ECE 321 - Quiz #4 - Name ______________________

Filters. Fall 2019

1) X and Y are related by the following transfer function

$$Y = \left(\frac{100(s+2)}{(s+4)(s+5)}\right)X = \left(\frac{100s+200}{s^2+9s+20}\right)X$$

What is the differential equation relating X and Y?

$(s^2+9s+20)y = (100s+200)x$

$y'' + 9y' + 20y = 100x' + 200x$

4

Find y(t) assuming

$$x(t) = 6 + 7\sin(8t)$$

$X = 6$

$s = 0$

$X = 6$

$= \left(\frac{100(s+2)}{(s+4)(s+5)}\right)_{s=0} \cdot X$

$= (10)(6)$

$= 60$

$x = 7\sin(8t)$

$s = j8$

$X = 0 - j7$

$= \left(\frac{100(s+2)}{(s+4)(s+5)}\right)_{s=j8} \cdot X$

$= (6.854 - j6.966)(0 - 7j)$

$= -48.76 - j47.97$

$y(t) = 60 - 48.76\cos(8t) + 47.97\sin(8t)$

3 3

$60 + 68.4\cos(8t + 135°)$

2) The transfer function for a 4th-order Butterworth filter with a DC gain of 1.00 and a corner at 1 rad/sec is

$$G(s) = \left(\frac{1}{(s+1\angle 22.5^0)(s+1\angle -22.5^0)(s+1\angle 67.5^0)(s+1\angle -67.5^0)} \right) = \left(\frac{1}{(s+1\angle \pm 22.5^0)(s+1\angle \pm 67.5^0)} \right)$$

What is the transfer function for a 4th-order Butterworth filter with a DC gain of 1.00 and a corner at 100 rad/sec?

$$\frac{100^4}{(s+100\angle \pm 22.5^\circ)(s+100\angle \pm 67.5^\circ)}$$

The transfer function for a 4th-order Chebychev filter with a DC gain of 1.00 and a corner at 1 rad/sec is

$$G(s) = \left(\frac{0.6387}{(s+0.72\angle \pm 38.5^0)(s+1.11\angle \pm 77.8^0)} \right)$$

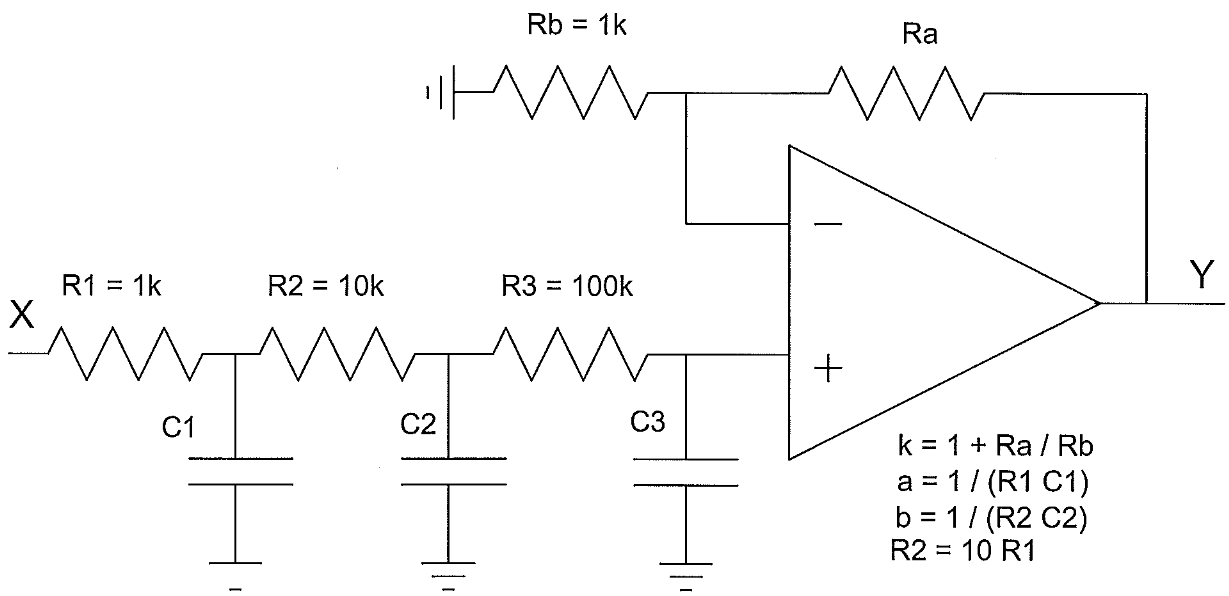
What is the transfer function for a 4th-order Chebychev filter with a DC gain of 1.00 and a corner at 100 rad/sec?

$$\frac{100^4 \cdot 0.6387}{(s+72\angle \pm 38.5^\circ)(s+111\angle \pm 77.8^\circ)}$$

3) Find R and C to implement the following transfer function with real poles

$$Y = \left(\frac{200}{(s+2)(s+5)(s+10)}\right)X$$

C1	C2	C3	Ra
500μF	20 μF	1μF	1k



$\frac{1}{RC} = 2$

$C = 500\mu F$

$$G(s) \approx \left(\frac{k \cdot abc}{(s+a)(s+b)(s+c)}\right)$$

$\frac{1}{RC} = 5$

$C = 20\mu F$

$\frac{1}{RC} = 10$

$C = 1\mu F$

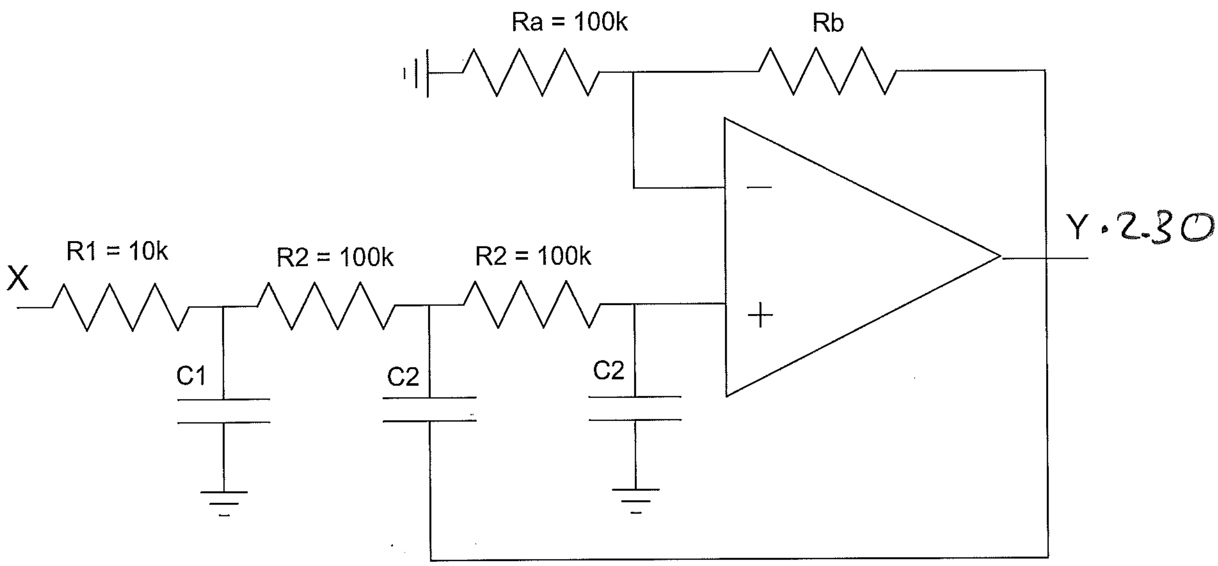
DC gain $= 2 = 1 + \frac{Ra}{Rb}$

$Ra = 1k$

4) Determing R and C to implement the following filter

$$Y = \left(\frac{1{,}244{,}000}{(s+85)\left(s+121\angle\pm 69.5^0\right)} \right) X$$

C1	C2	Rb	resulting DC gain
1.17uF	82nF	130k	2.30



$$Y = \left(\frac{\left(\frac{1}{R_1C_1}\right)}{s+\left(\frac{1}{R_1C_1}\right)} \right) \left(\frac{k\left(\frac{1}{R_2C_2}\right)^2}{s^2+\left(\frac{3-k}{RC}\right)s+\left(\frac{1}{RC}\right)^2} \right) X \qquad k = 1 + \frac{R_b}{R_a}$$

$$\frac{1}{R_1C_1} = 85$$

$$C_1 = 1.17\mu F$$

$$\frac{1}{R_2C_2} = 121$$

$$C_2 = 82nF$$

$$3 - k = 2\cos(69.5^\circ)$$

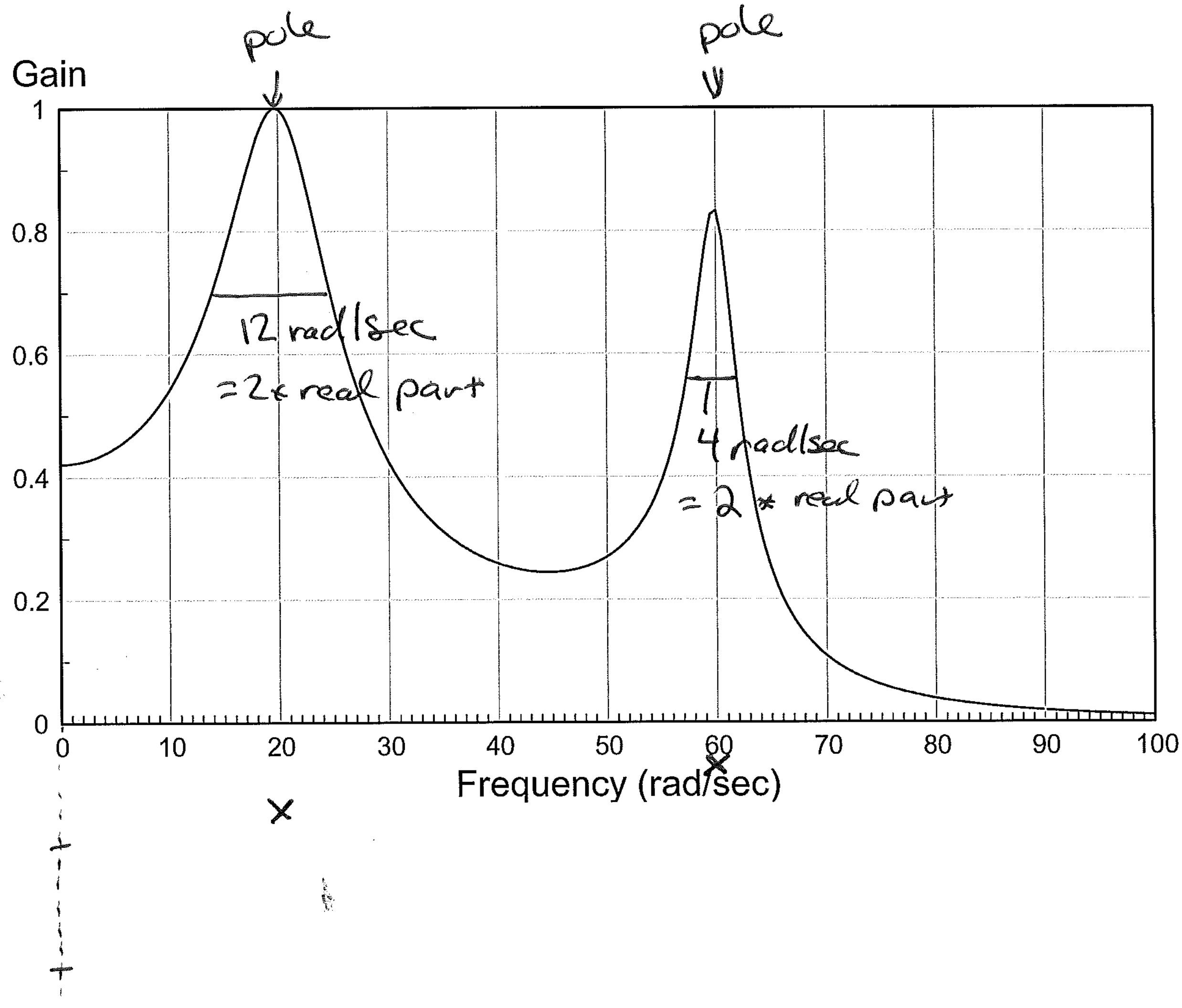
$$k = 2.30 = 1 + \frac{R_b}{R_a}$$

$$R_b = 130k$$

5) A 4th-order filter has the following gain vs. frequency. Determine the location of the poles (real and complex part)

$$Y = \left(\frac{k}{(s+p_1)(s+p_1^*)(s+p_2)(s+p_2^*)} \right) X$$

Pole #1		Pole #2	
real part	complex part	real part	complex part
-6	±j20	-2	j60



Phinneas and Ferb Bonus! What was the purpose of the Tree-Falls-In-The-Woods-Inator?

- Knock down all of the trees in the Tri-State area so Dr. Doofenschmirtz's kites wont get stuck anymore
- Makes is so that anything that falls makes the sound "Doofenschmirtz"
- Get rid of the trees that are blocking Dr. Doofenschmirtz's view of the ocean
- Create a wind-storm so the ceremony for Dr. Doofenschmirtz's brother has to be cancelled