ECE 321 - Quiz #3. Name _____

Filter Design. April 19, 2018

1) The transfer function for a 4th-order Butterworth filter with a corner at 1 rad/sec is

$$G(s) = \left(\frac{1}{\left(s+1\angle\pm 22.5^0\right)\left(s+1\angle\pm 67.5^0\right)} \right)$$

$$G(s) = \left(\frac{1}{\left(s^2+1.8478s+1\right)\left(s^2+0.7654s+1\right)} \right)$$

Give the transfer function for a 4th-order Butterworth filter with

- A corner at 100 rad/sec, and
- A DC gain of 1.000

$$\frac{100^4}{(s+100\angle\pm 22.5^\circ)(s+100\angle\pm 67.5^\circ)}$$

2) Design a low-pass filter to meet the following requirements:

- $0.9 < \text{gain} < 1.1 \quad \omega \leq 10$ rad/sec
- $\text{gain} < 0.2 \quad \omega \geq 30$ rad/sec

2a) How many poles do you need for this filter?

$$\left(\frac{1}{3}\right)^n < 0.2$$

$$n > 1.46$$

$$\boxed{n = 2}$$

2b) Suggest the transfer function for a filter which meets (or comes close to meeting) these requirements.

Pick Butterworth

Corner ≈ 10

angle $= 180°/2 = \pm 90°$

$$\frac{10^2}{(s+10\angle\pm 45°)}$$

3) Design a low-pass filter to meet the following requirements:

- $0.9 < \text{gain} < 1.1 \quad \omega \leq 10$ rad/sec
- $\text{gain} < 0.02 \quad \omega \geq 30$ rad/sec

2a) How many poles do you need for this filter?

$$\left(\frac{1}{3}\right)^n < 0.02$$

$$n > 3.56$$

$$\boxed{n = 4}$$

2b) Suggest the transfer function for a filter which meets (or comes close to meeting) these requirements.

4th order Butterworth

Corner = 10 rad/sec (a-la problem #1)

$$\frac{10^4}{(s+10\angle\pm 22.5^\circ)(s+10\angle\pm 67.5^\circ)}$$

4) The frequency content of a 10 rad/sec (1.6Hz) square wave is

$$x(t) = 6\sin(10t) + 2\sin(30t) + \ldots$$

To turn this into a sine wave, the following low-pass filter is proposed:

$$Y = \left(\frac{2000}{(s+8.5)\left(s+12.1\angle 69.5^0\right)\left(s+12.1\angle -69.5^0\right)}\right)X = \left(\frac{2000}{(s+8.5)\left(s^2+8.475s+146.41\right)}\right)X$$

Find y(t)

$S = j10$

$(\quad)_{j10} = 1.577\angle -110^\circ$

$y = (1.577\angle -110^\circ)(0-j6)$

$y = 9.46\angle 159^\circ$

$S = j30$

$(\quad)_{j30} = .0806\angle 124^\circ$

$y = (.0806\angle 124^\circ)(0-j2)$

$y = .161\angle 34^\circ$

$y(t) = 9.46\cos(10t + 159^\circ) + 0.161\cos(30t + 34^\circ)$

mostly a pure sine wave

5) The frequency content of a 10 rad/sec (1.6Hz) square wave is

$$x(t) = 6\sin(10t) + 2\sin(30t) + \ldots$$

To turn this into a sine wave, the following band-pass filter is proposed:

$$Y = \left(\frac{2s}{s^2+2s+100}\right)X = \left(\frac{2s}{(s+1+j9.95)(s+1-j9.95)}\right)X$$

Find y(t)

$s = j10$

$( )_{j10} = 1$

$y = (1)(0-j6)$

$y = -j6 = 6\angle -90°$

$y = 6\cdot\cos(10t - 90°)$

$s = j30$

$( )_{j30} = .0748\angle -85°$

$y = (.0748\angle -85°)(0-j2)$

$y = .149\angle -175°$

$y = 0.149\cos(30t - 175°)$

$y = 6\sin(10t) + 0.149\cos(30t - 175°)$

mostly 1st harmonic

Bernie Sanders vs. Godzilla Bonus!!! Which is more (i.e. who's more popular)?

- The number of people who voted for Bernie Sanders in 2016, or — 1.6 million
- The number of people who saw Godzilla 2014 in the theater? — 30+ million

$300 million in ticket sales