ECE 321 - Homework #3

1) Give the transfer function for a low-pass filter which meets the followníg design requirements:

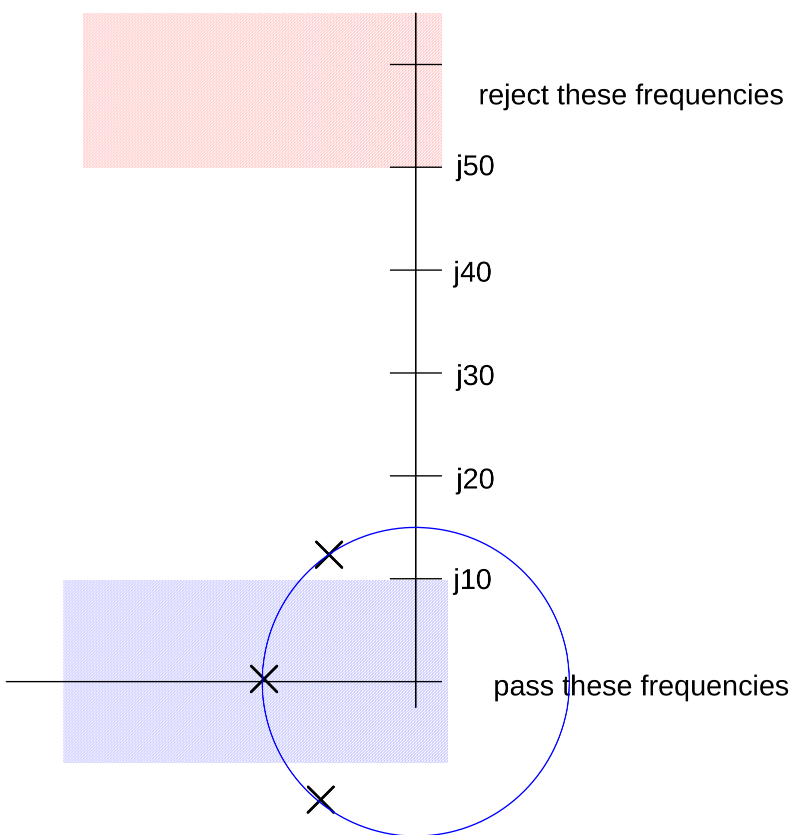
- Gain > 0.9 $\quad$ 0 < w < 10 rad/sec
- Gain < 0.1 $\quad$ w > 50 rad/sec

There are many solutions.

Guess a 3nd-order low pass (because it isn't that hard to built: only needs one op-amp).

Pick a corner frequency between 10 and 50. Let the corner be 15 rad/sec.

Guess a Butterworth filter because they work well and are easy to compute.



After some trial and error with the corner, one solution is

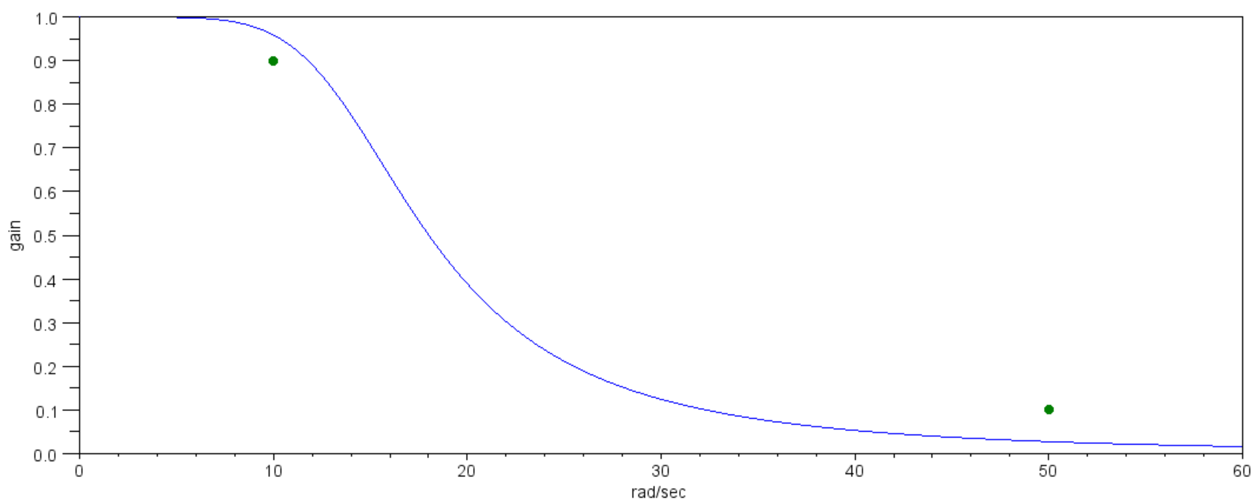
$$G = \left(\frac{15^3}{(s+15)(s+15\angle 60^0)(s+15\angle -60^0)}\right)$$

Checking the gain at 10 and 50 rad/sec:

	Gain	Requirement	Requirement Met?
10 rad/sec	0.9587	> 0.9	yes
50 rad/sec	0.0269	< 0.1	yes

2) Plot the gain vs. frequency for your filter (actually, this is what I did first)

```
-->p1 = -15;
-->p2 = -15*exp(j*%pi/3)
-->p3 = -15*exp(-j*%pi/3)

-->G = 15^3 ./ ( (s-p1).*(s-p2).*(s-p3) );

-->plot(w,abs(G),[10,50],[0.9,0.1],'.');
-->xlabel('rad/sec');
-->ylabel('gain');
```



$$G = \left(\frac{15^3}{(s+15)(s+15\angle 60^0)(s+15\angle -60^0)} \right)$$

Note

- This filter meets the design requirements
- There is room for slop. You can adjust the corner (shift the curve left and right) and still meet the design requirements.

3) Design an op-amp circuit to implement the filter you chose.

The following circuit has the transfer function

$$Y = \left(\frac{k\left(\frac{1}{RC}\right)^3}{\left(s+\frac{1}{RC}\right)\left(s^2+\left(\frac{3-k}{RC}\right)s+\left(\frac{1}{RC}\right)^2\right)} \right) X \qquad k = 1 + \frac{R_2}{R_1}$$

This should be

$$\left(\frac{15^3}{(s+15)(s+15\angle 60^0)(s+15\angle -60^0)} \right) = \left(\frac{15^3}{(s+15)(s^2+15s+225)} \right)$$

Matching terms

$$\left(\frac{1}{RC}\right) = 15$$

R = 1M, C = 1/15 uF

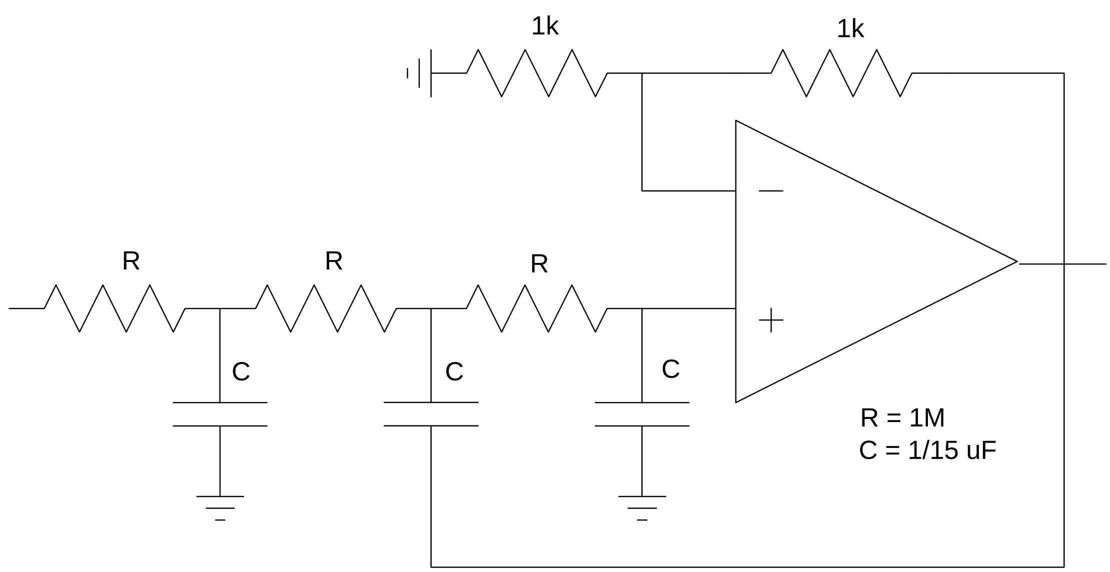
$$\left(\frac{3-k}{RC}\right) = 15$$

$$3 - k = 1$$

$$k = 2$$

The actual gain is twice what it should be. Normally you don't worry about this: you usually need to amplify the signal. The filter provides a gain of two. The rest comes from the rest of the circuit.

$$Y = \left(\frac{2 \cdot 15^3}{(s+15)(s+15\angle 60^0)(s+15\angle -60^0)} \right) X$$



4) Give the transfer function for a low-pass filter which meets the following design criteria:

- Gain > 0.9 w = 10 rad/sec
- Gain < 0.1 w < 5 rad/sec or w > 15 rad/sec

Let's make the bandwidth 10 rad/sec +/- 1 rad/sec

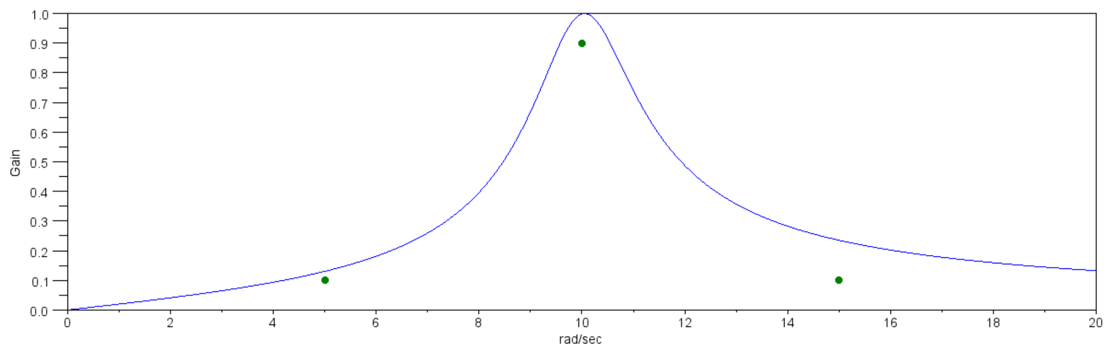
- The poles are centered at j10
- They are one away from j10

```
-->w = [0:0.01:20]';
-->j = sqrt(-1);
-->s = j*w;
```

```
-->p1 = 1 + j*10;
-->p2 = conj(p1);
-->G = s ./ ( (s+p1) .* (s+p2) );
```

The peak gain is off, so adjust G(s) so that the max is one:

```
-->G = G / max(abs(G));

-->plot(w,abs(G),[5,10,15],[0.1,0.9,0.1],'.');
```



The gain is too high at 5 and 15, so slide the pole in (reduce the real part from -1 to -0.4) to make it more selective. After some trial and error,

```
-->p1 = 0.4 + j*10;
-->p2 = conj(p1);
-->G = s ./ ( (s+p1) .* (s+p2) );
-->G = G / max(abs(G));

-->plot(w,abs(G),[5,10,15],[0.1,0.9,0.1],'.');
-->xlabel('rad/sec');
-->ylabel('Gain');
```

This works, so

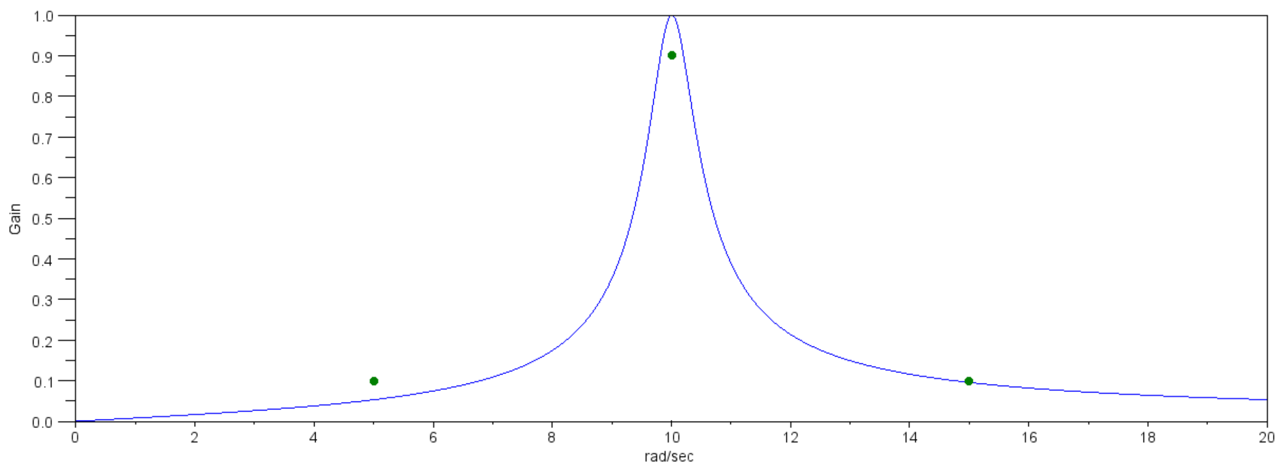
$$Y = \left(\frac{0.8s}{(s+0.4+j10)(s+0.4-j10)}\right)X$$

Checking the gain at 5, 10, and 15 rad/sec

```
-->s = j*5;
-->abs( 0.8*s ./ ( (s+p1) .* (s+p2) ) )

    0.0531446

-->s = j*10;
-->abs( 0.8*s ./ ( (s+p1) .* (s+p2) ) )

    0.9998001

-->s = j*15;
-->abs( 0.8*s ./ ( (s+p1) .* (s+p2) ) )

    0.0956820
```

	Gain	Requirement	Requirement Met?
5 rad/sec	0.0531	< 0.1	yes
10 rad/sec	0.9998	> 0.9	yes
15 rad/sec	0.0956	< 0.1	yes

5) Plot the gain vs. Frequency



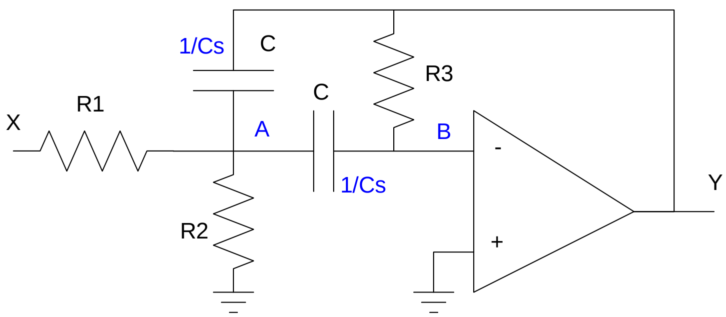
Gain vs. Frequency for Problem 5:
The gain is less than 0.1 at 5 and 15 rad/sec, more than 0.9 at 10 rad/sec

6) Design a filter to implement this transfer funciton

$$Y = \left(\frac{0.8s}{(s+0.4+j10)(s+0.4-j10)}\right)X$$

$$Y = \left(\frac{0.8s}{s^2+0.8s+100.16}\right)X$$

A filter of this form (zero at s = 0, two poles) can be implemented with the following circuit:



$$Y = \left(\frac{-\left(\frac{1}{R_1C}\right)s}{s^2+\left(\frac{2}{R_3C}\right)s+\left(\frac{R_1+R_2}{R_1R_2}\right)\left(\frac{1}{R_3C^2}\right)}\right)X$$

Matching terms:

Let C1 = C2 = 1uF

$\left(\frac{1}{R_1C}\right) = 0.8$ R1 = 1.25M

$\left(\frac{2}{R_3C}\right) = 0.8$ R3 = 2.5M

$\left(\frac{R_1+R_2}{R_1R_2}\right)\left(\frac{1}{R_3C^2}\right) = 100.16$ R2 = 4.06k