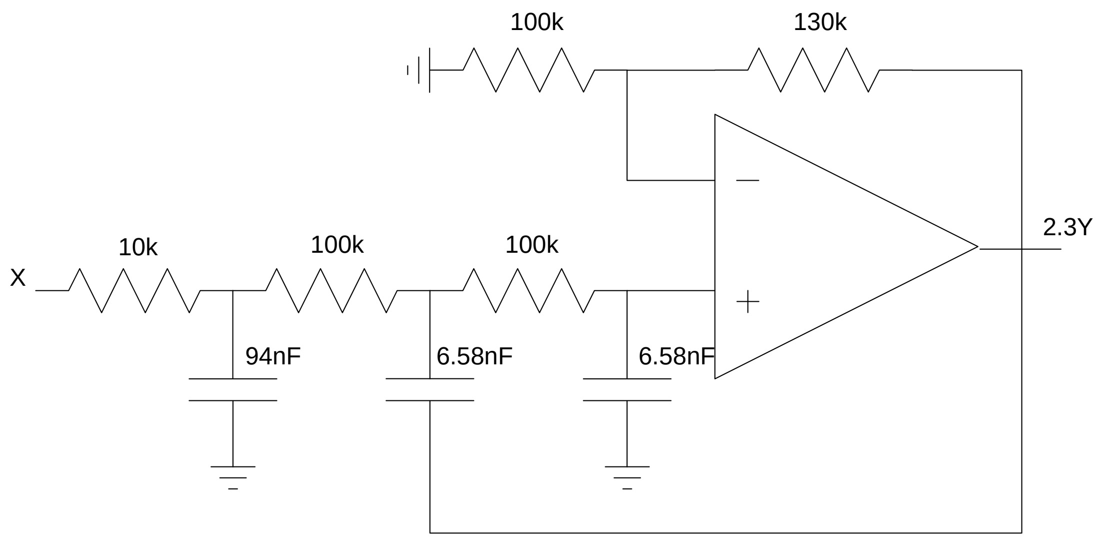
$$(3-k) = 2\cos(69.5^0)$$

$$k = 2.3$$

Let R3 = 100k, R4 = 130k

This results in the circuit having a gain of 2.3.  Either

- Live with it.  Call the ouput 2.3Y and adjust later stages, or
- Drop the gain by a factor of 2.3

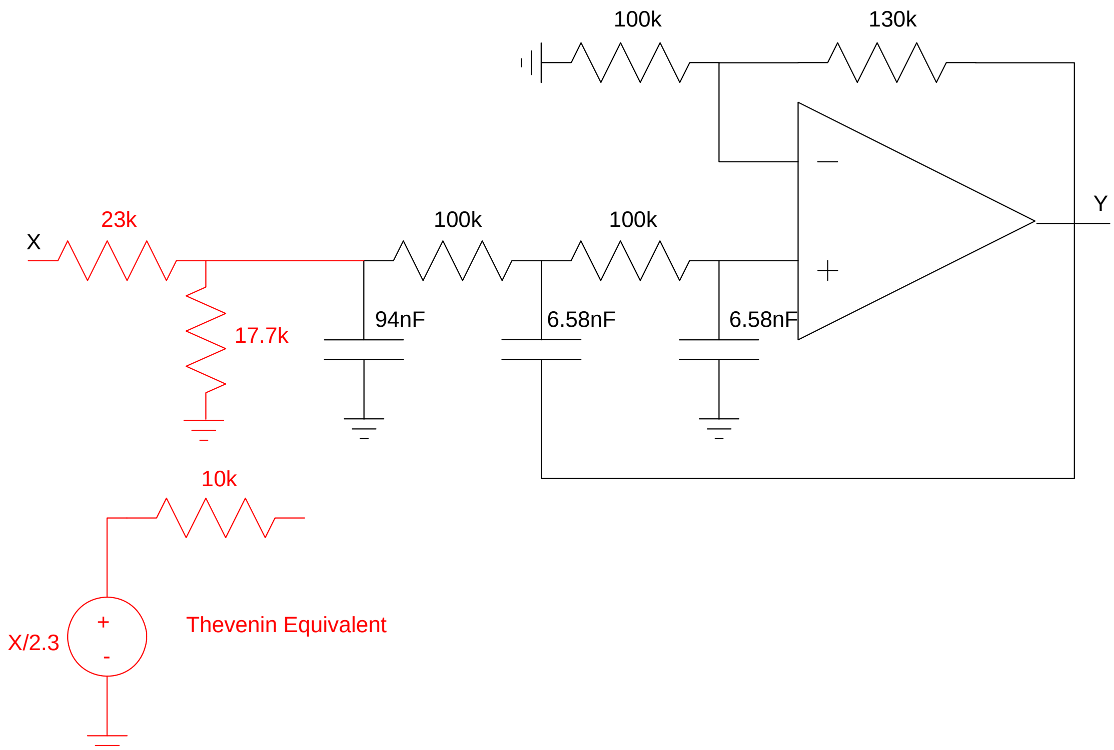



Using the latter and Thevenin equivalents.

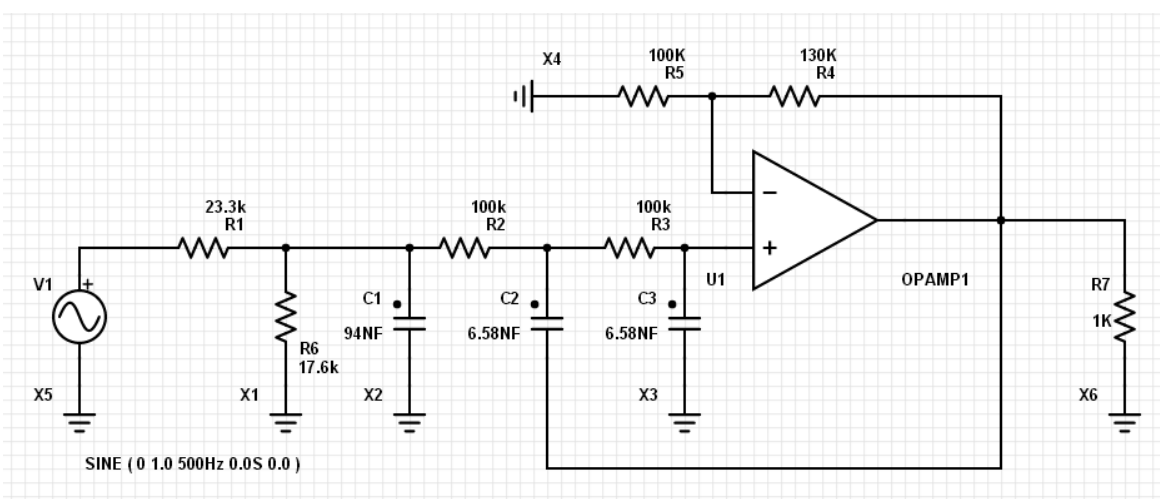
$$R_a || R_b = R_{th} = 10k$$

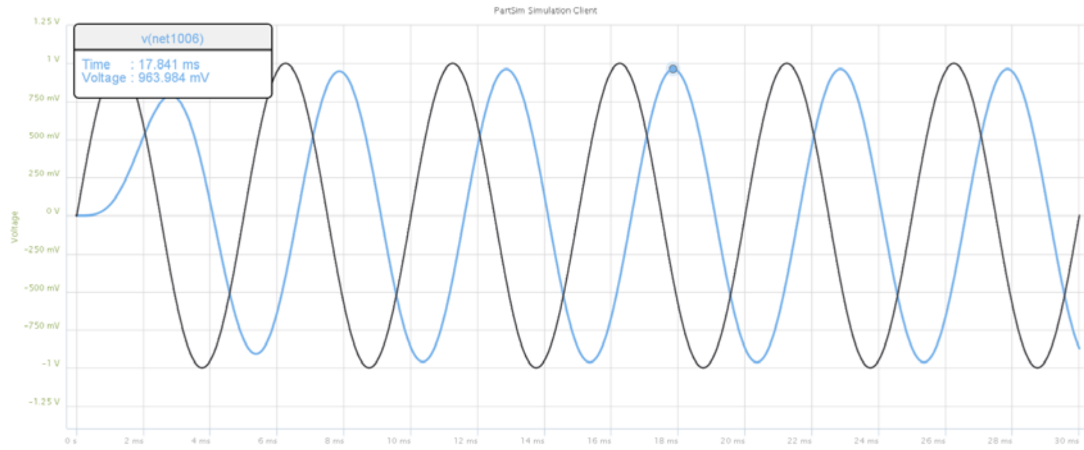
$$\left(\frac{R_b}{R_a + R_b}\right) X = \frac{X}{2.3}$$

which gives Ra = 23k, Rb = 17.7k

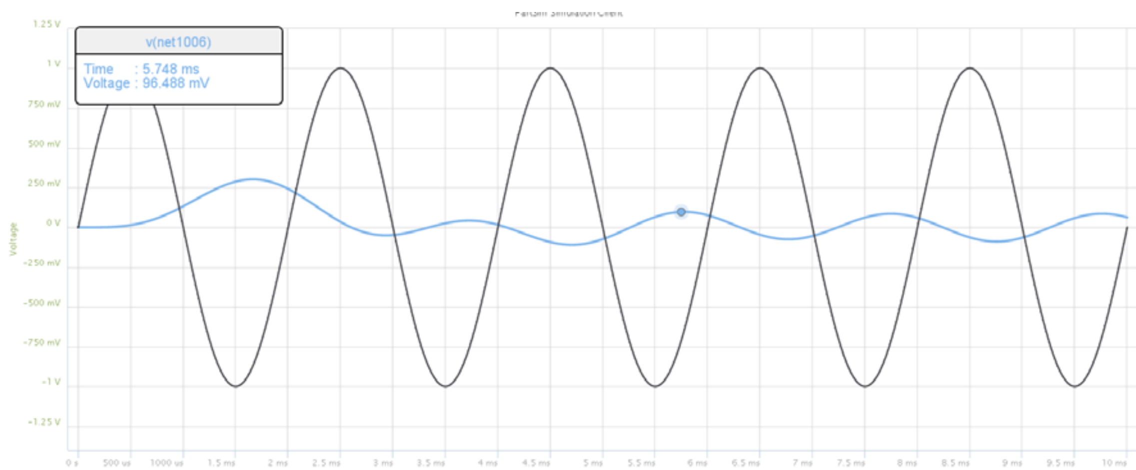



3) Test your design in PartSim: Assume a load of 1k Ohms is added (should be part of the requirements: capable of driving a 1k Ohm load)






200Hz:  Gain = 0.963

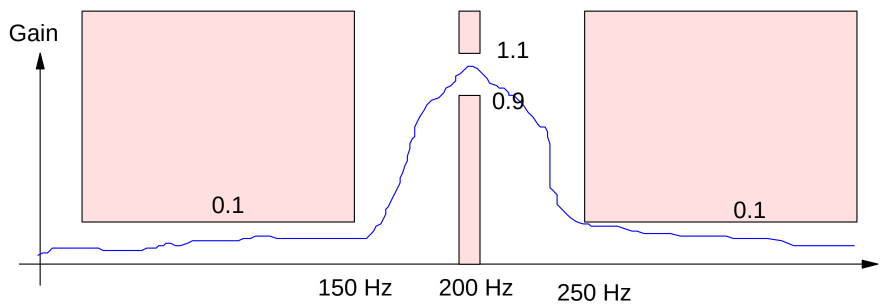



500Hz:  Gain = 0.09648

| | 200 Hz | 500 Hz |
|---|---|---|
| Gain (calc) | 0.9815 | 0.0899 |
| Gain (sim) | 0.963 | 0.09648 |
| Spec | 0.9 < G < 1.1 | < 0.1 |
| Meet Spec? | yes | yes |

2) Design a band-pass filter to meet the following requirements:

- Input: +/- 10V, capable of 20mA
- Output: +/- 10V capable of 20mA
- Relationship:
  - 1.1 < Gain < 0.9 f = 200 Hz
  - Gain < 0.1 f > 250 Hz
  - Gain < 0.1 f < 150 Hz




a) Give a filter, G(s), which meets these requirements. Plot the gain vs. frequency for your G(s) in Matlab.

- The pass frequency is 200Hz: the complex part of the pole is 200Hz (1257 rad/sec)
- Assume the band width is +/- 10Hz: the real part of the pole is 10Hz away from the resonance (62.8 rad/sec)

Pick a filter with poles at

$$G(s) = \left(\frac{ks}{(s+62.8+j1255)(s+62.8-j1255)}\right) = \left(\frac{125s}{s^2+125s+1257^2}\right)$$

Check this in Matlab:

```
-->G = tf([125,0],[1,125,1257^2])

    125s
--------------------
s^2 + 125s + 11580000
```

Check the gain at 150Hz, 200Hz, 250Hz:

```
-->abs(evalfr(G,j*150*2*pi))

    0.1678813

-->abs(evalfr(G,j*200*2*pi))

    0.9999831

-->abs(evalfr(G,j*250*2*pi))

    0.2160498
```

The gain is 2.1x too large at 250Hz. To fix this, make the real part of the pole smaller (make the 's' term smaller). A little trian and error results in

$$G(s) = \left(\frac{ks}{(s+28+j1257)(s+28-j1257)}\right) = \left(\frac{56s}{s^2+56s+1257^2}\right)$$

```
-->G = tf2ss2([56,0],[1,56,1257^2]);

-->abs(evalfr(G,j*150*2*pi))

    0.0760725
```

```
-->abs(evalfr(G,j*200*2*pi))

    0.9999160

-->abs(evalfr(G,j*250*2*pi))

    0.0986480
```

Plotting the graph:

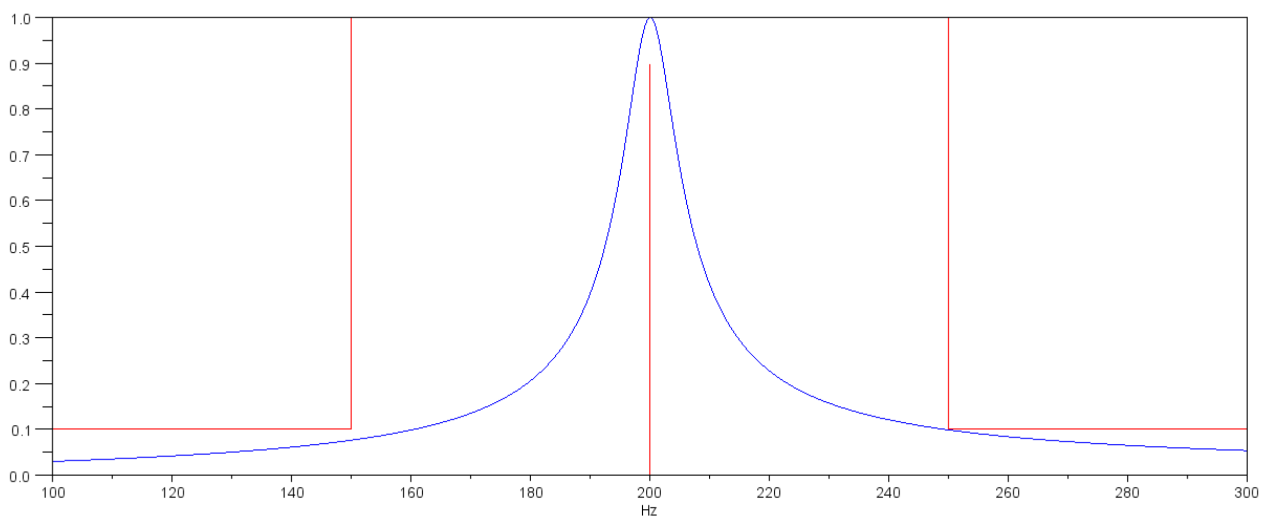
```
-->f = [100:0.1:300]';
-->w = 2*pi*f;
-->s = j*w;
-->G = 56*s ./ (s.^2 + 56*s + 1257^2);

-->plot(f,abs(G))

-->hold on
-->plot([200,200],[0,0.9],'r')
-->plot([100,150,150],[0.1,0.1,1],'r');
-->plot([250,250,300],[1,0.1,0.1],'r');
-->xlabel('Hz');
```

ull the real part of the pole in by a factor of 2.1

$$real(s) = \frac{62.5}{2.1} = 29.76$$



b) Design a circuit to implement this circuit

$$\left(\frac{56s}{s^2+56s+1257^2}\right) = \left(\frac{-\left(\frac{1}{R_1C}\right)s}{s^2+\left(\frac{2}{R_3C}\right)s+\left(\frac{R_1+R_2}{R_1R_2}\right)\left(\frac{1}{R_3C^2}\right)}\right)$$

Let C = 1uF

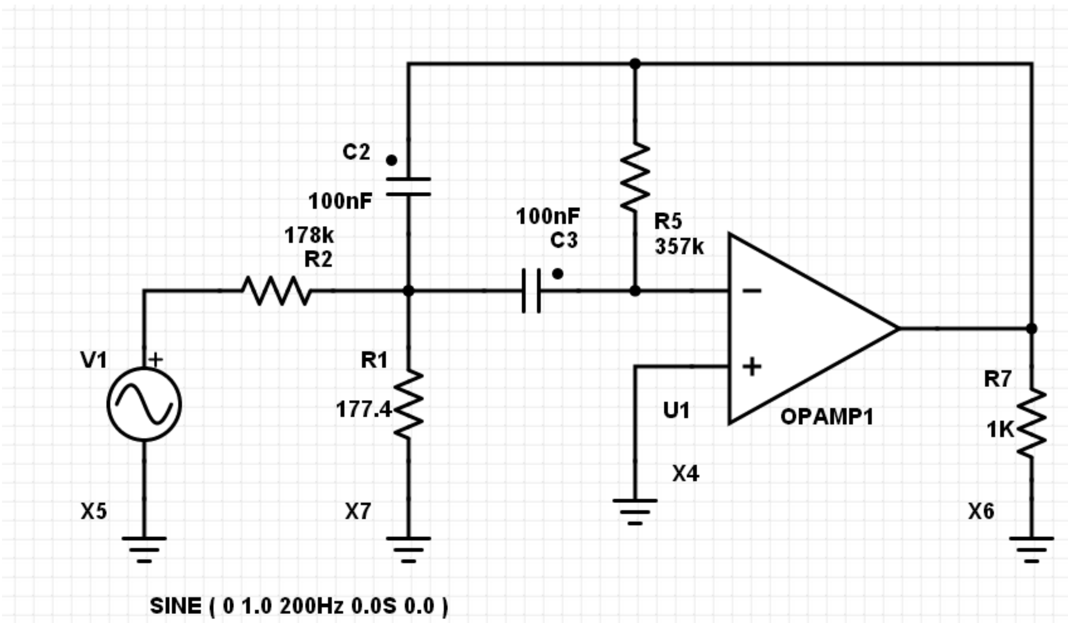
$$\left(\frac{1}{R_1C}\right) = 56 \qquad R1 = 17.8k$$

$$\left(\frac{2}{R_3C}\right) = 56 \qquad R3 = 35.7k$$

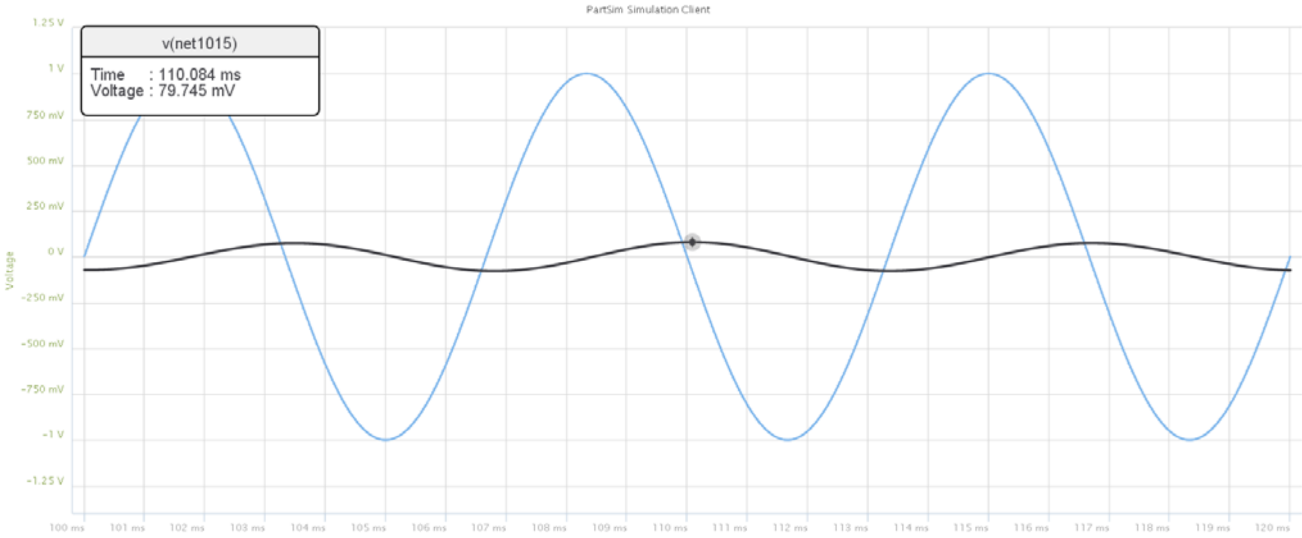
$$\left(\frac{R_1+R_2}{R_1R_2}\right)\left(\frac{1}{R_3C^2}\right) = 1257^2 \qquad R2 = 17.74$$

17.74 Ohms is a little small, so make

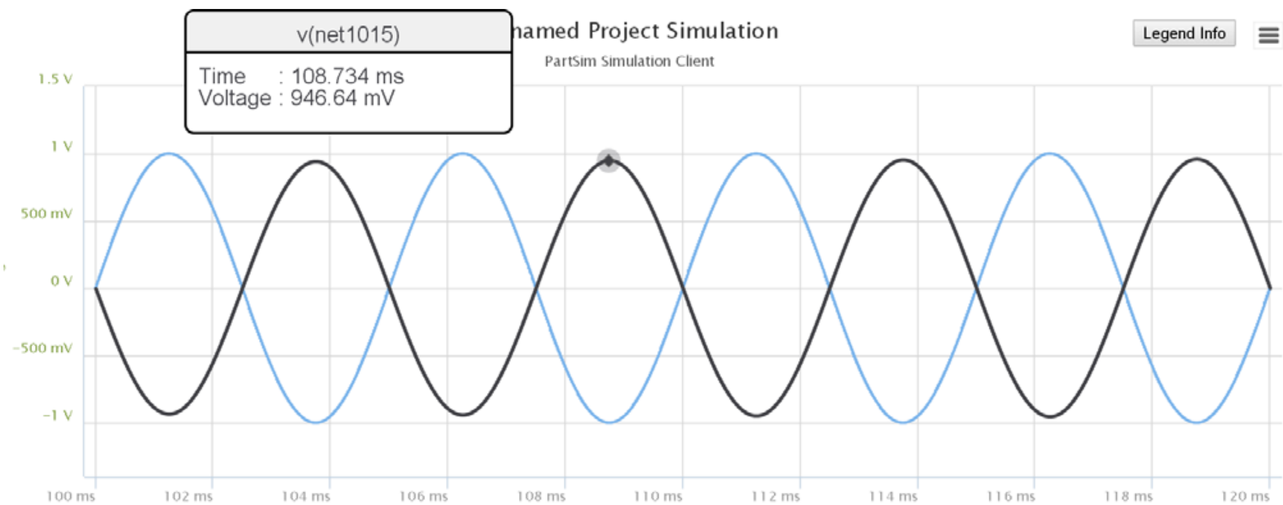
- All resistors 10x larger
- All capacitors 10x smaller



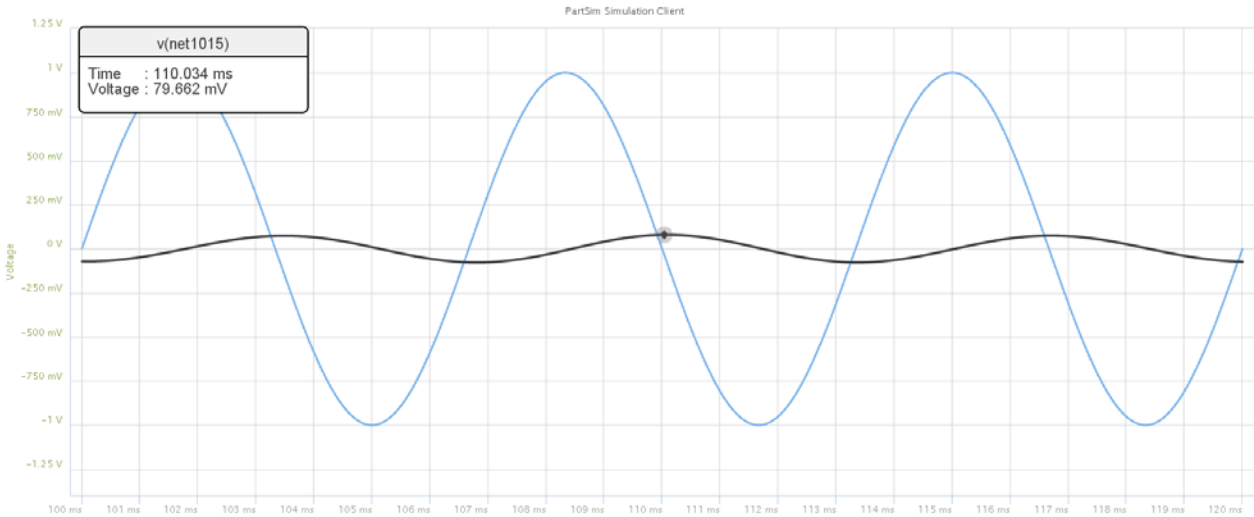

c) Test your design in PartSim




150Hz: Gain = 0.0797 (0.076 computed in Matlab)




200Hz: Gain = 0.9466 (1.00 computed in Matlab)




250Hz: Gain = 0.0796 (0.098 computed in Matlab)

| | 200 Hz | 200 Hz | 250 Hz |
|---|---|---|---|
| Gain (calc) | 0.076 | 1.000 | 0.098 |
| Gain (sim) | 0.0797 | 0.9466 | 0.0796 |
| Spec | < 0.1 | 0.9< g < 1.1 | < 0.1 |
| Meet Spec? | yes | yes | yes |

3) Build one of these circuits and check that it meets the requirements