## ECE 321 - Solutions to the Final

## Spring 2018

1) Determine the voltages and currents for the following push (pull) amplifier. Assume for the transistor

- $\beta=1000$
- $V_{b e}=1.4 \mathrm{~V}$

Assume for the LED

- $\mathrm{Vf}=3.0 \mathrm{~V} @ 300 \mathrm{~mA}$ :

| V 1 | 12 | 13 | V 4 |
| :---: | :---: | :---: | :---: |
| 7.4 V | 124.9 uA | 120 mA | 6 V |


$\mathrm{V}+$ equals $\mathrm{V}-$. This means V 4 must be 6 V for V - to be +2 V
$\mathrm{V} 1=\mathrm{V} 4+1.4 \mathrm{~V} \quad(\mathrm{Vbe})=7.4 \mathrm{~V}$
$I_{3}=\left(\frac{6 V-3.0 \mathrm{~V}}{25 \Omega}\right)=120 \mathrm{~mA}$
$I_{e}=I_{3}+\frac{6 V}{3 k}=125 \mathrm{~mA}$
$I_{b}=\frac{I_{e}}{1+\beta}=\frac{125 m \mathrm{~A}}{1001}=124.9 \mu \mathrm{~A}$
2) Determine the voltages and currents for the following push (pull) amplifier. Assume for the transistor

- $\beta=1000$
- $V_{b e}=1.4 \mathrm{~V}$

Assume for the LED

- $\mathrm{Vf}=3.0 \mathrm{~V} @ 300 \mathrm{~mA}$ :

| $\mathrm{v}_{1}$ | ${ }^{12}$ | $\mathrm{~V}_{3}$ | 14 |
| :---: | :---: | :---: | :---: |
| 6.4 V | 99.9 uA | 5.0 V | 100 mA |


$\mathrm{V}+=\mathrm{V}-=2 \mathrm{~V}$
$I_{4}=\frac{2 \mathrm{~V}}{20 \Omega}=100 \mathrm{~mA}$
$V_{3}=2 V+3 V=5 V$
$I_{2}=I_{b}=\frac{I_{e}}{1+\beta}=\frac{100 \mathrm{~mA}}{1001}=99.9 \mu \mathrm{~A}$
$V_{1}=V_{3}+V_{b e}=6.4 V$
3) Design an instrumentation amplifier so that the output is

- -10 V when $\mathrm{R}=1000$ Ohms
- +10 V when $\mathrm{R}=1400$ Ohms

For your circuit, what is the output voltage (Vout) when $\mathrm{R}=1100$ Ohms?

Vout when $\mathrm{R}=1100$ : Vout $=\mathbf{- 4 . 2 8 6 5 V}$


$$
\begin{aligned}
\mathrm{R}=1000 \text { Ohms } \quad(\mathrm{Vo}=-10 \mathrm{~V}) & \mathrm{R}=1400 \mathrm{Ohms} \quad(\mathrm{Vo}=+10 \mathrm{~V}) \\
V_{a}=\left(\frac{R}{R+1000}\right) 10 \mathrm{~V}=5.00 \mathrm{~V} & V_{a}=\left(\frac{R}{R+1000}\right) 10 \mathrm{~V}=5.833 \mathrm{~V}
\end{aligned}
$$

Gain:

$$
\text { gain }=\left(\frac{10 V-(-10 V)}{5.833 V-5.000 V}\right)=24.0
$$

The output goes up as the input get larger. Connect to the plus input

$$
\begin{aligned}
& Y=\operatorname{gain}\left(V_{p}-V_{m}\right) \\
& +10 V=24.0\left(5.8333 V-V_{m}\right) \\
& V_{m}=5.4167 V
\end{aligned}
$$

At 1100 Ohms

$$
V_{p} \approx\left(\frac{R}{R+1000}\right) \cdot 10 V=5.2381 V \quad Y=24(5.2183-5.4167)=-4.2865 V
$$

4) Give the transfer function for a low-pass filter which comes close to meeting the following requirements

- $0.9<$ Gain $<1.1$ frequencies less than $500 \mathrm{rad} / \mathrm{sec}$
- Gain $<0.2$ frequencies above $600 \mathrm{rad} / \mathrm{sec}$

You are free to choose any type of filter you like (Chebychev, Butterworth, Elliptic, etc.)

The number of poles you need are

$$
\begin{aligned}
& \left(\frac{500 \mathrm{~Hz}}{600 \mathrm{~Hz}}\right)^{N}=0.2 \\
& N=8.82
\end{aligned}
$$

Round up to $\mathrm{N}=9$

Assume a Butterworth filter. For 9 poles, the angle between poles is

$$
\theta=\left(\frac{180^{0}}{N}\right)=20^{0}
$$

Place the corder at $500 \mathrm{rad} / \mathrm{sec}$ (may need to adjust with Matlab)
Make the numerator whatever it takes so that the DC gain is 1.000

$$
G(s)=\left(\frac{500^{9}}{(s+500)\left(s+500 \angle \pm 20^{0}\right)\left(s+500 \angle \pm 40^{0}\right)\left(s+500 \angle \pm 60^{0}\right)\left(s+500 \angle \pm 80^{0}\right)}\right)
$$

5) A 3rd-order Butterworth low-pass filter has the following transfer function:

$$
Y=\left(\frac{250}{(s+5)\left(s^{2}+5 s+25\right)}\right) X=\left(\frac{250}{s^{3}+10 s^{2}+50 s+125}\right) X
$$

a) What is the differential equation relating X and Y ?

$$
\left(s^{3}+10 s^{2}+50 s+125\right) Y=250 X
$$

'sY' means 'the derivative of Y

$$
y^{\prime \prime \prime}+10 y^{\prime \prime}+50 y^{\prime}+125 y=250 x
$$

b) Determine $y(t)$ assuming

$$
x(t)=3 \sin (7 t)+5 \cos (8 t)
$$

Use super position

$$
\begin{array}{ll}
\mathrm{x}(\mathrm{t})=3 \sin (7 \mathrm{t}) & \mathrm{x}(\mathrm{t})=5 \cos (8 \mathrm{t}) \\
X=0-j 3 & X=5+j 0 \\
s=j 7 & x=j 8 \\
\left(\frac{250}{s^{3}+10 s^{2}+50 s+125}\right)_{s=j 7}=1.8492 \angle-3.30^{0} & (-)_{s=j 8}=1.1849 \angle 32.03^{0} \\
Y=\left(1.8492 \angle-3.30^{0}\right)(0-j 3) & Y=\left(1.1849 \angle 32.03^{0}\right)(5+j 0) \\
Y=5.5476 \angle-93.30^{0} & Y=5.9199 \angle 32.03^{0} \\
y(t)=5.5476 \cos \left(7 t-93.30^{0}\right) & y(t)=5.9199 \cos \left(8 t+32.03^{0}\right)
\end{array}
$$

The total answer is the sum of the two parts:

$$
y(t)=5.5476 \cos \left(7 t-93.30^{0}\right)+5.9199 \cos \left(8 t+32.03^{0}\right)
$$

6) Find the Thevenin equivalent of R1, R2 (Vb, Rb), and the Q-point (Ic, Vce) for the following transistor circuit. Assume a 3904 transistor:

- $\beta=200$
- $V_{b e}=0.7 \mathrm{~V}$

| Vb | Rb | lo | Vce |
| :---: | :---: | :---: | :---: |
| 3.5294 V | 17,657 | 688.7 uA | 5.788 V |


$R_{b}=R_{1} \| R_{2}=17,647 \Omega$
$V_{b}=\left(\frac{R_{2}}{R_{1}+R_{2}}\right) 12 \mathrm{~V}=3.5294 \mathrm{~V}$
$I_{b}=\left(\frac{V_{b}-0.7 V}{R_{b}+(1+\beta) R_{e}}\right)=3.4436 \mu A$
$I_{c}=\beta I_{b}=688.7 \mu \mathrm{~A}$
$V_{e}=R_{e}\left(I_{b}+I_{c}\right)=2.769 \mathrm{~V}$
$V_{c}=12-5000 I_{c}=8.556 \mathrm{~V}$
$V_{c e}=V_{c}-V_{e}=5.788 \mathrm{~V}$

7) Find the 2-port model for the following circuit:

| Rin | Ai | Rout | Ao $^{\text {A }}$ |
| :---: | :---: | :---: | :---: |
| 667 Ohms | 0.667 | 186.3 Ohms | 0.02417 |



Rin: Short Vout. Apply 1V to Vin. Compute Rin.

$$
R_{\text {in }}=2 k \| 1 k=667 \Omega
$$

Ai: Apply 1V tgo Vout. Measure Vin

$$
A_{i}=\left(\frac{2 k}{2 k+1 k}\right) 1 V=0.667 V
$$

Rout: Short Vin. Apply 1V to Vout. Comptue the current draw

$$
\begin{aligned}
& I=\left(\frac{1 V}{1 k}\right)+\left(\frac{1 V}{5 k}\right)+200\left(\frac{1 V}{5 k}\right)+\left(\frac{1 V}{6 k}\right)=5.367 m A \\
& R_{\text {out }}=\left(\frac{1 V}{5.367 \mathrm{~mA}}\right)=186.3 \Omega
\end{aligned}
$$

Ao: Apply 1V to Vin. Compute Vout

$$
\begin{aligned}
& \left(\frac{V_{\text {out }}-1}{1 k}\right)+\left(\frac{V_{\text {out }}}{5000}\right)+200\left(\frac{V_{\text {out }}}{5000}\right)+\left(\frac{V_{\text {out }}}{6000}\right)=0 \\
& V_{\text {out }}=0.02417
\end{aligned}
$$

8) Find the 2-port model for the following CE : CC amplifier

| Rin | Ai | Rout | Ao |
| :---: | :---: | :---: | :---: |
| $3 k$ | 0 | 14.72 Ohms | -43.70 |



Rout: Short V1. Apply 1V to V3. Compute the current

$$
\begin{aligned}
& V_{2}=\left(\frac{3 k}{3 k+5 k}\right) \cdot 0.9=0.3375 V \\
& I=\left(\frac{1 V-0.95 \cdot 0.3375 V}{10 \Omega}\right)=67.94 m A
\end{aligned}
$$

$$
\text { Rout }=\frac{1 V}{67.94 m A}=14.72 \Omega
$$

Ao: Apply 1V to Vin. Compute V3

$$
\begin{aligned}
& V_{2}=\left(\frac{5 k}{3 k+5 k}\right)(-50 \mathrm{~V})+\left(\frac{3 k}{3 k+5 k}\right)\left(0.9 V_{3}\right) \\
& V_{3}=0.95 V_{2} \\
& V_{2}=-46.00 \mathrm{~V} \\
& V_{3}=0.95 V_{2}=-43.70 \mathrm{~V}
\end{aligned}
$$

