ECE 321 - Homework #2

Filters. Due Monday April 3, 2017

Problem 1) DC Block: For the following filter:

$$Y = \left(\frac{2s}{s+6}\right)X$$

a) Determine the differential equation relating X and Y

Cross multiply

$$(s+6)Y = (2s)X$$

'sY' means 'the derivartive of Y

$$\frac{dy}{dt} + 6y = 2\frac{dx}{dt}$$

or using dot notation

$$\dot{y} + 6y = 2\dot{x}$$

b) Determine y(t) assuming

x(t) = 2 + 3 sin(10t) + 4 sin(100t)

Use superposition

x(t) = 2	x(t) = 3 sin(10t)	x(t) = 4 sin(100t)
s = 0	s = j10	s = j100
$\left(\frac{2s}{s+6}\right)_{s=0} = 0$	$\left(\frac{2s}{s+6}\right)_{s=j10} = 1.715\angle 31^0$	$\left(\frac{2s}{s+6}\right)_{s=j100} = 1.996\angle 3^0$
$y = (0)\cdot 2$	$(1.715\angle 31^0)\cdot 3\sin(10t)$	$y = (1.996\angle 3^0)\cdot 4\sin(100t)$
$y(t) = 0$	$y(t) = 5.145\sin(10t - 31^0)$	$y(t) = 7.98\sin(100t + 3^0))$
block	pass	pass

This is a DC block

Add up the three inputs to get x(t). Add up the three outputs to get y(t)

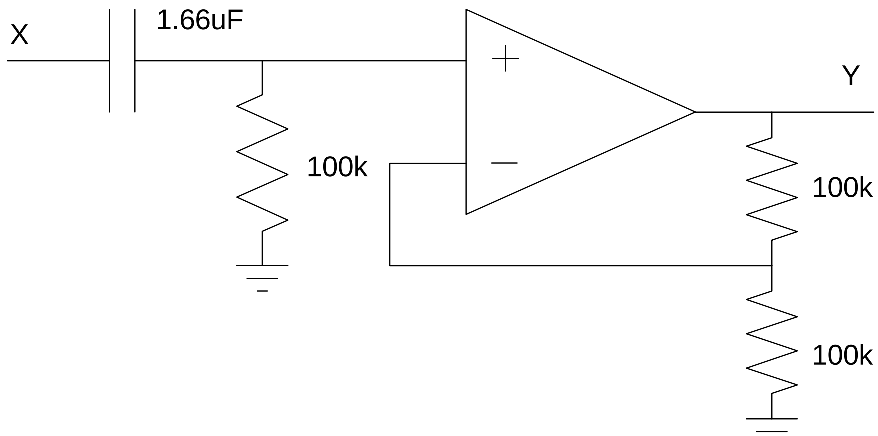
$$y(t) = 0 + 5.145\sin(10t - 31^0) + 7.98\sin(100t + 3^0)$$

c) Design an op-amp circuit to implement this filter.

Gain = 2.   Use two 100k resistors.

1/RC = 6. Let

- R = 100k
- C = 1.667uF



One possible implementation of a DC Block

Problem 2) Low Pass Filter with Real Poles: For the following filter:

$$Y = \left(\frac{400}{(s+10)(s+20)}\right) X$$

a) Determine the differential equation relating X and Y

Multiply out and cross multiply

$$(s^2 + 30s + 200)Y = 400X$$

which means

$$\frac{d^2y}{dt^2} + 30\frac{dy}{dt} + 200y = 400x$$

or

$$\ddot{y} + 30\dot{y} + 200y = 400x$$

b) Determine y(t) assuming

x(t) = 2 + 3 sin(10t) + 4 sin(100t)

Use superposition

x(t) = 2	x(t) = 3 sin(10t)	x(t) = 4 sin(100t)
s = 0	s = j10	s = j100
$\left(\frac{400}{(s+10)(s+20)}\right)_{s=0} = 2$	$\left(\frac{400}{(s+10)(s+20)}\right)_{s=j10} = 1.26\angle -71.6^0$	$\left(\frac{400}{(s+10)(s+20)}\right)_{s=j100} = 0.039\angle -163^0$
$y = (2) \cdot 2$	$y = (1.26\angle -71.6^0) \cdot 3\sin(10t)$	$y = (0.039\angle -163^0) \cdot 4\sin(100t)$
$y(t) = 4$	$y(t) = 3.79\sin(10t - 71.6^0)$	$y(t) = 0.156\sin(100t - 163^0)$
pass	mostly pass	block

This is a low-pass filter.

Add up the three outputs to get y(t)

$$y(t) = 4 + 3.79\sin(10t - 71.6^0) + 0.156\sin(100t - 163^0)$$

c) Design an op-amp circuit to implement this filter.

Use two RC filters with

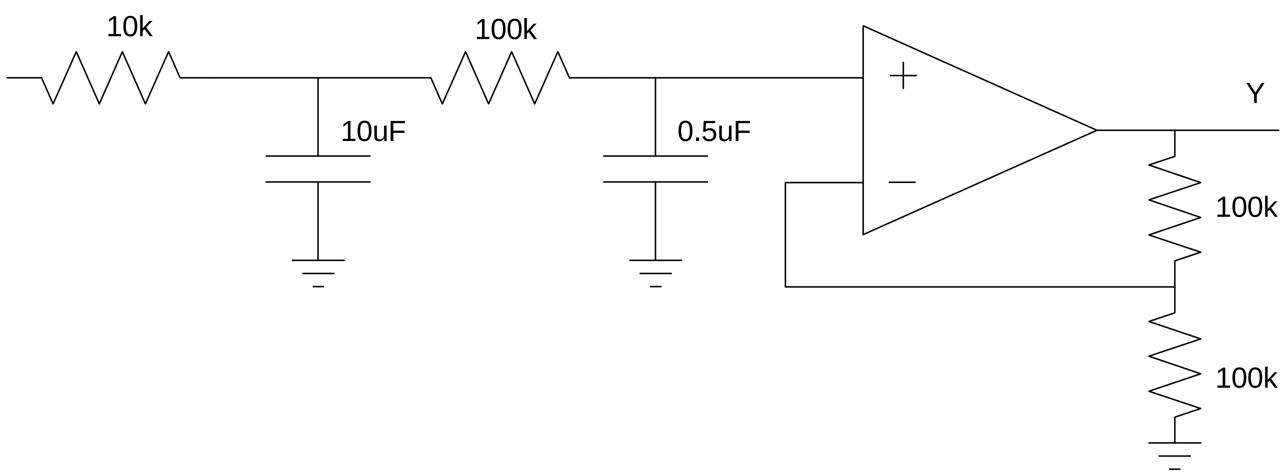
1/RC = 10

- R = 10k
- C = 10uF

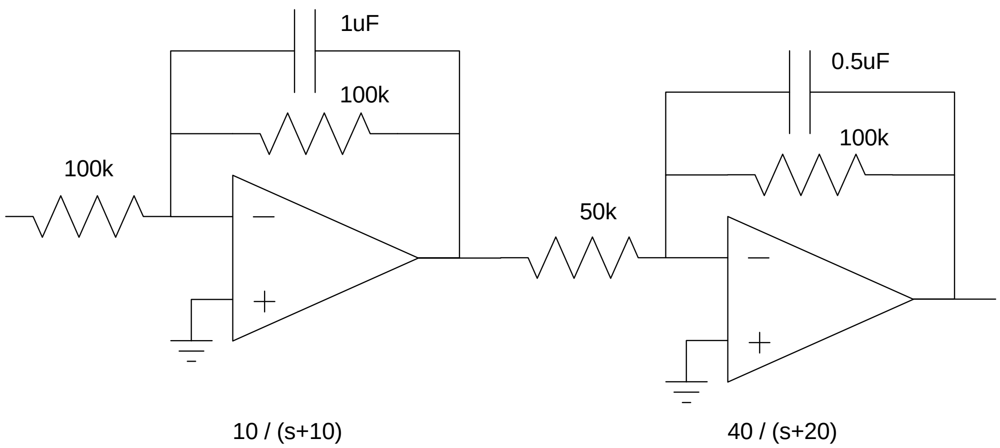
1/RC = 20

- R = 100k
- C = 0.5uF

Add a gain of two so that the DC gain is two



One implementation of a low-pass filter with two real poles



Another implementation of a low-pass filter with two real poles

Problem 3) Low Pass Filter with Complex Poles: For the following filter:

$$Y = \left(\frac{225}{s^2+15s+225}\right)X$$

a) Determine the differential equation relating X and Y

$$(s^2 + 15s + 225)Y = 225X$$

$$\frac{d^2y}{dt^2} + 15\frac{dy}{dt} + 225y = 225x$$

$$\boxed{\ddot{y} + 15\dot{y} + 225y = 225x}$$

b) Determine y(t) assuming

x(t) = 2 + 3 sin(10t) + 4 sin(100t)

x(t) = 2	x(t) = 3 sin(10t)	x(t) = 4 sin(100t)
s = 0	s = j10	s = j100
$\left(\frac{225}{s^2+15s+225}\right)_{s=0} = 1$	$\left(\frac{225}{s^2+15s+225}\right)_{s=j10} = 1.34\angle -116^0$	$\left(\frac{225}{s^2+15s+225}\right)_{s=j100} = 0.022\angle -171^0$
$y = (1) \cdot 2$	$y = (1.34\angle -116^0) \cdot 3\sin(10t)$	$y = (0.022\angle -171^0) \cdot 4\sin(100t)$
$y(t) = 2$	$y(t) = 4.02\sin(10t - 116^0)$	$y(t) = 0.089\sin(100t - 171^0)$
pass	pass	block

This is a better low-pass filter. The gain at 10 is closer to the DC gain (1.00)

$$\boxed{y(t) = 2 + 4.02\sin(10t - 116^0) + 0.089\sin(100t - 171^0)}$$

c) Design an op-amp circuit to implement this filter.

$$\left(\frac{1}{RC}\right)^2 = 225$$

$$\left(\frac{1}{RC}\right) = 15$$

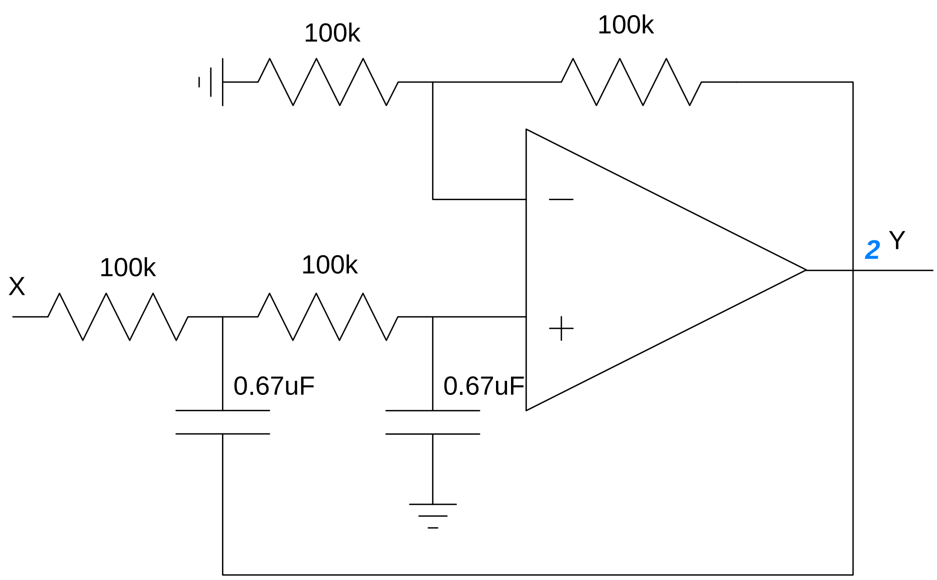
Let R = 100k, C = 0.67uF

$$\left(\frac{3-k}{RC}\right) = 15$$

$$3 - k = 1$$

$$k = 2$$

Let R1 = R2 = 100k



Problem 4) Band Pass Filter: For the following filter:

$$Y = \left(\frac{2s}{s^2+2s+10}\right)X$$

a) Determine the differential equation relating X and Y

$$(s^2 + 2s + 10)Y = (2s)X$$

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 10y = 2\frac{dx}{dt}$$

$$\ddot{y} + 2\dot{y} + 10y = 2\dot{x}$$

b) Determine y(t) assuming

x(t) = 2 + 3 sin(10t) + 4 sin(100t)

Use superposition:

x(t) = 2	x(t) = 3 sin(10t)	x(t) = 4 sin(100t)
s = 0	s = j10	s = j100
$\left(\frac{2s}{s^2+2s+10}\right)_{s=0} = 0$	$\left(\frac{2s}{s^2+2s+10}\right)_{s=j10} = 0.2169\angle -77^0$	$\left(\frac{2s}{s^2+2s+10}\right)_{s=j100} = 0.02\angle -88^0$
$y = (0) \cdot 2$	$y = (0.2169\angle -77^0) \cdot 3\sin(10t)$	$y = (0.02\angle -88^0) \cdot 4\sin(100t)$
$y(t) = 0$	$y = 0.6508\sin(10t - 77^0)$	$y(t) = 0.08\sin(100t - 88^0)$
block	pass	block

This is a band-pass filter. Frequencies close to 3.16 rad/sec get passed, those far away get blocked.

Add up all the outputs to get y(t)

$$y(t) = 0 + 0.6508\sin(10t - 77^0) + 0.08\sin(100t - 88^0)$$

c) Design an op-amp circuit to implement this filter.

Use the band-pass filter:

$$\left(\frac{2s}{s^2+2s+10}\right) = \left(\frac{\left(\frac{1}{R_1C}\right)s}{s^2+\left(\frac{2}{R_3C}\right)s+\left(\frac{R_1+R_2}{R_1R_2}\right)\left(\frac{1}{R_3C^2}\right)}\right)$$

Matching terms:

$$\left(\frac{1}{R_1C}\right) = 2$$

Let C = 1uF

$$R_1 = 500k$$

$$\left(\frac{2}{R_3C}\right) = 2$$

$$R_3 = 500k$$

$$\left(\frac{R_1+R_2}{R_1R_2}\right)\left(\frac{1}{R_3C^2}\right) = 10$$

$$R_2 = 300k$$