## ECE 321 - Homework \#3

Filters \& 2-Port Models. Due Monday, April 24th, 2017

1) Determine the Q-point for the following circuit (shown on the left). Assume $\beta=100$


First, redraw the circuit using the Thevening equivalent of the 50 k and 10 k resistor (shown in brown).

$$
\begin{aligned}
& V_{t h}=V_{b b}=\left(\frac{10 k}{10 k+50 \mathrm{k}}\right) 12 \mathrm{~V}=2.00 \mathrm{~V} \\
& R_{t h}=R_{b}=10 \mathrm{k}| | 50 \mathrm{k}=8.33 \mathrm{k}
\end{aligned}
$$

Next, find Ib:

$$
\begin{aligned}
& -10 V+8.33 k \cdot I_{b}+0.7 V+1 k\left(I_{b}+100 I_{b}\right)=0 \\
& I_{b}=\left(\frac{2 V-0.7 V}{8.33 k+(101) 1 k}\right)=11.89 \mu A
\end{aligned}
$$

Ic is 100Ib

$$
I_{c}=100 I_{b}=1.189 \mathrm{~mA}
$$

This gives

$$
\begin{aligned}
& V_{e}=1 k\left(I_{b}+I_{c}\right)=1.20 \mathrm{~V} \\
& V_{c}=12-4 k \cdot I_{c}=7.24 \mathrm{~V} \\
& V_{c e}=V_{c}-V_{e}=6.043 \mathrm{~V}
\end{aligned}
$$

2) Find R1, R2, and Re so that

- The Q-point is stabilized for variations in $\beta$, and
- Vce $=6 \mathrm{~V}$

Well, what are the odds that random values of resistors would result in a Q-point that is 6.00 V ?

Step 1: Find Rb. To stabilize the Q-point

$$
\begin{aligned}
& (1+\beta) R_{e} \gg R_{b} \\
& 101 k \gg R_{b}
\end{aligned}
$$

Let

$$
R_{b}=10 k
$$

Step 2: Find Vb.
For Vce $=6 \mathrm{~V}$,

$$
\begin{aligned}
& I_{c}=\frac{12 V-6 V}{4 k+(1.01) 1 k}=1.198 \mathrm{~mA} \\
& I_{b}=\frac{I_{c}}{\beta}=11.98 \mu A \\
& V_{b}=I_{b} R_{b}+0.7+\left(I_{b}+I_{c}\right) R_{e} \\
& V_{b}=2.029 V
\end{aligned}
$$

Step 3: Find R1 and R2

$$
\begin{aligned}
& \left(\frac{R_{2}}{R_{1}+R_{2}}\right) 12 \mathrm{~V}=2.029 \mathrm{~V} \\
& \left(\frac{R_{1} R_{2}}{R_{1}+R_{2}}\right)=10 \mathrm{k} \\
& R_{1}=\left(\frac{12 \mathrm{~V}}{2.029 \mathrm{~V}}\right) 10 \mathrm{k} \\
& R_{1}=59.14 \mathrm{k} \Omega \\
& R_{2}=12.03 \mathrm{k} \Omega
\end{aligned}
$$



3) Requirements:

- Input: $+/-10 \mathrm{~V}$ analog signal, 100 Hz to 1 kHz , capable of 20 mA (i.e. a LM833 op-amp)
- Output: $+/-10 \mathrm{~V}$ analog signal capable of 20 mA (i.e. another LM833 op-amp)
- Relationship: Specify how the input and output relate. For a low-pass filter, for example, this might be - $0.9<$ Gain < 1.1 for frequencies below 250 Hz
- Gain < 0.2 for frequencies above 600 Hz

Let's go with this set of requirements.
4) Matlab Analysis:

- Give the transfer function for a filter which meets your requirements
- Plot the gain vs. frequency for your filter along with the requirements.
i) The number of poles you need is
$\left(\frac{250 \mathrm{~Hz}}{600 \mathrm{~Hz}}\right)^{n}<\left(\frac{0.2}{1.0}\right)$
$n>1.838$
Let $\mathrm{n}=3$

Assume a 3rd order Chebychev filter with a corner at 250 Hz . For a corner at 1 Hz ,

$$
G(s)=\left(\frac{1}{(s+0.85)\left(s+1.21 \angle \pm 69.5^{0}\right.}\right)
$$

Change this to $250 \mathrm{~Hz}(1570 \mathrm{rad} / \mathrm{sec})$

- $0.85 \Rightarrow 0.85 \cdot 1570=1334$
- $1.21 \Rightarrow 1.21 \cdot 1570=1899$
so

$$
G(s)=\left(\frac{(1334)(1899)(1899)}{(s+1334)\left(s+1899 \angle \pm 69.5^{0}\right.}\right)
$$

In Matlab, check if this meets the requiremetns:
Input the poles, scaled to 250 Hz ( $1570 \mathrm{rad} / \mathrm{sec}$ )

```
-->p1 = 0.85*1570;
-->p2 = 1.21*exp(j*69.5*%pi/180) * 1570;
-->p3 = conj(p2);
```

Plot the gain vs. frequency, scaled so that the DC gain is one:

```
\(-->f=[0: 0.1: 1000]^{\prime} ;\)
-->w = 2*\%pi*f;
-->s = j*W;
\(-->G=1570 \wedge 3\)./ ( \((\mathrm{s}+\mathrm{p} 1) . *(\mathrm{~s}+\mathrm{p} 2) . *(\mathrm{~s}+\mathrm{p} 3))\);
\(-->G=G / G(1) ;\)
-->plot(f,abs(G), [250, 600],[0.9, 0.2],'.');
-->xlabel('Frequency (Hz)');
-->ylabel('Gain');
```



Well, that's pretty sweet: this meets the specs right off the bat. If it didn't, adjust the corner left and right so that the gain is more than 0.9 at 250 Hz and less than 0.2 at 600 Hz .

Net:

$$
G(s)=\left(\frac{1334 \cdot 1899^{2}}{(s+1334)\left(s+1899 \angle 69.5^{0}\right)\left(s+1899 \angle-69.5^{0}\right)}\right)
$$

5) Analysis: R and C Calculations. Give the schematic along with calculations for R and C for a circuit to implement the transfer function you found in problem \#3.

For the circuit below

$$
\left(\frac{1}{R C}\right)=1344
$$

Let $\mathrm{R}=10 \mathrm{k}$

$$
\mathrm{C}=74.4 \mathrm{nF}
$$

$$
\left(\frac{1}{R C}\right)=1899
$$

Let $\mathrm{R}=100 \mathrm{k}$

$$
C=5.3 n F
$$

$$
(3-k)=2 \cos \left(69.5^{0}\right)
$$

$$
k=2.2996
$$

Let R1 $=100 \mathrm{k}$
R2 = 129.96k


If you want the gain to be one, change the 10 k resistor so that

- The Thevenin resistance is 10 k
- The Thevenin voltage is X / 2.296

$$
\begin{aligned}
\mathrm{Rb} & =22.96 \mathrm{k} \\
\mathrm{Ra} & =17.716 \mathrm{k}
\end{aligned}
$$


6) Test: Simulate your circuit in PartSim. Check the simulated gain at

- The two endpoints
- The corners, and
- A few other points


| Frequency (Hz) | Gain (Matlab) | Gain (PartSim) | Gain (Lab) |
| :---: | :---: | :---: | :---: |
| 100 Hz | 1.000 | 0.9814 | - |
| 250 Hz | 0.998 | 0.9768 | - |
| 400 Hz | 0.400 | 0.3670 | - |
| 600 Hz | 0.104 | 0.0968 | - |
| 1000 Hz | 0.021 | 0.0199 | - |

7) Validation (Lab): Build your filter. Measure the gain at several frequencies to see if it agrees with your analysis
