## ECE 321 - Homework \#4

2-Port Models, DC Bias for Transistors, Common Emitter Amplifier. Due Monday, April 23rd, 2018

## 2-Port Model

1) Find the 2-port model for the following circuit


Rin: Set Vout $=0 \mathrm{~V}$, measure the resistance at Vin

$$
\begin{aligned}
& R_{i n}=100+200 \| 500 \\
& R_{\text {in }}=242 \Omega
\end{aligned}
$$

Ain: Set Vout = 1V, measure Vin

$$
\begin{aligned}
& A_{i}=\left(\frac{500}{500+200}\right) \cdot 1 V \\
& A_{i}=0.7143
\end{aligned}
$$

Rout: Set Vin $=0$. Measure the resistance at the output

$$
\begin{aligned}
& R_{\text {out }}=200+100 \| 500 \\
& R_{\text {out }}=283 \Omega
\end{aligned}
$$

Ao: Set Vin = 1V, measure Vout

$$
\begin{aligned}
& A_{o}=\left(\frac{500}{500+100}\right) \cdot 1 V \\
& A_{o}=0.833
\end{aligned}
$$


2) Find the 2-port model for the following circuit


Rin: Set Vout $=0 \mathrm{~V}$, measure the resistance at the input. Since this isn't obvious, apply a 1 V test voltage at Vin and compute Iin:


Writing the voltage node equation at Vx

$$
\begin{aligned}
& \left(\frac{V_{x}-1}{1 k}\right)+\left(\frac{V_{x}}{2 k}\right)+100\left(\frac{V_{x}}{2 k}\right)=0 \\
& V_{x}=0.0194 V \\
& I_{i n}=\left(\frac{1-V_{x}}{1000}\right)=980 \mu A \\
& R_{\text {in }}=\frac{V_{i n}}{I_{i n}}=\frac{1 V}{980 \mu A}=1020 \Omega
\end{aligned}
$$

Ai: Apply 1V at Vout, compute Vin: This works out ot $\mathrm{Vx}=1 \mathrm{~V}(\mathrm{Ib}=100 \mathrm{Ib}=0) . \mathrm{Ai}=1$.


Rout: Short Vin, measure the resistance at Vout. Since this isn't obvious, add a 1V test voltage at the output and computer Iout


Solve for $\mathrm{V}_{\mathrm{x}}$ :

$$
\begin{aligned}
& \left(\frac{V_{x}}{1 k}\right)+\left(\frac{V_{x}-1}{2 k}\right)+100\left(\frac{V_{x}-1}{2 k}\right)=0 \\
& V_{x}=0.9806 V \\
& I_{\text {out }}=\frac{1}{3 k}+\left(\frac{1-0.9806}{2 k}\right)=343 \mu A \\
& R_{\text {out }}=\frac{1 V}{343 \mu A}=2915 \Omega
\end{aligned}
$$

Ao: Apply 1 V at the input, find the output voltage


Solve for Vx

$$
\begin{aligned}
& \left(\frac{V_{x}}{5 k}\right)+100\left(\frac{V_{x}}{5 k}\right)+\left(\frac{V_{x}-1}{1 k}\right)=0 \\
& V_{x}=0.04717 \mathrm{~V} \\
& V_{\text {out }}=\left(\frac{3 k}{3 k+2 k}\right) V_{x}=0.0283 \mathrm{~V}
\end{aligned}
$$



## Q-Point Design

3) Determine the Q-point for the following circuit. Assume ideal silicon transistor with

- $\beta=100$
- $\left|V_{b e}\right|=0.7 \mathrm{~V}$
- $\min \left(\left|V_{c e}\right|\right)=0.2 V$

First, redraw the circuit using the Thevenin equivalent for R1, R2, and a 12 V source

$$
\begin{aligned}
& R_{t h}=R_{b}=400 k| | 500 k=222 k \\
& V_{t h}=V_{b}=\left(\frac{400 k}{400 k+500 k}\right) 12 V=5.333 \mathrm{~V}
\end{aligned}
$$



Next, solve for Ib . Writing the loop equation around Ib

$$
\begin{aligned}
& -5.33+222 k \cdot I_{b}+0.7+1 k \cdot\left(I_{b}+100 I_{b}\right)=0 \\
& I_{b}=\left(\frac{5.33-0.7}{222 k+101 \cdot 1 k}\right)=14.33 \mu A \\
& I_{c}=100 I_{b}=1.433 m A
\end{aligned}
$$

The Q-point is then

$$
\begin{aligned}
& V_{c}=12-2000 \cdot I_{c}=9.133 \mathrm{~V} \\
& V_{e}=1000 \cdot\left(I_{b}+I_{c}\right)=1.448 \mathrm{~V} \\
& V_{c e}=7.686 \mathrm{~V}
\end{aligned}
$$

4) Change this circuit so that the Q-point is

- Vce $=6 \mathrm{~V}$, and
- Stabilized for variations in $\beta$

Start with Vce $=6 \mathrm{~V}$. This means

$$
\begin{aligned}
& 12 V=2000 \cdot I_{c}+6 V+1000 \cdot\left(I_{b}+I_{c}\right) \\
& I_{c}=1.993 m A \\
& I_{b}=19.93 \mu A
\end{aligned}
$$

To stabilize the Q-point

$$
(1+\beta) R_{e} \gg R_{b}
$$

$$
101 k \gg R_{b}
$$

Let $\mathrm{Rb}=10 \mathrm{k} . \mathrm{Vb}$ is then

$$
\begin{aligned}
& V_{b}=10 k \cdot I_{b}+0.7+1 k \cdot\left(I_{b}+I_{c}\right) \\
& V_{b}=2.913 V
\end{aligned}
$$

Solving for R1 and R2

$$
\begin{aligned}
& R_{1}=\left(\frac{12 \mathrm{~V}}{2.913 V}\right) 10 \mathrm{k}=41.2 \mathrm{k} \Omega \\
& R_{1} \| R_{2}=10 \mathrm{k} \\
& R_{2}=13.21 \mathrm{k}
\end{aligned}
$$



## Common Emitter

5) Determine the 2-port model for the following common emitter amplifier

Redraw the AC (small signal) model:

rf comes from the DC operating point ( problem \#3: $\mathrm{Ib}=14.33 \mathrm{uA}$ )

$$
r_{f}=\left(\frac{0.052}{I_{b}}\right)=\left(\frac{0.052}{14.33 \mu \mathrm{~A}}\right)=3629 \Omega
$$

Then:

$$
\begin{aligned}
& R_{\text {in }}=400 \mathrm{k}| | 500 \mathrm{k}| | r_{f}=3570 \Omega \\
& A_{i}=0 \\
& R_{\text {out }}=2 \mathrm{k} \Omega \\
& A_{o}=-\frac{\beta \cdot R_{c}}{r_{f}}=-55.12
\end{aligned}
$$


6) Check your analysis in PartSim

DC Operating Point (Problem 3 )

|  | Analysis (problem 3) | Simulation <br> Run - DC Bias |
| :---: | :---: | :---: |
| Vce | 7.68 V | 6.32 V |
| Ic | 1.43 mA | 1.91 mA |



AC Operating Point ( 1 M load): $\quad$ Ao $=-123.3$ (vs. -55 computed)


AC Operating Point: 2 k Load. Gain $=-62.6$ (half of what it was for a 1 M load. The output impedance is 2 k )


AC Operating Point with a 3570 Ohm resistor added in series with Vin:


The output voltage drops from

- 628 mV when Rin $=0$
- 277 mV when Rin $=3570$

This means

$$
\left(\frac{R_{i n}}{R_{i n}+3570}\right)=\left(\frac{277 m V}{628 m V}\right)
$$

$$
R_{i n}=2817 \Omega
$$

## Lab (over)

## Lab

7a) Specify the overall requirements for a circuit which incorporates the hardware built in previous homework assignments. For example:

- Input: +/- 1V AC signal capable of driving 1 mA (i.e. a cell phone)
- Output: 8 Ohm Speaker
- Relationship:
- $9<$ gain < 11 for frequencies less than 200 Hz
- gain $<1$ for frequencies above 600 Hz

7b) Specify how this design is split into three secions and the requirements for each section. For example:
Secion 1: Amplfier

- Input: +/- 1V AC signal capable of driving 1 mA (i.e. a cell phone)
- Output: +/- 10V AC signal capable of driving 1 kOhm
- Relationship: $y=10 x$ ( $+/-10 \%$ )

Section 2: Filter

- Input: +/- 10V AC signal capable of driving 1 kOhm
- Output: $+/-10 \mathrm{~V}$ AC signal capable of driving 1 kOhm
- Relationship:
- 9 < gain < 11 for frequencies less than 200 Hz
- gain $<1$ for frequencies above 600 Hz


## Section 3: Push-Pull Amplifier

- Input: +/- 10V AC signal capable of driving 1 kOhm
- Output: 8 Ohm speaker
- Relationship: $\mathrm{y}=\mathrm{x}(+/-10 \%)$
(next week - homework \#5): Assembler your three circuits together and collect data to validate
- Each section works separately
- The entire circuit works together

