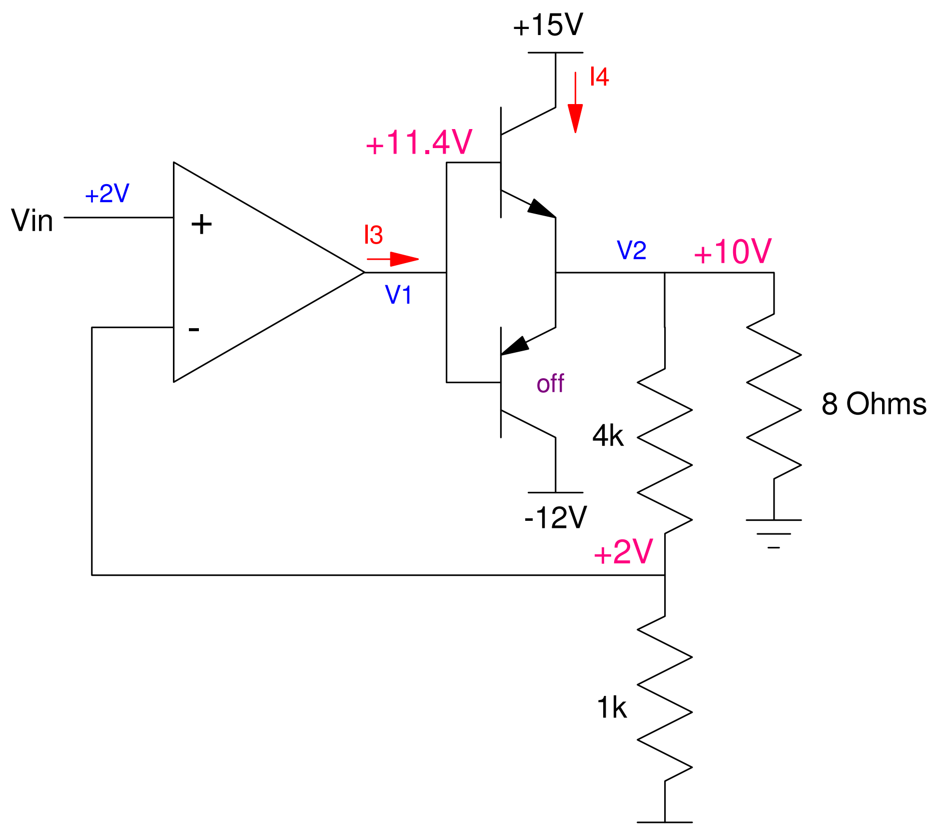
ECE 321 - Solution to Final

Fall 2018

1) Push/Pull: Determine the voltages for the following push-pull amplifier. Assume transistors with the following characteristics:

- $|V_{be}| = 1.4V$ (Darlington pairs)
- $\beta = 1000$
- $\min(|V_{ce}|) = 0.9V$

V1	V2	I3	I4
11.4V	**10.0V**	**1.2531mA**	**1.2531A**



Total Current

$$I_e = \left(\frac{10V}{8\Omega}\right) + \left(\frac{10V}{5k\Omega}\right) = 1.252A$$

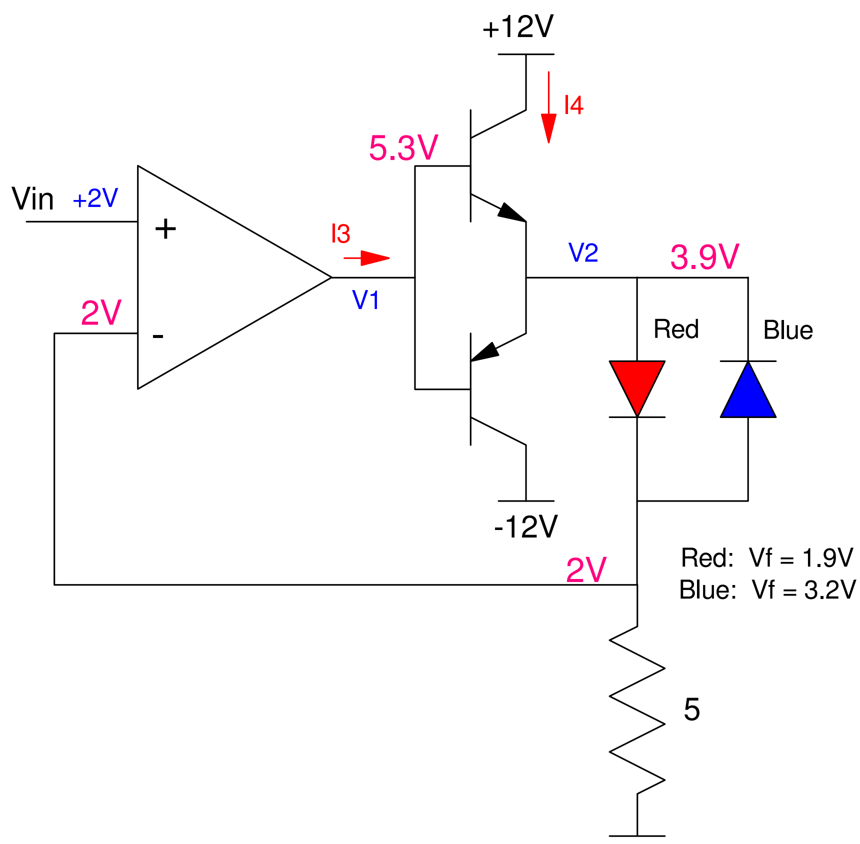
$$I_b = \left(\frac{I_e}{1+\beta}\right) = 1.2531mA$$

$$I_c = \beta I_b = 1.2531A$$

2) Push/Pull: Determine the voltages for the following push-pull amplifier. Assume transistors with the following characteristics:

- $|V_{be}| = 1.4V$ (Darlington pairs)
- $\beta = 1000$
- $\min(|V_{ce}|) = 0.9V$

V1	V2	I3	I4
5.3V	**3.9V**	**399.6uA**	**399.6mA**



total current

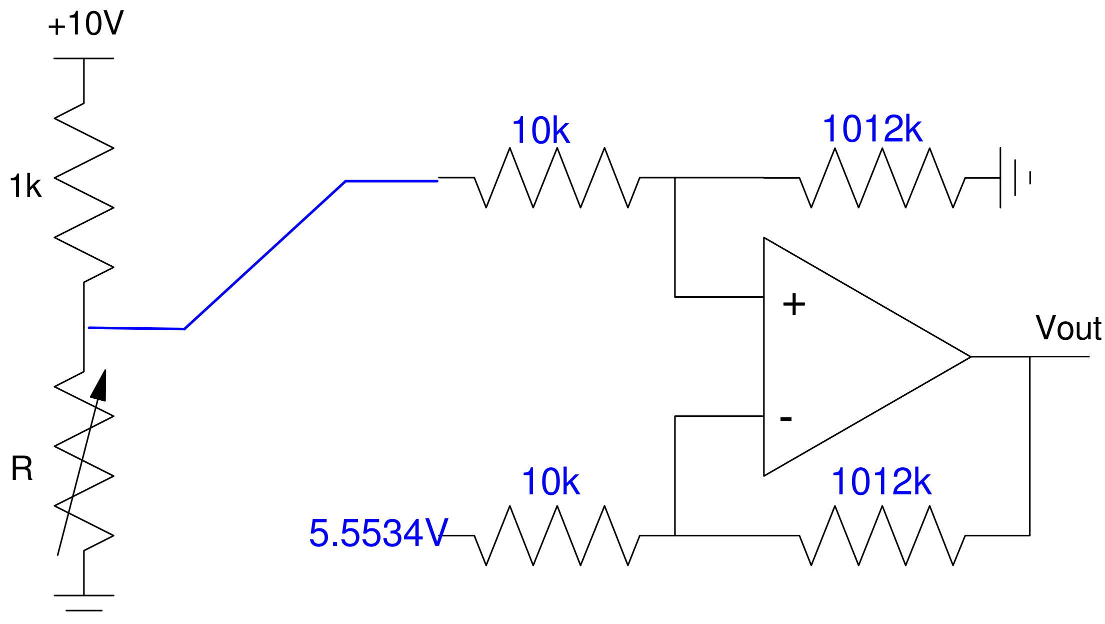
$$I_e = \left(\frac{2V}{5\Omega}\right) = 400mA$$

$$I_b = \left(\frac{I_e}{1+\beta}\right) = 399.6\mu A$$

$$I_c = \beta I_b = 399.6mA$$

3) Instrumentation Amplifier: Design a circuit which converts resistance to voltage:

- Vout = -10V when R = 1200 Ohms
- Vout = +10V when R = 1300 Ohms



R = 1200 Ohms (Vout = -10V)

$$V_a = \left(\frac{R}{R+1000}\right) 10V = 5.4545V$$

R = 1300 Ohms (Vout = +10V)

$$V_a = \left(\frac{R}{R+1000}\right) 10V = 5.6522V$$

As the input goes up, the output goes up. Connect to the + input

$$gain = \left(\frac{10V-(-10V)}{5.6522V-5.4545V}\right) = 101.2$$

To find the offset, plug in one of the endpoints

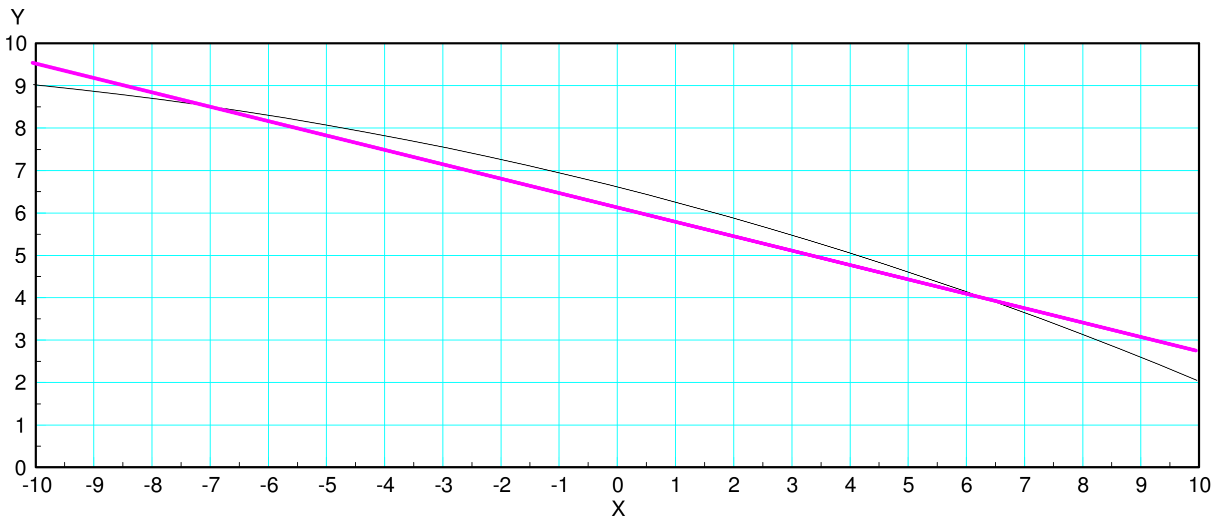
$$V_{out} = gain \cdot (V_p - V_m)$$

$$+10V = 101.2 \cdot (5.6522 - V_m)$$

$$V_m = 5.5534V$$

4) Amplifier: X and Y are related with the following graph. Determine a straight-line approximation to this curve:

y = ax + b



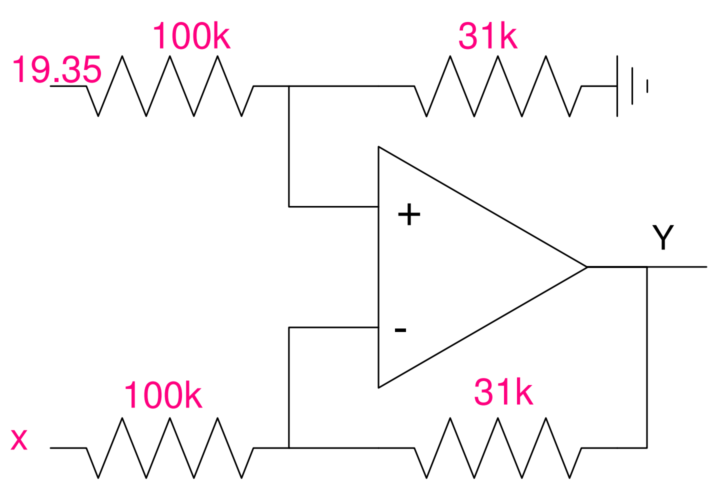
$$slope = \left(\frac{2.8V - 9.5V}{10V - (-10V)} \right) = -0.31$$

offset = 6.0V

$$y = -0.31x + 6$$

4b) Design a circuit to implement y = ax + b

$$y = 0.31(19.35V - x)$$



5) Filter Analysis: X and Y are related by the following transfer function

$$Y = \left(\frac{20}{(s+3)(s+7)}\right)X$$

5a) What is the differential equation relating X and Y?

$$((s+3)(s+7))Y = (20)X$$

$$(s^2 + 10s + 21)Y = 20X$$

'sY' means 'the derivative of Y'

$$\boxed{y'' + 10y' + 21y = 20x}$$

5b) Find y(t) assuming

$$x(t) = 3 + 7\cos(10t)$$

x(t) = 3

X = 3

s = 0

$$\left(\frac{20}{(s+3)(s+7)}\right)_{s=0} = 0.9524$$

y = gain * input

$$Y = (0.9524)(3)$$

$$Y = 2.8571$$

meaning

$$y(t) = 2.8571$$

x(t) = 7 cos(10t)

X = 7 + j0

s = j10

$$\left(\frac{20}{(s+3)(s+7)}\right)_{s=j10} = 0.1569\angle -128.3^0$$

y = gain * input

$$Y = (0.1569\angle -128.3^0)(7 + j0)$$

$$Y = 1.0986\angle -128.3^0$$

meaning

$$y(t) = 1.0986\cos(10t - 128.3^0)$$

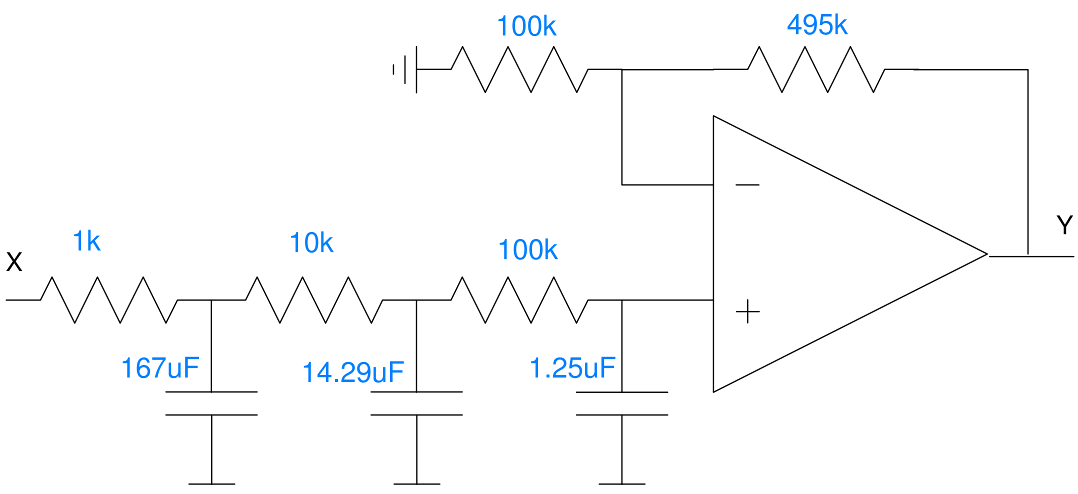
To get the total input, add up both terms

To get the total output, add up both terms

$$\boxed{y(t) = 2.8571 + 1.0986\cos(10t - 128.3^0)}$$

6) (Real Poles): Find R and C so that the following filter has the transfer function of

$$Y = \left(\frac{2000}{(s+6)(s+7)(s+8)}\right) X$$



To prevent loading, increase R by 10x for each stage (1k, 10k, 100k)

The pole is then 1/RC

Stage 1:

$$\frac{1}{RC} = 6$$

$$R = 1k$$

$$C = 167uF$$

Stage 2:

$$\frac{1}{RC} = 7$$

$$R = 10k$$

$$C = 14.29\mu F$$

Stage 3:

$$\frac{1}{RC} = 8$$

$$R = 100k$$

$$C = 1.25\mu F$$

The DC gain is

$$\left(\frac{2000}{(s+6)(s+7)(s+8)}\right)_{s=0} = 5.9520 = 1 + \frac{R_2}{R_1}$$

Let R1 = 100k

R2 = 495k

7) (Complex Poles): The transfer function for a 3rd-Order Chebychev Filter with a DC gain of one:

$$Y = \left(\frac{1.2445}{(s+0.85)\left(s+1.21\angle\pm 69.5^0\right)} \right) X$$

5a) Give the transfer function for a 3rd-order Chebychev filter with a corner at 100 rad/sec

Scale all poles by 100. (Do a change in variable: $s \rightarrow \frac{s}{100}$)

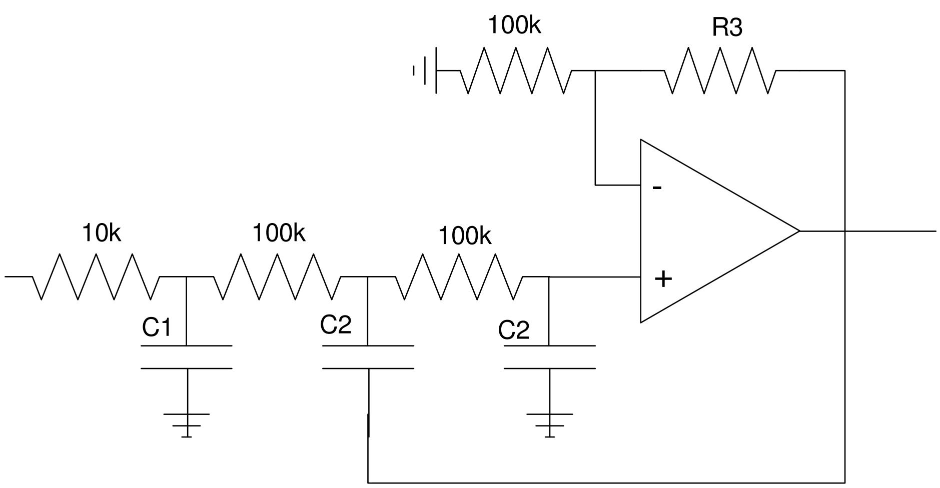
$$Y = \left(\frac{1.2445 \cdot 100^3}{(s+85)\left(s+121\angle\pm 69.5^0\right)} \right) X$$

5b) Find R and C to implement this filter

C1	C2	R3	Resulting DC gain
1.176uF	**82.64nF**	**130k**	**2.30**

$$\left(\frac{1}{10k \cdot C_1}\right) = 85 \qquad \left(\frac{1}{100k \cdot C_2}\right) = 121 \qquad 3 - k = 2\cos\left(69.5^0\right)$$

$$k = 2.30 = 1 + \frac{R_3}{100k}$$



$$Y = \left(\frac{\left(\frac{1}{R_1C_1}\right)}{s + \left(\frac{1}{R_1C_1}\right)} \right) \left(\frac{k\left(\frac{1}{R_2C_2}\right)^2}{s^2 + \left(\frac{3-k}{R_2C_2}\right)s + \left(\frac{1}{R_2C_2}\right)^2} \right) X \qquad k = 1 + \frac{R_3}{100{,}000} \qquad 3 - k = 2\cos\theta$$

8) A filter is to meet the following requirements:

- $0.9 < \text{gain} < 1.2$ $\quad\quad$ $w < 500$
- $\text{gain} < 0.1$ $\quad\quad$ $w > 700$

Determine how many poles this filter will need (N)

Give the transfer function for an N-order Butterworth low-pass filter with a corner at 500 rad/sec

N: # Poles Required	Transfer Function of Butterworth Filter: Corner = 500 rad/sec
7	$\left(\frac{500^7}{(s+500)\left(s+500\angle\pm 25.71^0\right)\left(s+500\angle\pm 51.43^0\right)\left(s+500\angle\pm 77.14^0\right)}\right)$

The number of poles you need are

$$\left(\frac{500}{700}\right)^N = 0.1$$

$$N = 6.843$$

Round up to N = 7

The angle between poles is

$$\theta = \frac{180^0}{N} = 25.71^0$$

Bonus! One acre of wheat produces approximately 3200 pounds of seed. How many pounds of seeds does industrial help produce per acre?

Only 700 pounds per acre.

You're not going to feed the world on hemp seed. But then, wheat has been modified for over 3000 years to get this sort of yield. Wheat is pretty impressive in terms of yield per acre. That's part of the reason that wheat has fed Europe for over 3000 years.