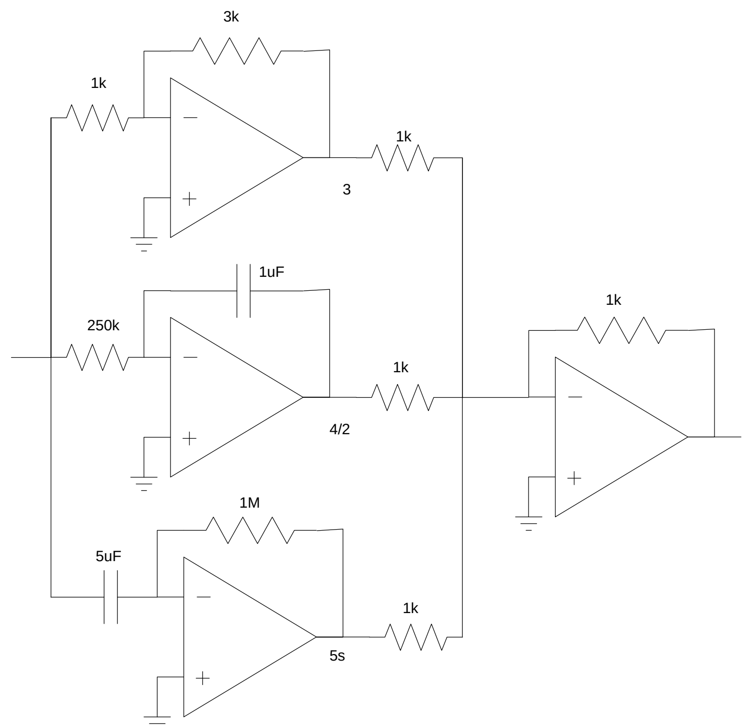
ECE 321 - Homework #2

Integrators, Differentiators, Filters and Phasors. Due Monday, April 11th

1) Design an op-amp circuit to implement a PID compensator:

$$Y = \left(3 + \frac{4}{s} + 5s\right)X$$



Problem 2-5) A filter has the following transfer function

$$Y = \left(\frac{100}{(s+8.5)\left(s+12.1\angle 69.5^0\right)\left(s+12.1\angle -69.5^0\right)} \right) X$$

2) What is the differential equation relating X and Y?

Multiply out the polynomial. In Matlab:

```
>> p1 = -8.5;
>> p2 = 12.1*exp(-j*69.5*pi/180);
>> p3 = conj(p2);
>> poly([p1, p2,p3])

    1    17    218.4    1244.5
```

Cross Multiply:

$$(s^3 + 17s^2 + 218.4s + 1244)Y = 100X$$

this means

$$\boxed{\frac{d^3y}{dt^3} + 17\frac{d^2y}{dt^2} + 218.4\frac{dy}{dt} + 1244y = 100x}$$

Another way to write this is

$$y''' + 17y'' + 218.4y' + 1244y = 100x$$

3) Determine y(t) assuming

$$x(t) = 2 + 3\cos(4t) + 5\cos(400t)$$

Use superposition

x(t) = 2

s = 0

$$\left(\frac{100}{(s+8.5)\left(s+12.1\angle 69.5^0\right)\left(s+12.1\angle -69.5^0\right)} \right)_{s=0} = 0.0804$$

Output = Gain * Input

$$y = (0.0804) * (2)$$

$$y = 0.1608$$

x(t) = 3cos(4t)

s = j4

$$\left(\frac{100}{(s+8.5)\left(s+12.1\angle 69.5^0\right)\left(s+12.1\angle -69.5^0\right)} \right)_{s=j4} = 0.079\angle -39^0$$

Output = Gain * Input

$$y = (0.079\angle -39^0) \cdot 3\cos(4t)$$

$$y(t) = 0.237\cos(4t - 39^0)$$

x(t) = 5cos(400t)

s = j400

$$\left(\frac{100}{(s+8.5)\left(s+12.1\angle 69.5^0\right)\left(s+12.1\angle -69.5^0\right)} \right)_{s=j400} = 0.00000156\angle 92^0$$

Output = Gain * Input

$$y = (0.00000156\angle 92^0) \cdot 5\cos(400t)$$

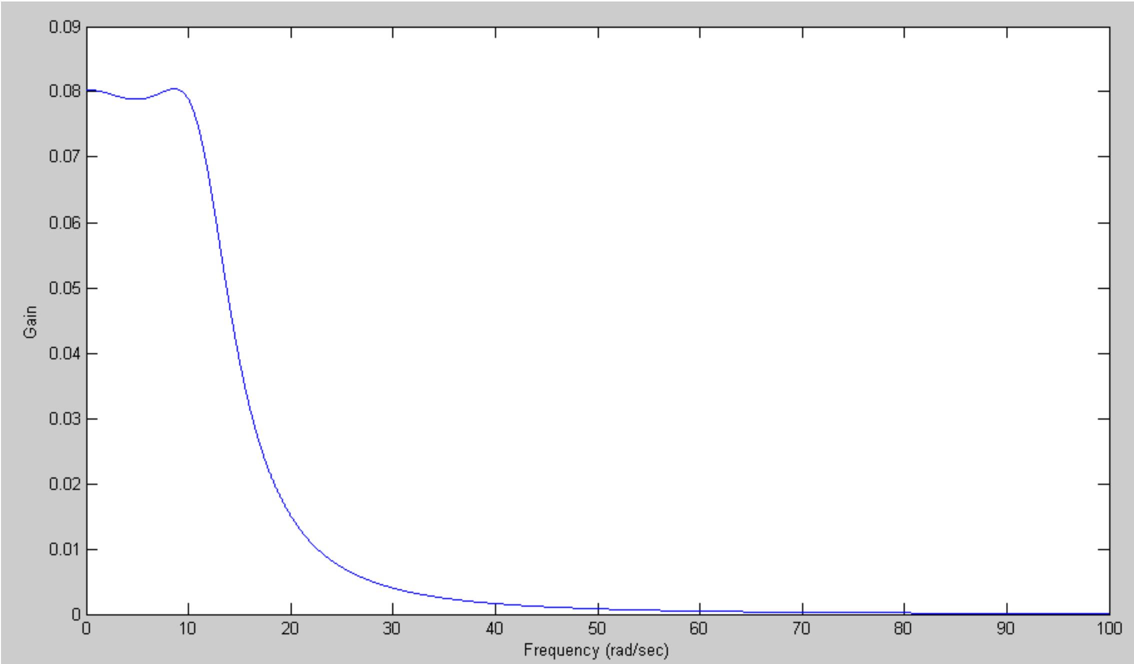
$$y(t) = 0.0000078\cos(400t + 92^0)$$

Add up all three terms to get x(t). Add up all three terms to get y(t)

$$\boxed{y = 0.1608 + 0.237\cos(4t - 39^0) + 0.0000078\cos(400t + 92^0)}$$

4) Plot the gain of this filter from 0 to 100 rad/sec

```
>> w = [0:0.01:100]';
>> s = j*w;

>> p1 = -8.5;
>> p2 = -12.1*exp(-j*69.5*pi/180);
>> p3 = conj(p2);
>> G = 100 ./  ( (s-p1).*(s-p2).*(s-p3) );

>> plot(w,abs(G))
>> xlabel('Frequency (rad/sec)');
>> ylabel('Gain');
```

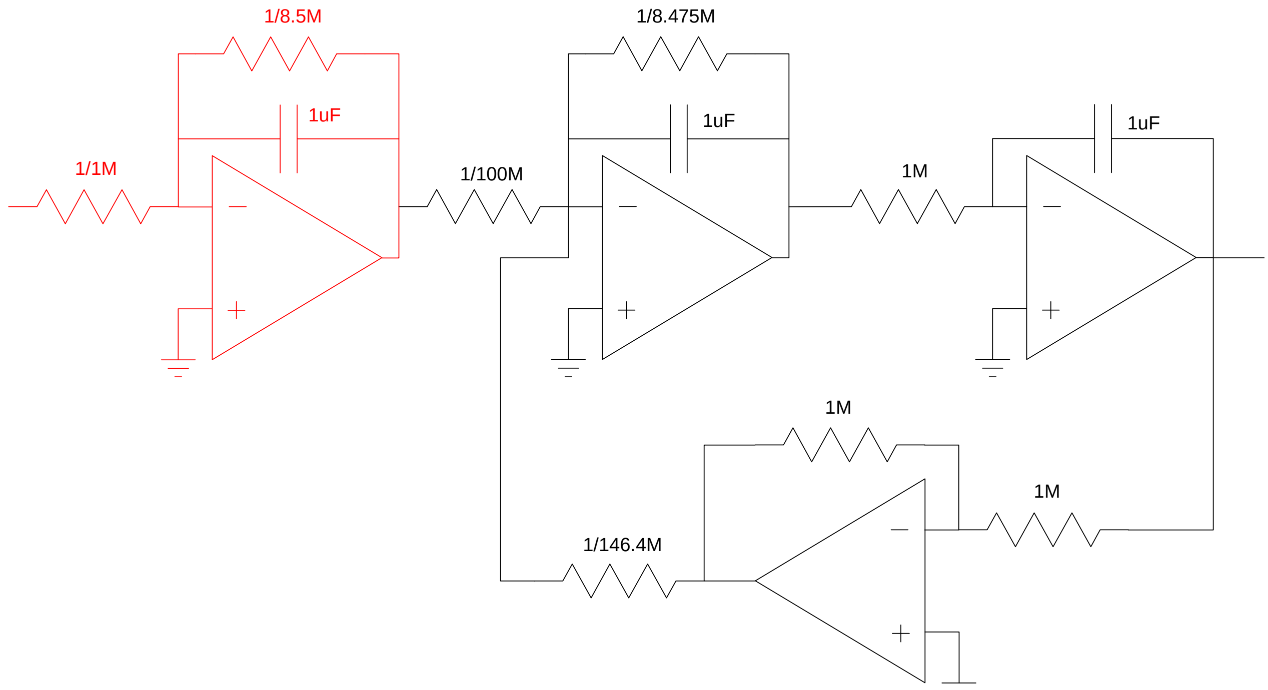


5) Design a circuit to implement this filter

Separate this into two parts: a 1st-order filter and a 2nd-order filter

$$\left(\frac{1}{s+8.5}\right)\left(\frac{100}{s^2+8.475s+146.4}\right)$$

Build each stage separately and cascade them:



6a) Design a filter which

- Has a gain between 0.9 and 1.1 at 10 rad/sec
- Has a gain less than 0.1 at 1 rad/sec, and
- Has a gain less than 0.1 at 100 rad/sec

Place the pole at j10 rad/sec to pass 10 rad/sec

Adjust the real part so that the gain at 1 and 100 rad/sec is low enough.

Guess #1: Make the real part '1' so that the filter passes frequencies at 10 rad/sec +/- 1 rad/sec

$$G(s) = \left(\frac{ks}{(s+1+j10)(s+1-j10)}\right)$$

Pick 'k' so that the gain ag j10 is one

```
>> G = zpk(0,[-1+j*10,-1-j*10],1);
>> evalfr(G,j*10)

   0.4988 + 0.0249i
```

Add a gain of 2 so that the gain at 10 is close to 1:

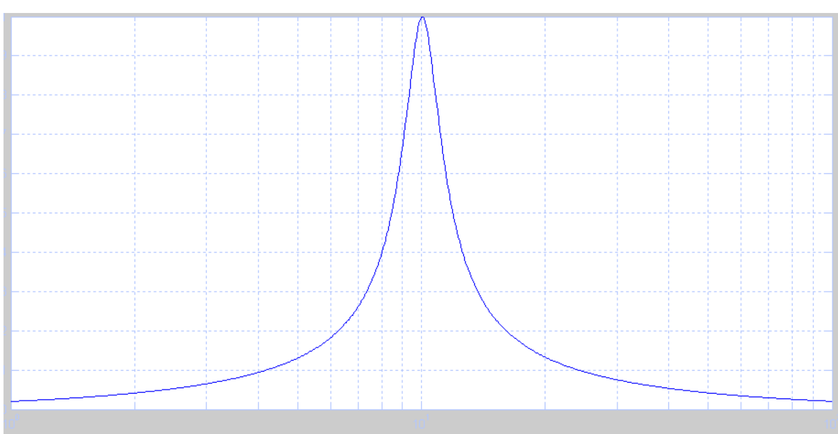
```
>> G = zpk(0,[-1+j*10,-1-j*10],2);
>> evalfr(G,j*10)

   0.9975 + 0.0499i
```

Now check the gain at 1 and 100 rad/sec

```
>> evalfr(G,j*1)

   0.0004 + 0.0200i

>> evalfr(G,j*100)

   0.0004 - 0.0202i
```

6b) Verify your filter in Matlab

Done. The plot from 1 to 100 rad/sec is



Lab:

7) Design a push-pull amplifier to drive an 8-Ohm speaker

- Input: -5V to +5V, capable of 10mA, 0 - 10kHz
- Output: 8 Ohm speaker
- Relationship: Vo = Vin, +/- 500mV

8) Build a circuit to implement this push-pull amplifier. Verify its operation at

- Vin = -4V
- Vin = -2V
- Vin = +2V
- Vin = +4V