ECE 321 - Homework #2

Active Filters, Poles, Zeros, & Frequency Response. Due Monday, April 9th, 2018

1) Assume X and Y are related by the following transfer function:

$$Y = \left(\frac{2000}{(s+5)(s+6)(s+7)}\right)X = \left(\frac{2000}{s^3+18s^2+107s+210}\right)X$$

1a) What is the differential equation relating X and Y?

Cross multiply

$$(s^3 + 18s^2 + 107s + 630)Y = (2000)X$$

'sY' means *the derivative of y*

$$y''' + 18y'' + 107y' + 210y = 2000x$$

1b) Determine y(t) assuming

$$x(t) = 2 + 3\cos(10t)$$

Treat this as two separate problems

$x(t) = 2$

s = 0

$$Y = \left(\frac{2000}{(s+5)(s+6)(s+7)}\right)_{s=0} \cdot X$$

$$Y = (9.52)\cdot(2)$$

$$y(t) = 19.04$$

$x(t) = 3\cos(10t)$

s = j10

$$X = 3 + j0$$

$$Y = \left(\frac{2000}{(s+5)(s+6)(s+7)}\right)_{s=j0} \cdot (3+j0)$$

$$Y = (-1.25 - j0.055)\cdot(3+j0)$$

$$Y = -3.76 - j0.16$$

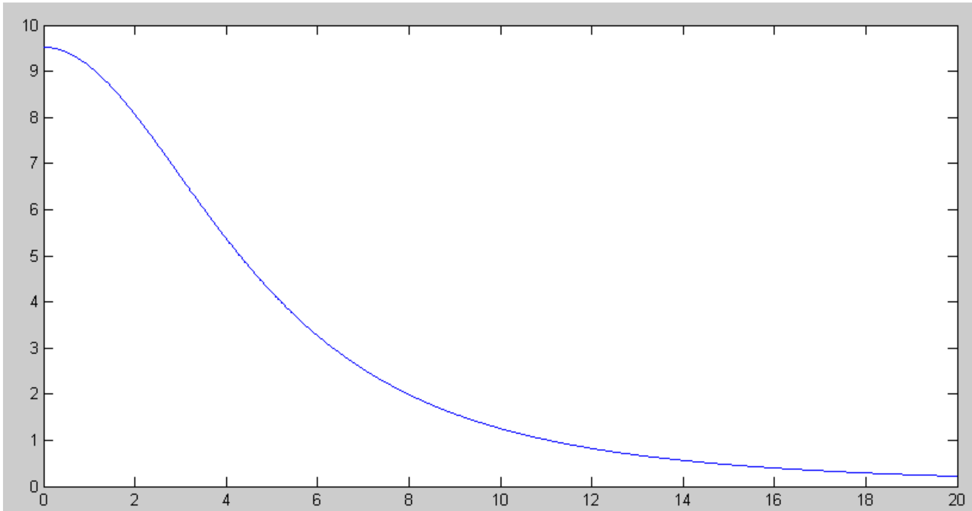
$$y(t) = -3.76\cos(10t) + 0.16\sin(10t)$$

Put them together:

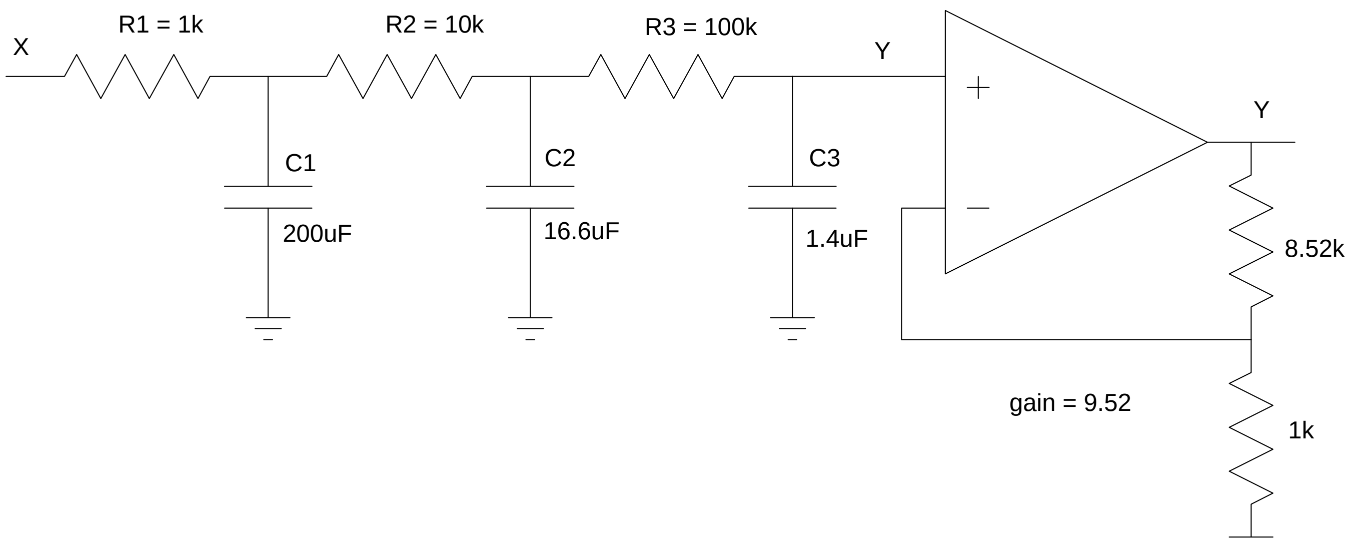
$$y(t) = 19.04 - 3.76\cos(10t) + 0.16\sin(10t)$$

1c) Plot the gain vs. frequency for this filter from 0 < w < 20 rad/sec

```
>> w = [0:0.01:20]';
>> s = j*w;
>> G = 2000 ./ ( (s+5).*(s+6).*(s+7) );
>> plot(w,abs(G))
```



1d) Design an op-amp circuit to implement this filter.



$\frac{1}{R_1C_1} = 5$ $\frac{1}{R_2C_2} = 6$ $\frac{1}{R_3C_3} = 7$

2) Assume X and Y are related by the following transfer function:

$$Y = \left(\frac{2000}{(s+10)(s+5+j8.67)(s+5-j8.67)}\right)X = \left(\frac{2000}{s^3+20s^2+200s+1000}\right)X$$

2a) What is the differential equation relating X and Y?

Cross multiply

$$(s^3 + 20s^2 + 200s + 1000)Y = (2000)X$$

This means

$$\boxed{y''' + 20y'' + 200y' + 1000y = 2000x}$$

2b) Determine y(t) assuming

$$x(t) = 2 + 3\cos(10t)$$

Treat this as two separate problems

x(t) = 2

$s = 0$

$$Y = \left(\frac{2000}{s^3+20s^2+200s+1000}\right)_{s=0} \cdot X$$

$$Y = (2) \cdot 2$$

$$y(t) = 4$$

x(t) = 3 cos(10t)

$s = j10$

X = 3 + j0

$$Y = \left(\frac{2000}{s^3+20s^2+200s+1000}\right)_{s=j10} \cdot (3 + j0)$$

$$Y = (-1 - j) \cdot (3 + j0)$$

$$Y = -3 - j3$$

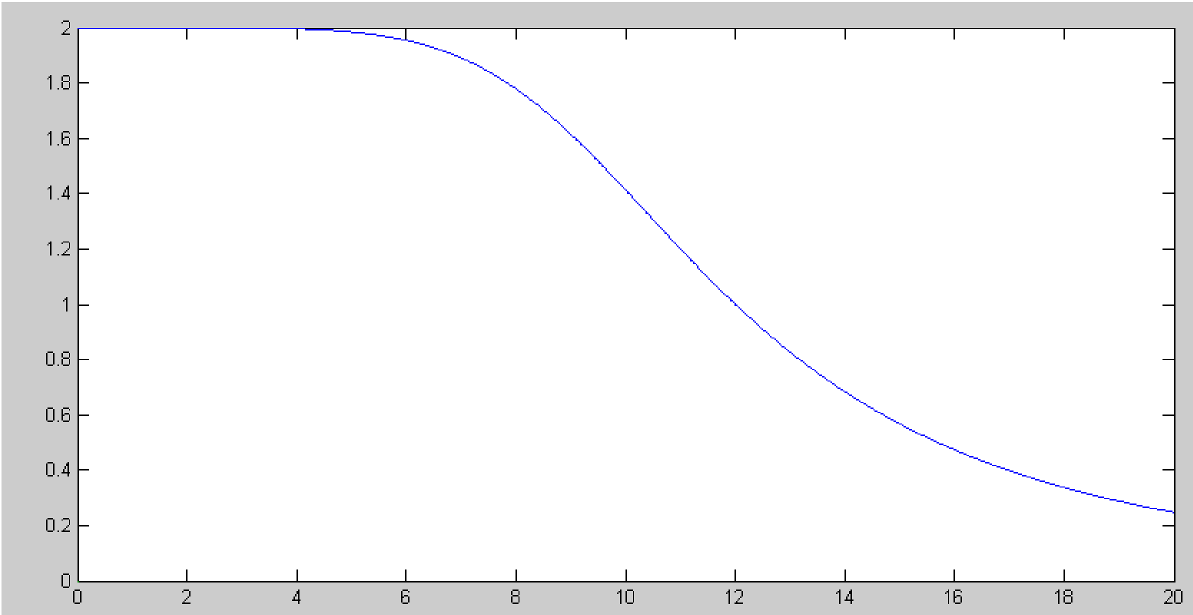
$$y(t) = -3\cos(10t) + 3\sin(10t)$$

Put it together

$$\boxed{y(t) = 4 - 3\cos(10t) + 3\sin(10t)}$$

2c) Plot the gain vs. frequency for this filter from 0 < w < 20 rad/sec

```
>> w = [0:0.01:20]';
>> s = j*w;
>> G = 2000 ./ ( s.^3 + 20*s.^2 + 200*s + 1000);
>> plot(w,abs(G))
```



2d) Design an op-amp circuit to impliment this filter.

Rewrite this as two filters cascaded

$$Y = \left(\frac{10}{s+10}\right)\left(\frac{200}{(s+5+j8.67)(s+5-j8.67)}\right)X$$

$$Y = \left(\frac{10}{s+10}\right)\left(\frac{200}{(s+10\angle 60^o)(s+10\angle -60^o)}\right)X$$

This is an RC filter (1/RC = 10) and a 2nd-order low-pass filter

$$\frac{1}{RC} = 10$$

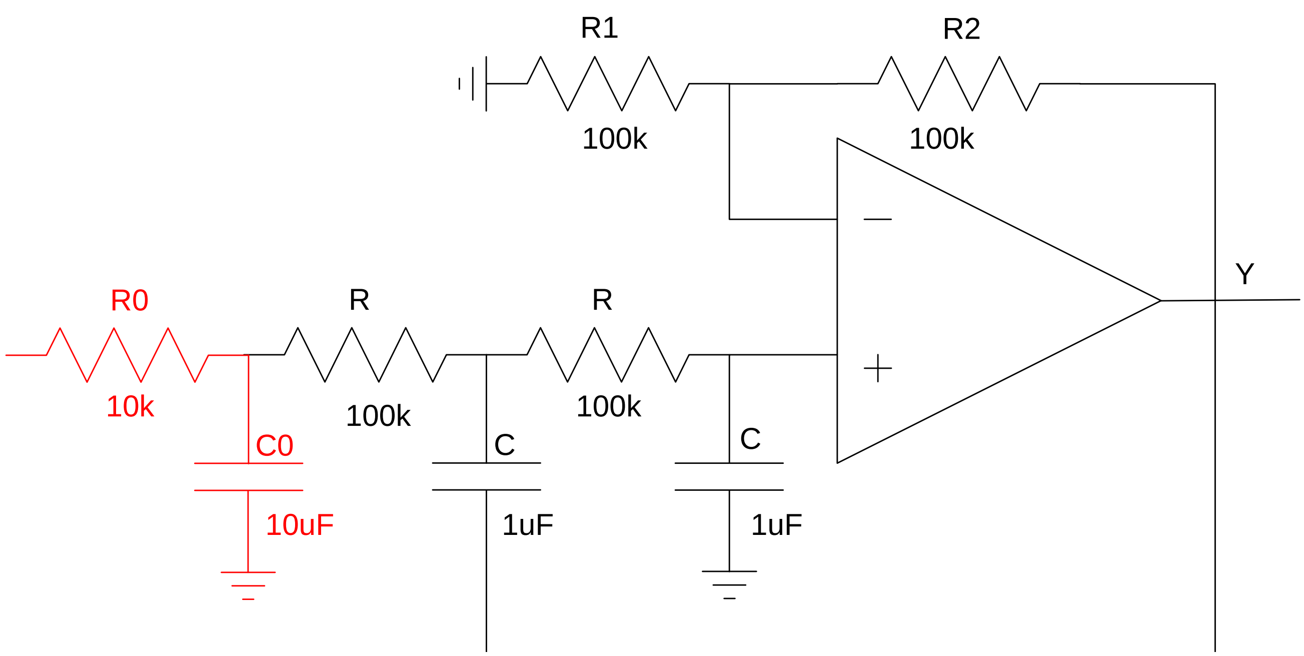
Let R = 100k, C = 1uF

$$3 - k = 2\cos\theta$$

$$3 - k = 2\cos(60^0)$$

$$k = 2 = \left(1 + \frac{R_2}{R_1}\right)$$

Let R1 = R2 = 100k



3) Assume X and Y are related by the following transfer function:

$$Y = \left(\frac{4s}{(s+2+j9.8)(s+2-j9.8)}\right) X = \left(\frac{4s}{s^2+4s+100}\right) X$$

3a) What is the differential equation relating X and Y?

Cross multiply

$$(s^2 + 4s + 100)Y = (4s)X$$

this means

$$y'' + 4y' + 100y = 4x'$$

3b) Determine y(t) assuming

$$x(t) = 2 + 3\cos(10t)$$

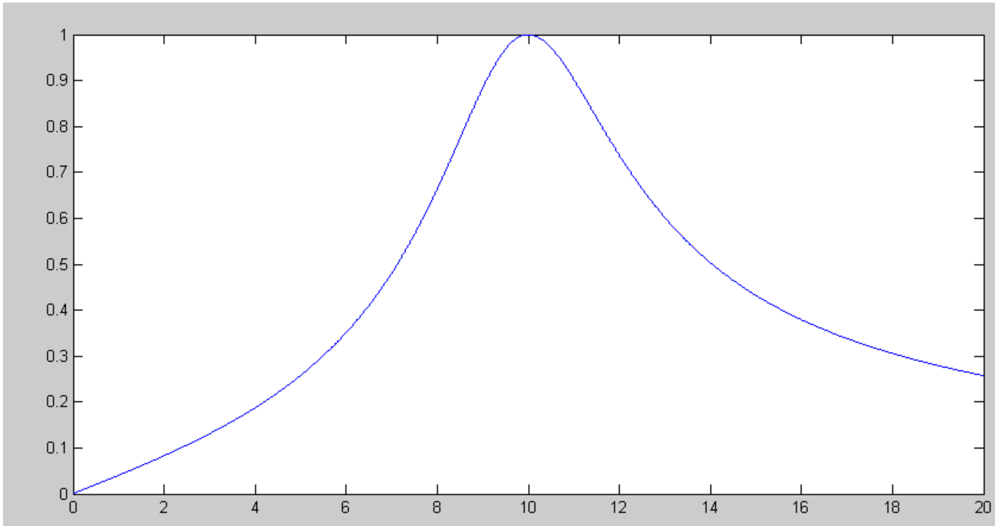
Treat this as two separate problems

x(t) = 2	x(t) = 3 cos(10t)
s = 0	s = j10
X = 2	X = 3 + j0
$Y = \left(\frac{4s}{s^2+4s+100}\right)_{s=0} \cdot X$	$Y = \left(\frac{4s}{s^2+4s+100}\right)_{s=j10} \cdot X$
$Y = (0) \cdot (2)$	$Y = (1 + j0) \cdot (3 + j0)$
$Y = 0$	$Y = 3 + j0$
$y(t) = 0$	$y(t) = 3\cos(10t)$

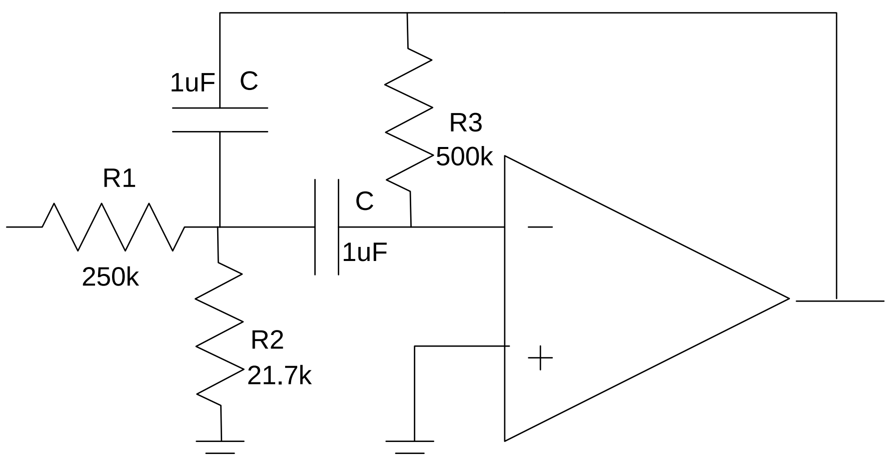
Put it together

$$y(t) = 0 + 3\cos(10t)$$

3c) Plot the gain vs. frequency for this filter from 0 < w < 20 rad/sec



3d) Design an op-amp circuit to impliment this filter.



$$Y = \left(\frac{-\left(\frac{1}{R_1 C}\right)s}{s^2 + \left(\frac{2}{R_3 C}\right)s + \left(\frac{R_1+R_2}{R_1 R_2 R_3 C^2}\right)} \right) X = \left(\frac{4s}{s^2+4s+100} \right) X$$

Let C = 1uF

$\left(\frac{1}{R_1 C}\right) = 4$ R1 = 250k

$\left(\frac{2}{R_3 C}\right) = 4$ R3 = 500k

$\left(\frac{R_1+R_2}{R_1 R_2 R_3 C^2}\right) = 100$ R2 = 21,739

4) Lab: Build a push-pull amplifier to meet the following requirements:

- Input: +/- 10V signal capable of driving 10mA (1 k Ohm)
- Output: 8 Ohm speaker
- Relationship: Y = X +/- 0.5 Volts from DC to 10kHz.

Collect data to verify your push-pull amplifier is working (save your circuit - we'll use it next week)