ECE 321 - Homework #3

Filters. Due Monday, December 2nd

1) X and Y are related by the following transfer function

$$Y = \left(\frac{20}{(s+3)(s+5)}\right)X$$

1a) What is the differential equation relating X and Y?

Multiply out and cross multiply

$$(s^2 + 8s + 15)Y = 20X$$

'sY' means 'the derivative of Y'

$$\frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 15y = 20x$$

or using different notation

$$\boxed{y'' + 8y' + 15y = 20x}$$

1b) Find y(t) for

$$x(t) = 4 + 5\sin(2t)$$

Use superpostion

x(t) = 4

s = j0

X = 4 + j0

$$Y = \left(\frac{20}{(s+3)(s+5)}\right)_{s=0} \cdot X$$

$$Y = (1.333) \cdot (4 + j0)$$

$$Y = 5.3333$$

$$y(t) = 5.333$$

x(t) = 5sin(2t)

s = j2

X = 0 - j5

$$Y = \left(\frac{20}{(s+3)(s+5)}\right)_{s=j2} \cdot X$$

$$Y = (0.5836 - j0.8488) \cdot (0 - j5)$$

$$Y = -4.2440 - j2.9178$$

y(t) = -4.2440 cos(2t) + 2.9178 sin(2t)

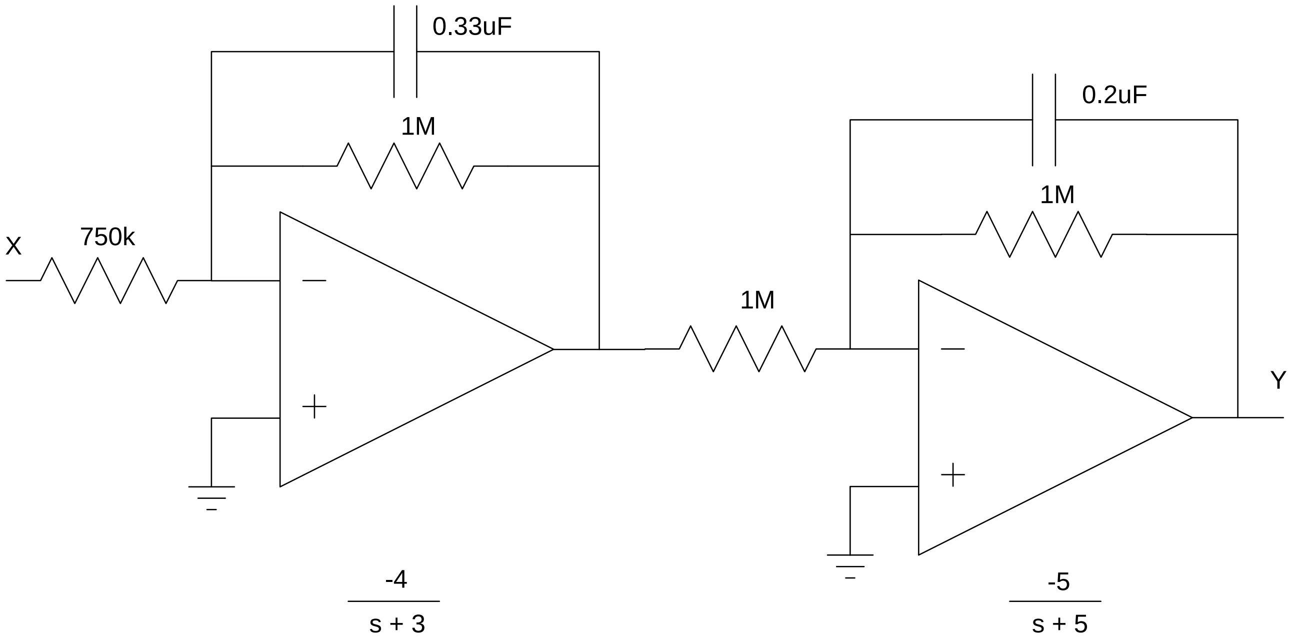
The total answer is then

y(t) = 5.333 - 4.2440 cos(2t) + 2.9178 sin(2t)

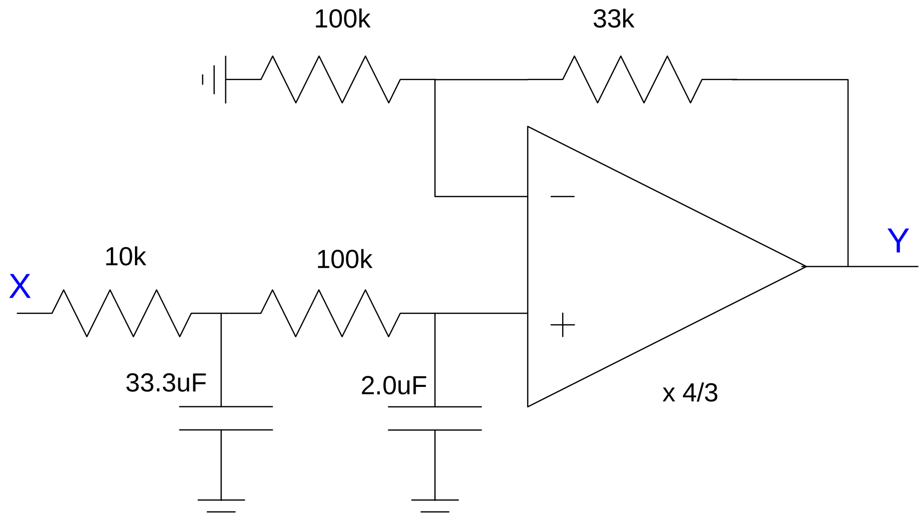
2) Design a circuit to implement

$$Y = \left(\frac{20}{(s+3)(s+5)} \right) X$$

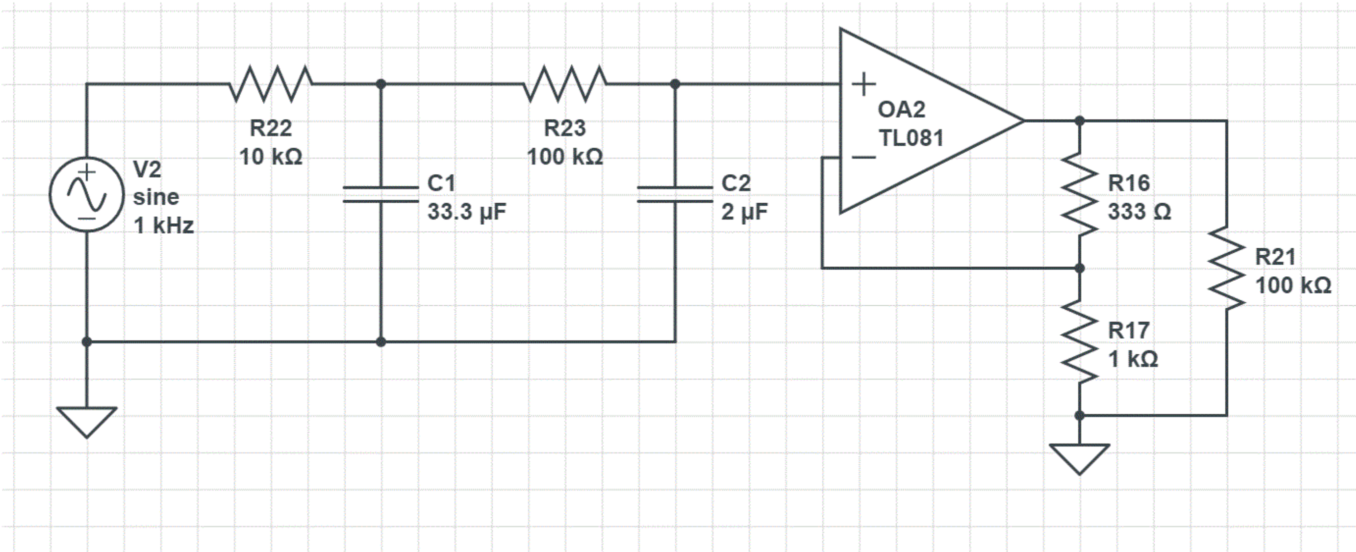
Option #1



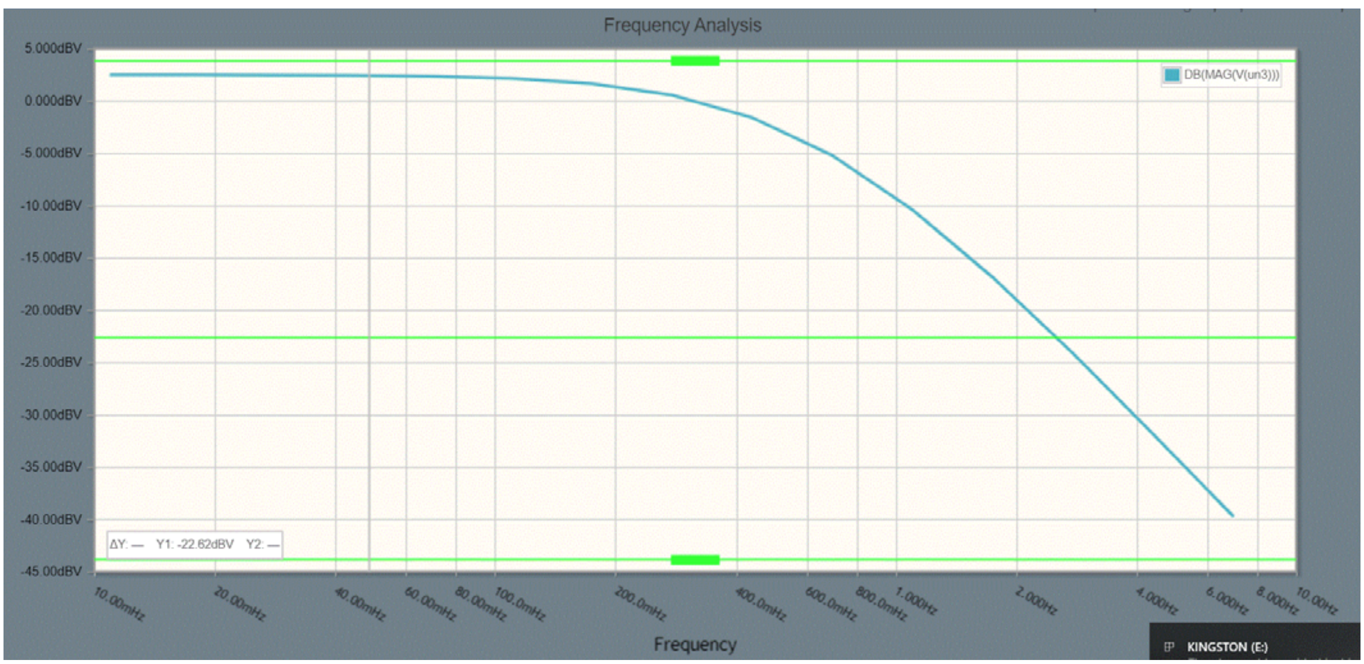
Option #2



Check your design in PartSim



CircuitLab Schematic

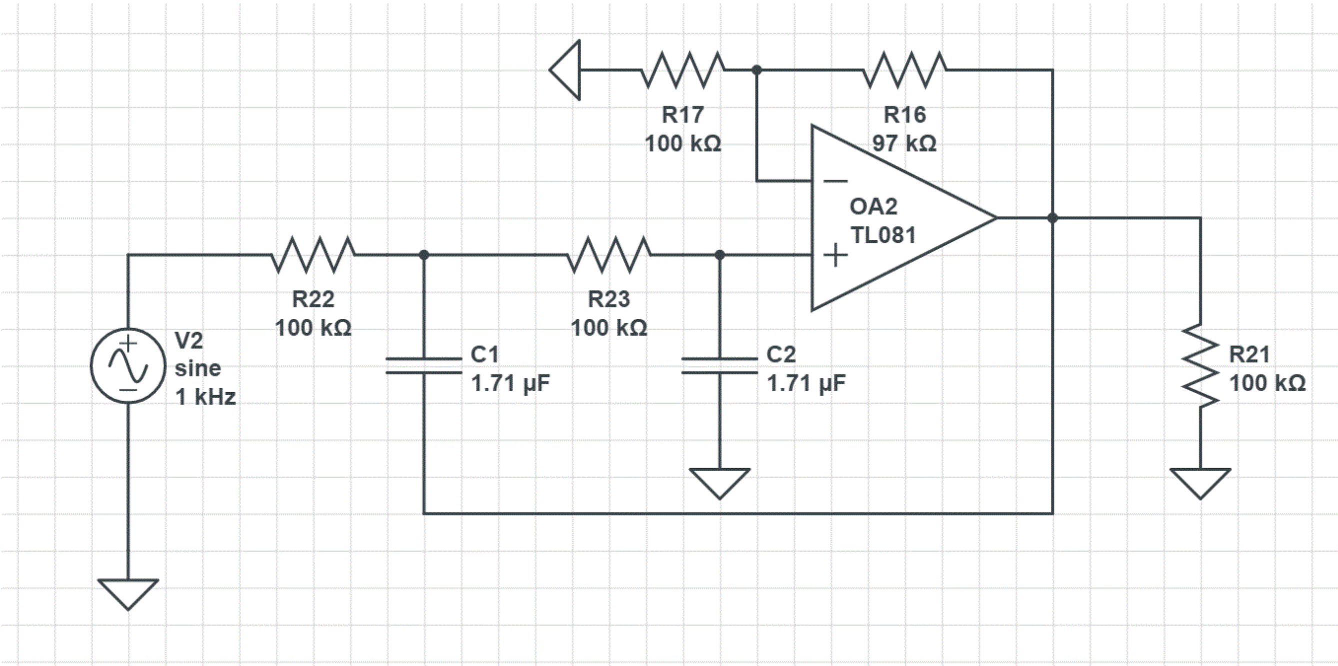


AC Sweep: This is a low-pass filter

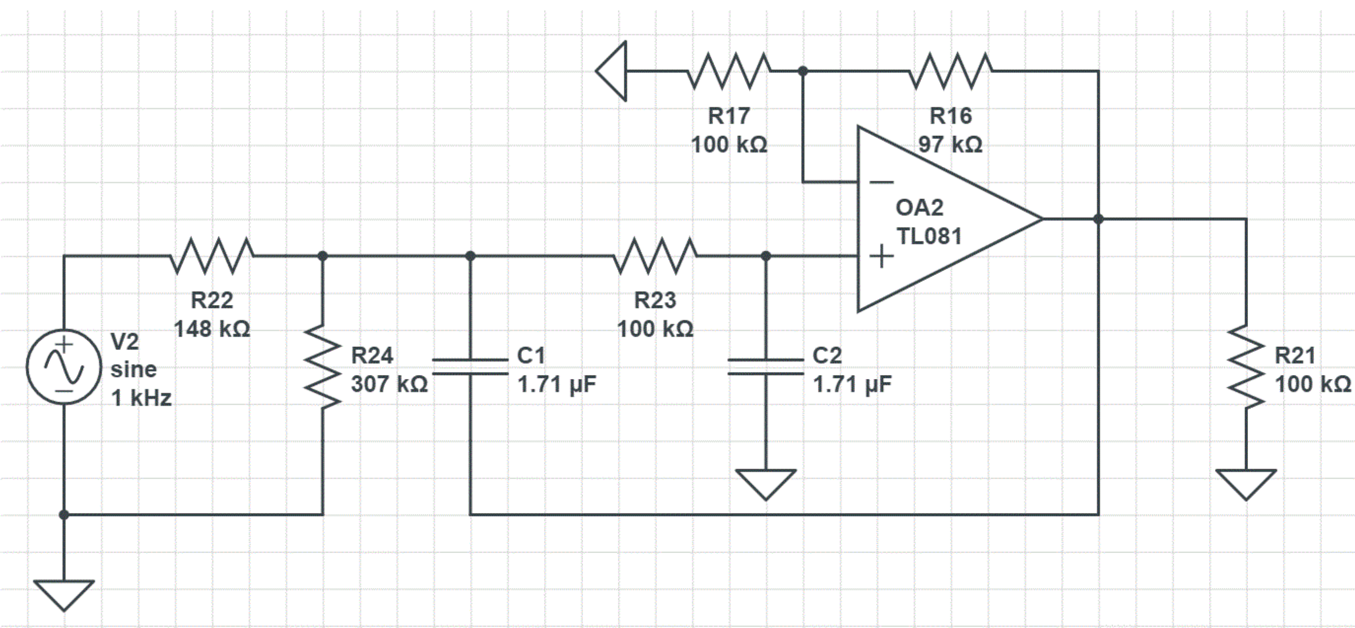
3) Design a circuit to implement

$$Y = \left(\frac{20}{(s+3+j5)(s+3-j5)} \right) X$$

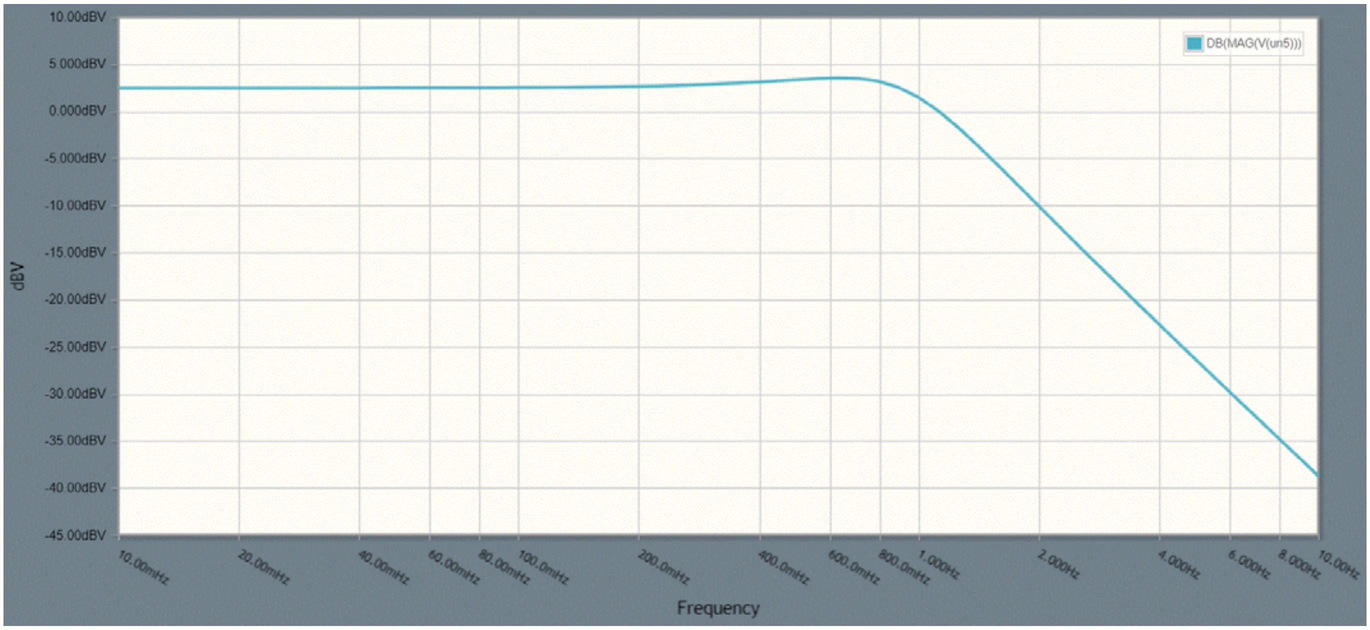
Check your design in PartSim



Circuit Option #1: DC Gain = 1.97 (vs. 1.3333). Output = 1.48Y



Circuit Option #2: DC Gain = 1.3333.
First 100k resistor is replaced with its Thevenin equivalent: Rth = 100k, Vth = 0.677Vin



10.00dBV
5.000dBV
0.000dBV
-5.000dBV
-10.00dBV
-15.00dBV
-20.00dBV
-25.00dBV
-30.00dBV
-35.00dBV
-40.00dBV
-45.00dBV
dBV
DB(MAG(V(un5)))
10.00mHz
20.00mHz
40.00mHz
60.00mHz
80.00mHz
100.0mHz
200.0mHz
400.0mHz
600.0mHz
800.0mHz
1.000Hz
2.000Hz
4.000Hz
6.000Hz
8.000Hz
10.00Hz
Frequency

Problem 4-8) You may use the following requirements or you may change them to match your term project

4) Requirements: Specify the requirements for a filter. As an example, a bass-boost could have the following requirements

Input:

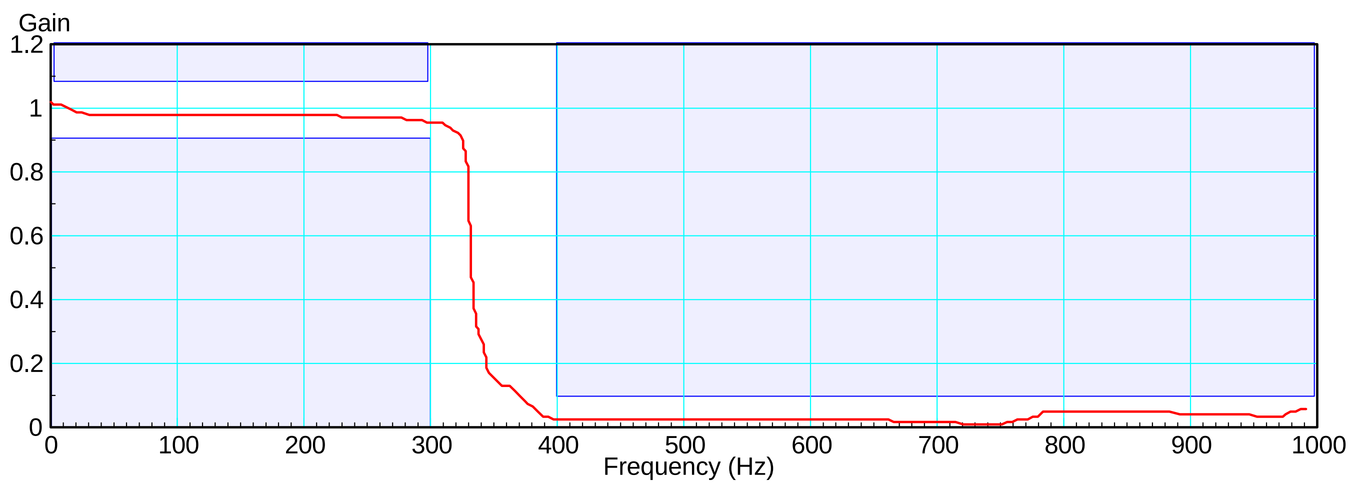
- 10Vpp sine wave
- 0 - 1kHz
- Capable of driving 10mA

Output:

- 10Vpp sine wave
- 0 - 1kHz
- Capable of drving 10mA

Relationship

- 0.9 < gain < 1.1 for frequencies between 20Hz and 300Hz
- gain < 0.1 for frequencies above 500Hz (changed from 400Hz so I don't need a 9th-order filter)



5) Analysis: Design a filter to meet these requirements. Include in your calculations

- The required number of poles
- The transfer function of your resulting design,
- A gain vs. frequency plot for your filter, and
- The gain at the design points (300Hz and 500Hz in the above example)

The number of poles you need is

$$\left(\frac{300hz}{500hz}\right)^n < \left(\frac{0.1}{1.0}\right)$$

$$n > \frac{\ln(0.1)}{\ln(0.60)} = 4.50$$

Let n = 5

A 5th-order Chebychev filter with a corner at 1 rad/sec has a transfer function of:

$$G(s) = \left(\frac{\alpha}{(s+0.48)\left(s+0.76\angle\pm 59.3^0\right)\left(s+1.06\angle\pm 82^0\right)}\right)$$

A 5th-order Chebychev filter with a corner at 2000 rad/sec has a transfer function of:

$$G(s) = \left(\frac{\alpha}{(s+0.960)\left(s+1520\angle\pm 59.3^0\right)\left(s+2120\angle\pm 82^0\right)}\right)$$

Checking the gain at 300Hz and 500Hz (design points)

$$\left(\frac{\alpha}{(s+960)\left(s+1520\angle\pm 59.3^0\right)\left(s+2120\angle\pm 82^0\right)}\right)_{s=0} = 1.0000$$

$$\left(\frac{\alpha}{(s+960)\left(s+1520\angle\pm 59.3^0\right)\left(s+2120\angle\pm 82^0\right)}\right)_{s=j300\cdot 2\pi} = 1.0174$$

$$\left(\frac{\alpha}{(s+960)\left(s+1520\angle\pm 59.3^0\right)\left(s+2120\angle\pm 82^0\right)}\right)_{s=j500\cdot 2\pi} = 0.059$$

Plotting the gain vs. frequency in Matlab

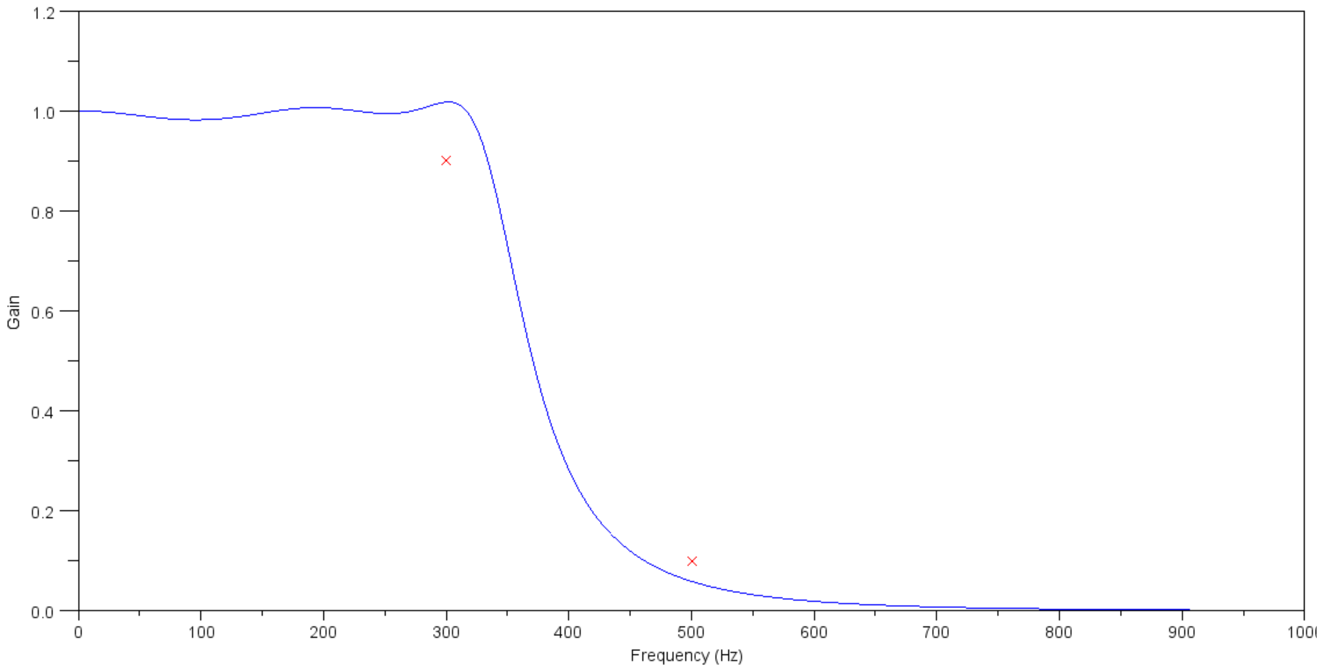
```
p1 = 960;
p2 = 1520 * exp(j*59.3*pi/180);
p3 = conj(p2);
p4 = 2120 * exp(j*82*pi/180);
p5 = conj(p4);

alpha = abs(p1 * p2 * p3 * p4 * p5)

     alpha  =   9.969D+15

f = [0:1000]';
w = 2*pi*f;
s = j*w;
```

```
Gs = alpha ./ ( (s+p1) .* (s+p2) .* (s+p3) .* (s+p4) .* (s+p5) );
plot(f,abs(Gs),[300,500],[0.9,0.1],'rx')
xlabel('Frequency (Hz)');
ylabel('Gain');
```



6) Simulation: Test your circuit design in PartSim (or similar program) to verify your design is correct

Circuit Design

$$G(s) = \left(\frac{\alpha}{(s+960)\left(s+1520\angle\pm59.3^0\right)\left(s+2120\angle\pm82^0\right)} \right)$$

Split in to three circuits

$$\left(\frac{a}{s+960}\right)\left(\frac{b}{s+1520\angle\pm59.3^0}\right)\left(\frac{c}{s+2120\angle\pm82^0}\right)$$

Stage 1:

Let R = 10k

$$\frac{1}{RC} = 960$$

$$C = 0.104\mu F$$

Stage 2:

Let R = 100k

$$\frac{1}{RC} = 1520$$

$$C = 6.57nF$$

$$3 - k = 2\cos(59.3^0)$$

$$k = 1.97$$

Stage 3: Let R = 100k

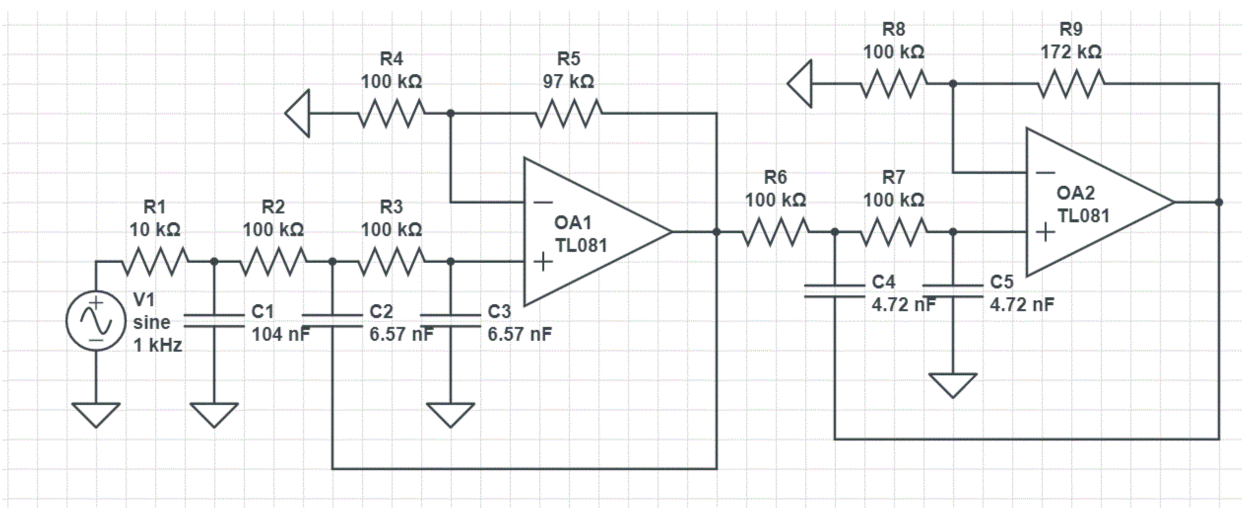
$$\frac{1}{RC} = 2120$$

$$C = 4.72nF$$

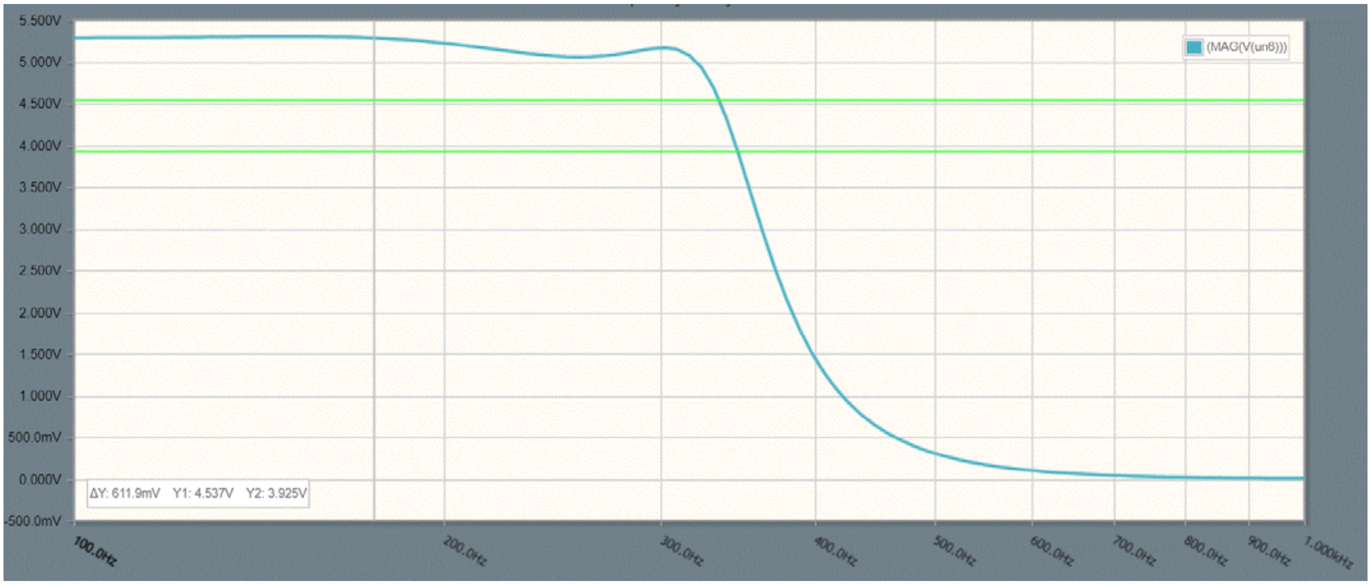
$$3 - k = 2\cos(82^0)$$

$$k = 2.72$$

This gives the following circuit



Running in CircuitLab



Checking some numbers

Frequency	Calculated Matlab	Simulated CircuitLab / 5.289	Measured ECE Lab
0Hz	1.000	1.000	
200Hz	1.006	0.9879	
300Hz	1.017	0.9758	
500Hz	0.059	0.0577	
1000Hz	0.001	0.0012	

7) Validation: Build your circuit and take measurement to show that it does (or does not) meet your requirements