ECE 321 - Homework #4

2-Port Models. CE Amplifiers (DC and AC). Due Monday, December 6th

CE Amplifiers (DC Analysis)

1) Determine the Q-point for the following circuits. Assume 3904 NPN transistors

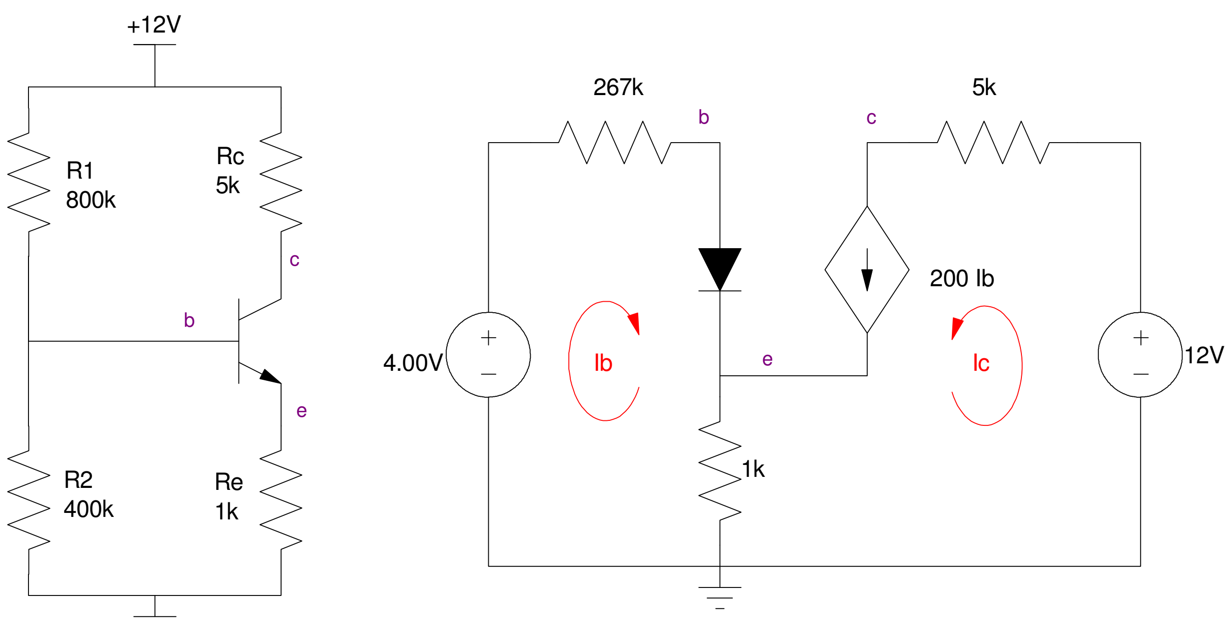
- $\beta = 200$
- $| V_{be} | = 0.7V$

Step 1: Replace R1 and R2 with their Thevenin equivalent

$$R_{th} = 800k || 400k = 267k$$

$$V_{th} = \left(\frac{400k}{400k+800k}\right) 12V = 4.00V$$

Step 2: Redraw the circuit replacing the transistor with its model in the active region:



Step 3: Find Ib

$$-4V + 267k \cdot I_b + 0.7V + 1k \cdot (I_b + I_c) = 0$$

$$I_b = \left(\frac{4.00V - 0.7V}{267k + 201 \cdot 1k}\right) = 7.056\mu A$$

$$I_c = 200 I_b = 1.411mA$$

Find Vce

$$V_c = 12 - 5k \cdot I_c = 4.944V$$

$$V_e = 1k \cdot (I_b + I_c) = 1.418V$$

$$V_{ce} = V_c - V_e = 3.525V$$

2) Modify this circuit so that

- Vce = 6.0V, and
- The Q-point is stabilized for variations in β

Going backwards

$$V_{ce} = 6V = 12V - 5k \cdot I_c - 1k \cdot (I_b + I_c)$$

$$I_c = \left(\frac{6V}{5000+\left(1+\frac{1}{200}\right)1000} \right) = 991.7\mu A$$

$$I_b = \frac{I_c}{200} = 4.959\mu A$$

To stabilize the Q-point

$$(1+\beta)R_e >> R_b$$

$$201k >> R_b$$

Let Rb 20k. Vb is then

$$V_b = I_b R_b + 0.7 + R_e(I_b + I_c)$$

$$V_b = 1.796V$$

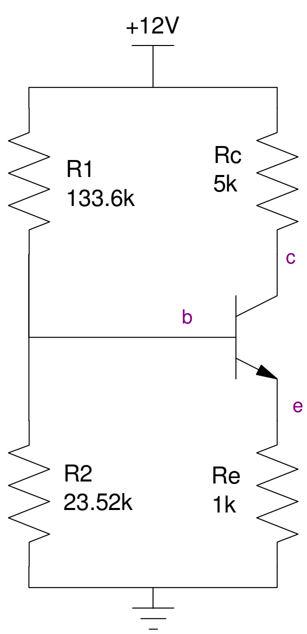
Solve for R1 and R2

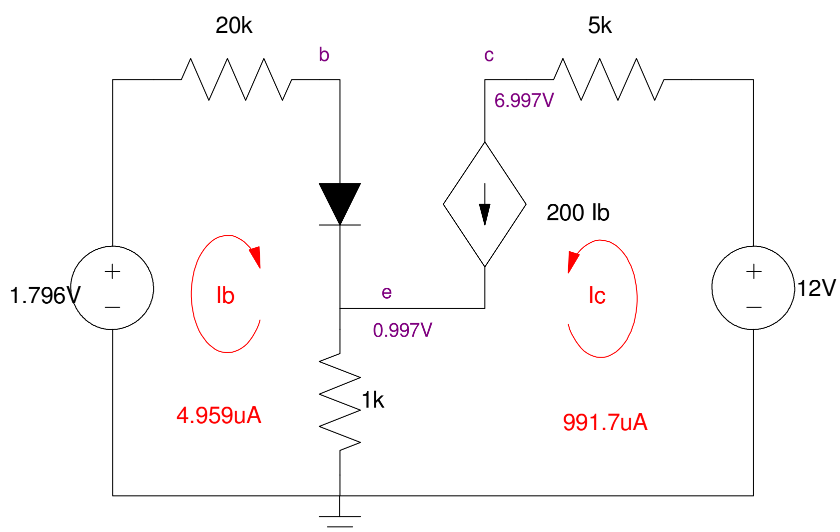
$$\left(\frac{R_1 R_2}{R_1+R_2} \right) = 20k$$

$$\left(\frac{R_2}{R_1+R_2} \right) 12V = 1.796V$$

$$R_1 = \left(\frac{12V}{1.796V} \right) 20k = 133.6k$$

$$R_2 = 23.53k$$

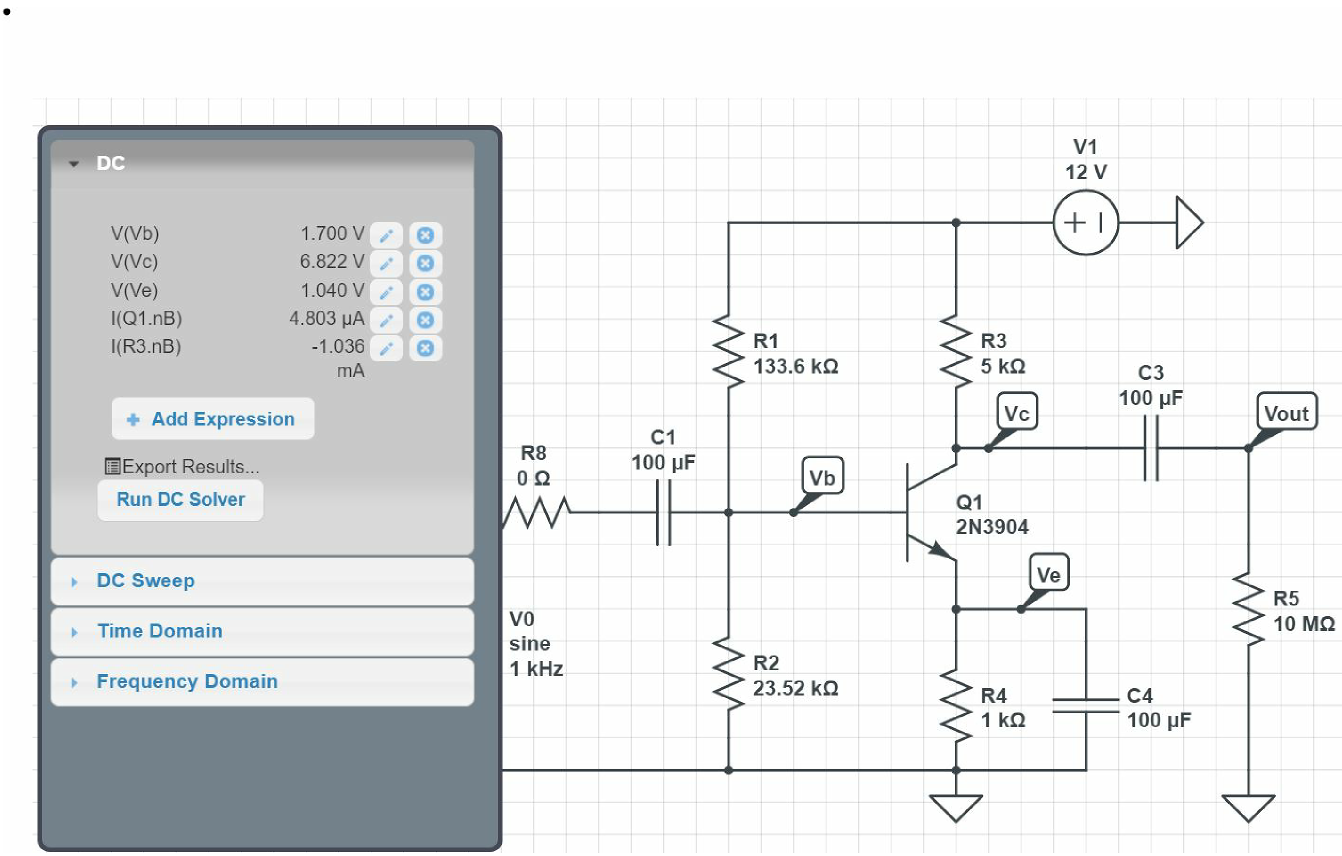




3) Check you answers in CircuitLab

	Vb	Vc	Ve	Vce
Calculated	1.697V	6.997V	0.997V	6.00V
Simulated	1.700V	6.882V	1.040V	5.842V

Note that CircuitLab uses $\beta = 215$ (Ic / Ib)

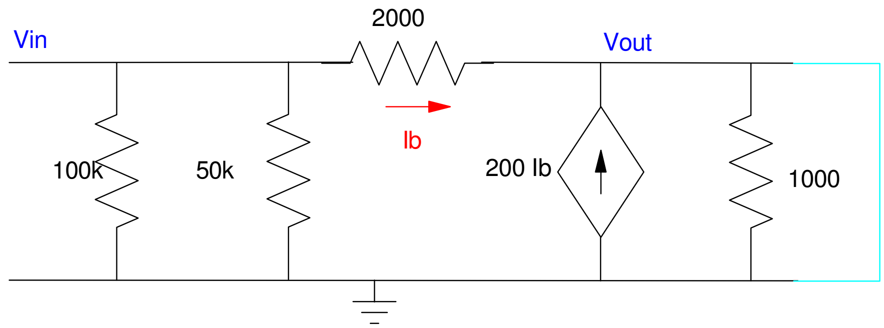
- 

2-Port Models

4) Determine the 2-port model for the following circuit

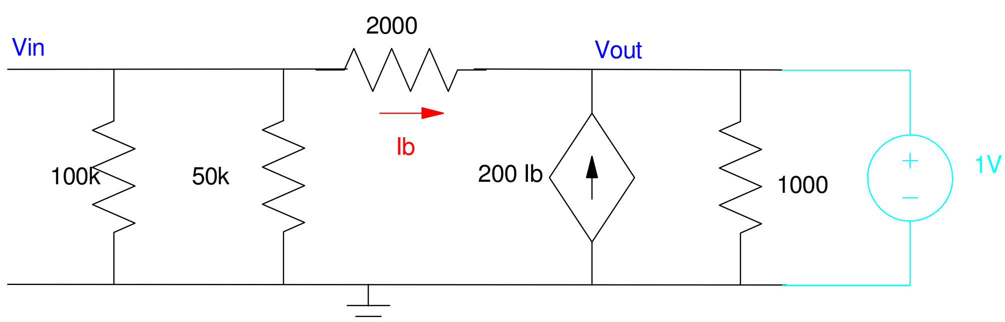
Rin: Short Vout, measure the resistance at the input

$$R_{in} = 100k||50k||2000 = 1887\Omega$$



Ain: Apply 1V at the ouput, measure Vin

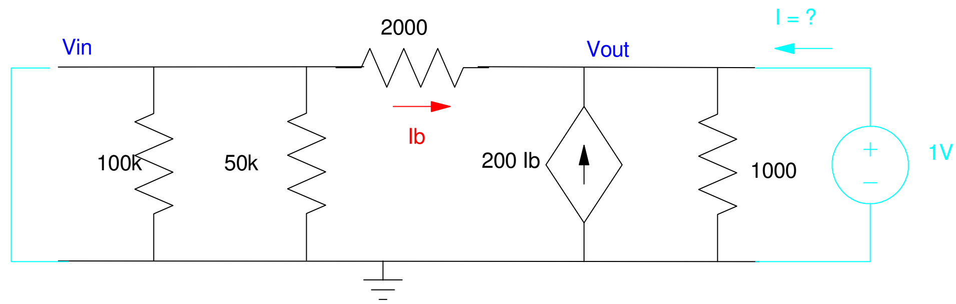
$$V_{in} = \left(\frac{100k||50k}{100k||50k+2000}\right)1V = 0.9434$$



Rout: Short Vin. Applu 1V at Vout and measure Iin. From that compute Rout

$$I = \frac{1V}{2000\Omega} + \frac{1V}{1000\Omega} + 200\left(\frac{1V}{2000\Omega}\right) = 101.5mA$$

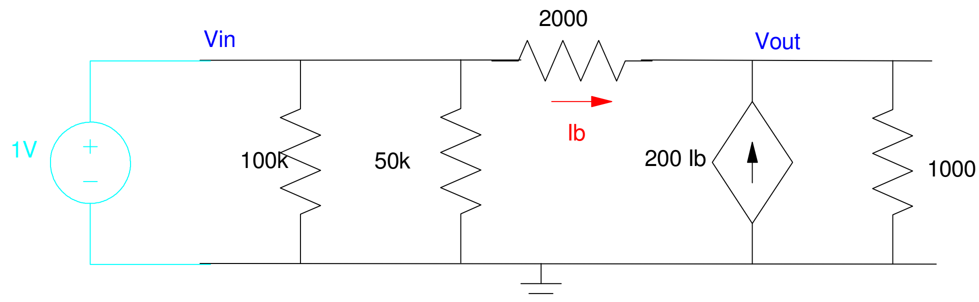
$$R_{out} = \frac{1V}{101.5mA} = 9.852\Omega$$



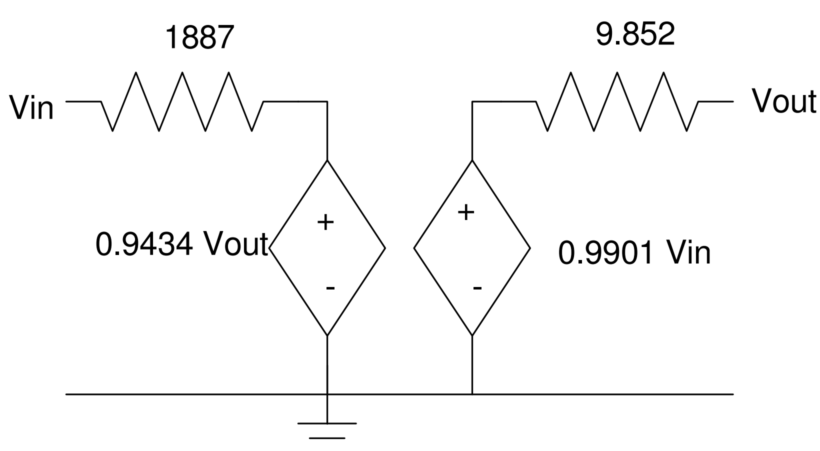
Ao: Apply 1V at Vin. Compute Vout

$$\left(\frac{V_{out}-1}{2000}\right) + \left(\frac{V_{out}}{1000}\right) + 200\left(\frac{V_{out}-1}{2000}\right) = 0$$

$$V_{out} = 0.9901V$$



So the 2-port model is

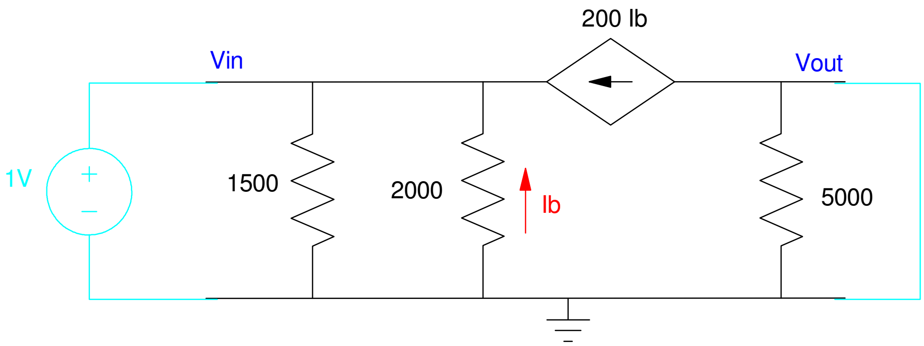


5) Determine the 2-port model for the following circuit

Rin: Short Vout. Apply 1V at the input

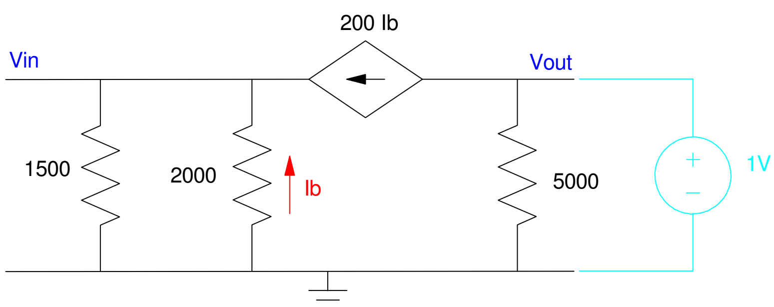
$$I_{in} = \frac{1V}{1500\Omega} + \frac{1V}{2000\Omega} + 200\left(\frac{1V}{2000\Omega}\right) = 101.2mA$$

$$R_{in} = \left(\frac{1V}{101.2mA}\right) = 9.885\Omega$$



Ai: Apply 1V at the output. Compute Vin

Vin = 0



Rout: Short Vin. Compute the resistance at the output

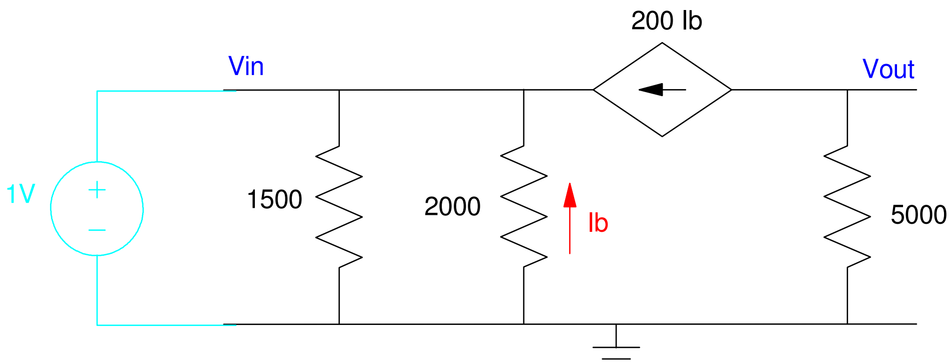
Rout = 5000

Ao: Apply 1V at the input. Compute Vout

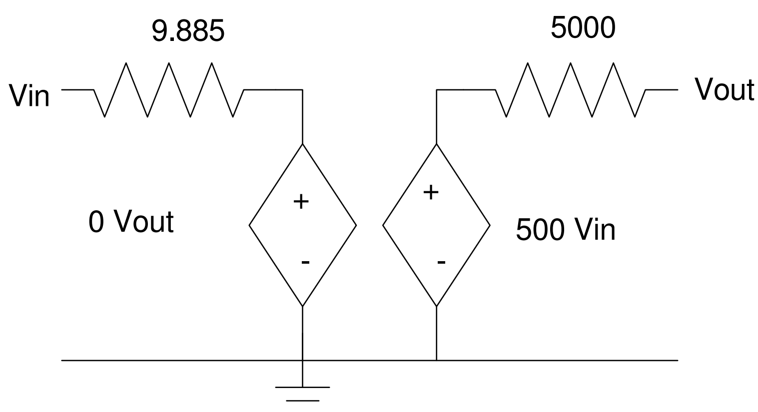
$$-I_b = \frac{1V}{2000\Omega} = 500\mu A$$

$$-200 I_b = 100mA$$

$$V_{out} = 5000\Omega \cdot 100mA = 500$$



so the 2-port model is



CE Amplifiers (AC Analysis)

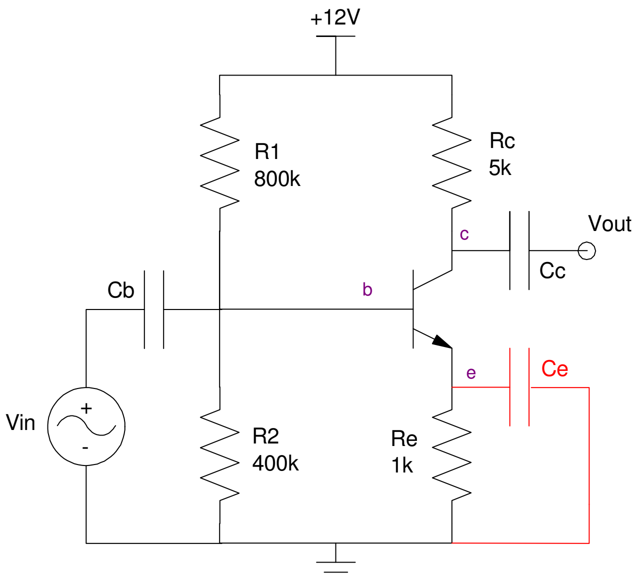
6) Draw the small signal model for the CE amplifier (below)

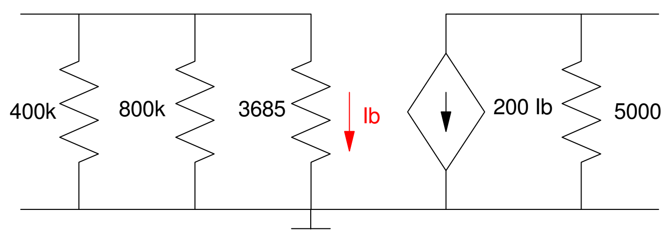
- Determine the resulting 2-port model

From problem #1

$$I_b = 7.056\mu A$$

$$r_f = \left(\frac{0.026}{I_b}\right) = 3685\Omega$$





This gives

$$R_{in} = 400k||800k||3685 = 3635\Omega$$

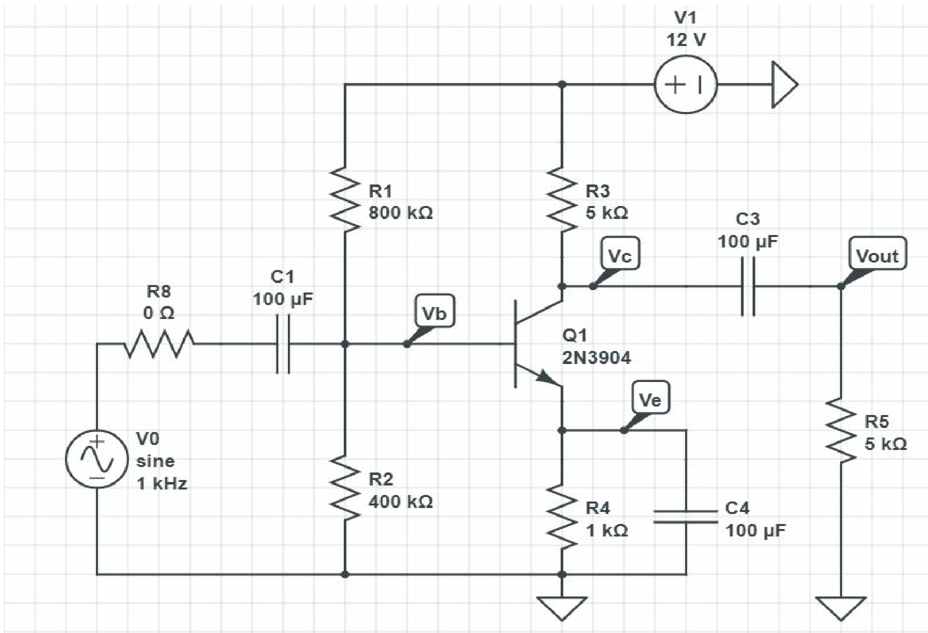
$$A_{in} = 0$$

$$R_{out} = 5000\Omega$$

$$A_{out} = -\left(\frac{200 \cdot 5000}{3685}\right) = -271.4$$

7) Check your answers for problem #6 in CircuitLab

- Rin: If you add a resistor in series with Vs equal to Rin, the output drops by half
- Rout: If you load Vout with a resistor equal to Rout, the output drops by half
- Ao: Apply a 1mV, 1kHz sine wave at Vin. The output should be Ao*Vin



V0, R8, and R5 are added so you can run some tests to find the 2-port parameters.

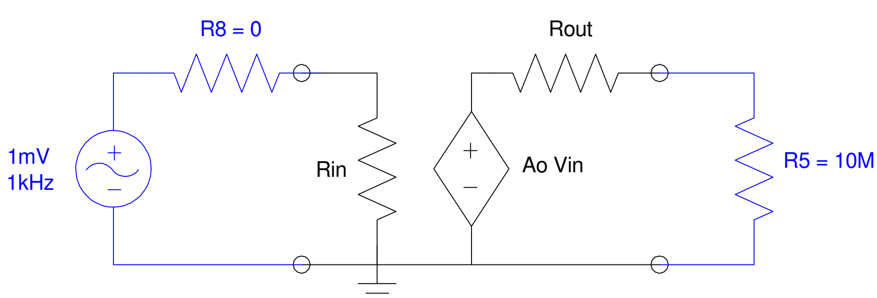
Ao: Set

- V0 = 1mV @ 1kHz
- R8 = 0
- R5 = 10M

Run a time domain simulation and measure the peak at Vout

- Vout = 257.9mV peak

The 2-port model is then



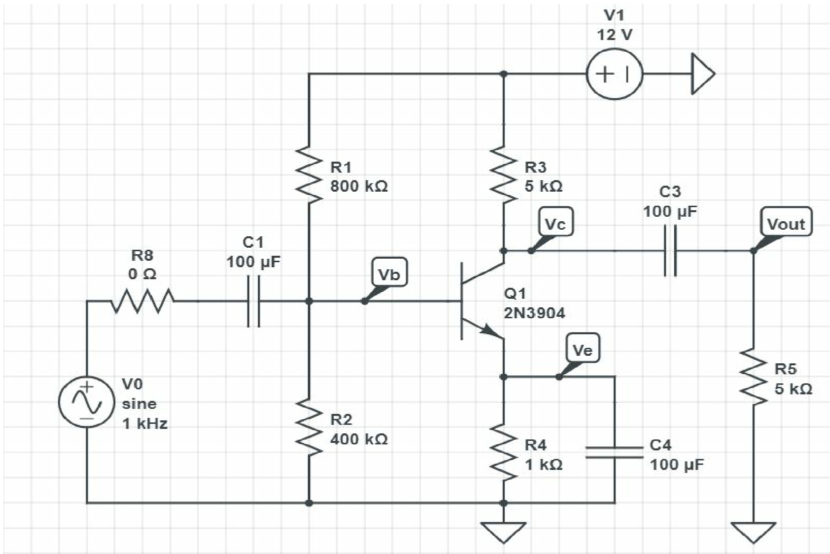
Ao = 257.9 (actually -257.9)

Rin: Set

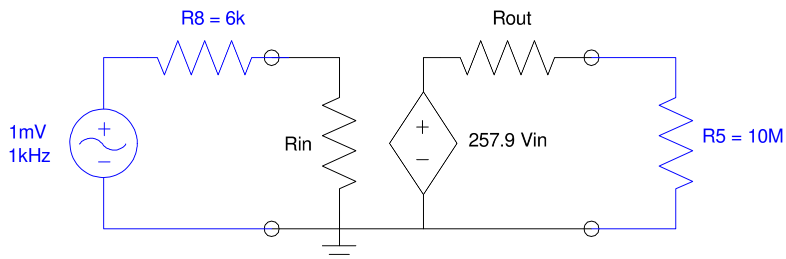
- R8 = 6000
- R5 = 10M

Measure Vout:

- Vout = 98.81mV



This tells you Rin. It's easier to see with the 2-port model:



$$V_{out} = 98.81mV = 257.9 \cdot \left(\frac{R_{in}}{R_{in}+6k}\right) \cdot 1mV$$

Solving for Rin

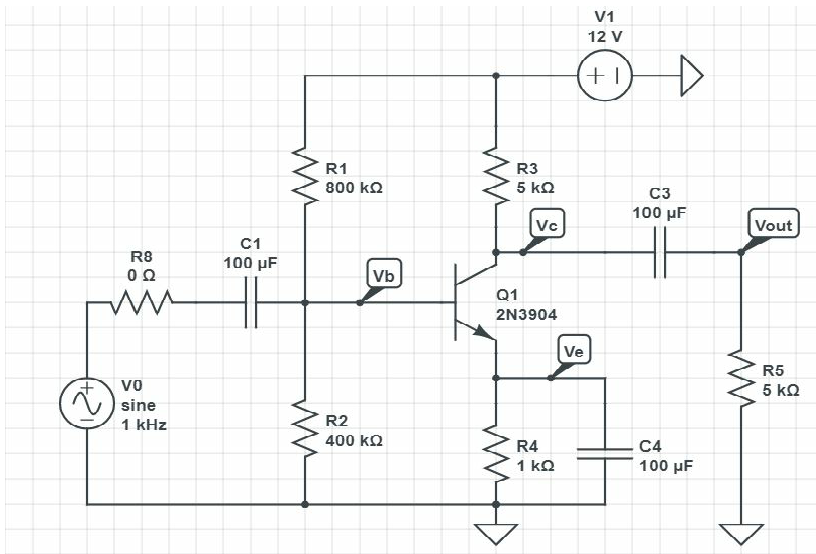
$$R_{in} = \left(\frac{98.81mV}{257.9mV - 98.81mV}\right) 6000\Omega = 3727\Omega$$

Rout: Set

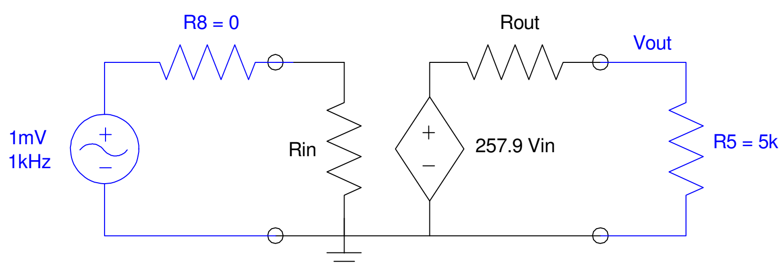
- R8 = 0
- R5 = 5000

Measure Vout:

- Vout = 134.6mV



This tells you Rout. The 2-port model is now:



Vout is

$$V_{out} = 134.6mV = \left(\frac{5k}{5k+R_{out}}\right) \cdot 257.9 \cdot 1mV$$

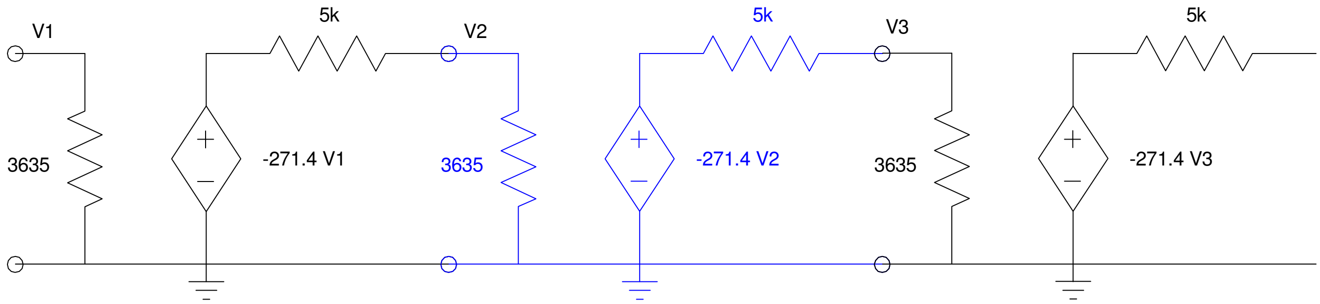
Solving for Rout:

$$R_{out} = \left(\frac{257.9mV - 134.6mV}{134.6mV}\right) 5000\Omega = 4580\Omega$$

	Rin	Ao	Rout
Calcualted	3,685	-271.4	5,000
Simulated	3,727	-257.9	4,580

8) Determine the 2-port model for cascading three of these CE amplifers (CE : CE : CE)

Use the 2-port models



By inspection

Rin = 3635

Ain = 0

Rout = 5000

Ao you need to work for. Set V1 = 1V

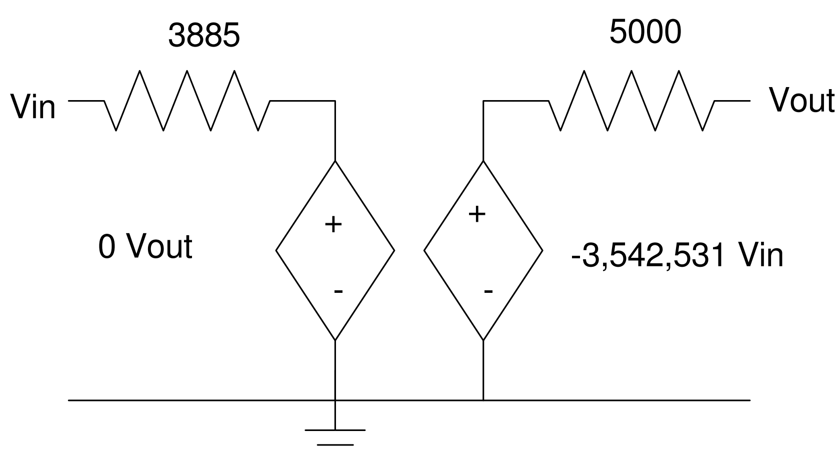
$$V_2 = \left(\frac{3635}{3635+5000}\right)(-271.4V) = -114.2V$$

$$-271.4 \cdot V_2 = 31.007kV$$

$$V_3 = \left(\frac{3635}{3635+5000}\right) \cdot (31,007V) = 13,052V$$

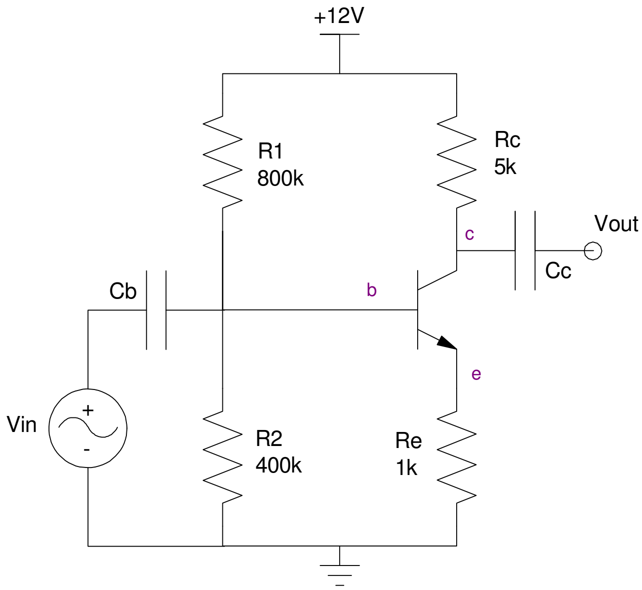
$$V_{out} = -271.4V_3 = -3.542MV$$

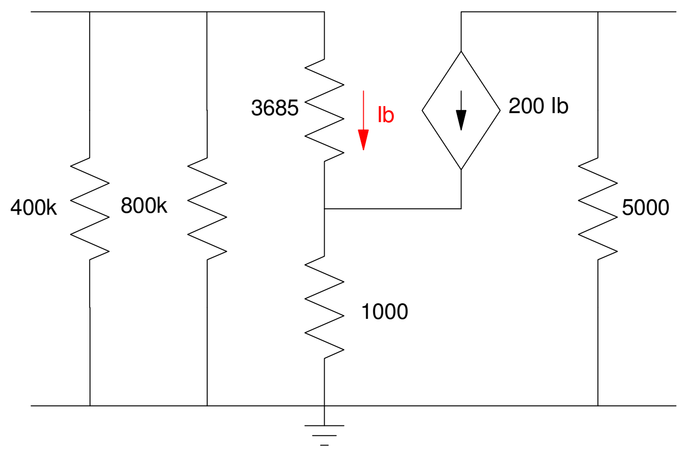
resulting in the 2-port model being



9) Remove Ce. Determine the 2-port model of this CE amplifiers

First draw the small-signal model





Now find the 2-port parameters. Note that the 1000 Ohm resistor looks like a 201k resistor (Ib + 200Ib current flows through it)

$$R_{in} = 400k||800k||(3685 + 201k)$$

$$R_{in} = 115.8k$$

$$A_{in} = 0$$

$$R_{out} = 5k$$

$$A_{out} = -\left(\frac{200 \cdot 5000}{3685+201k}\right) = -4.886$$