ECE 321 - Solution to Homework #3

Calibration and Filters. Due Monday, April 15th

A light sensor has a lux vs. resistance relationship of

$$R = \frac{100{,}000}{Lux}$$

1) Design a circuit which output -10V to +10V as the light level changes from 10 Lux to 100 Lux.

10 Lux = 10,000 Ohms

100 Lux = 1000 Ohms

Use a voltage divider with a 3000 Ohm resistor

10 Lux   (Vout = -10V)

$$X = \left(\frac{10000}{3000+10000}\right)10V = 7.6923V$$

100 Lux (Vout = +10V)

$$X = \left(\frac{1000}{3000+1000}\right)10V = 2.5000V$$

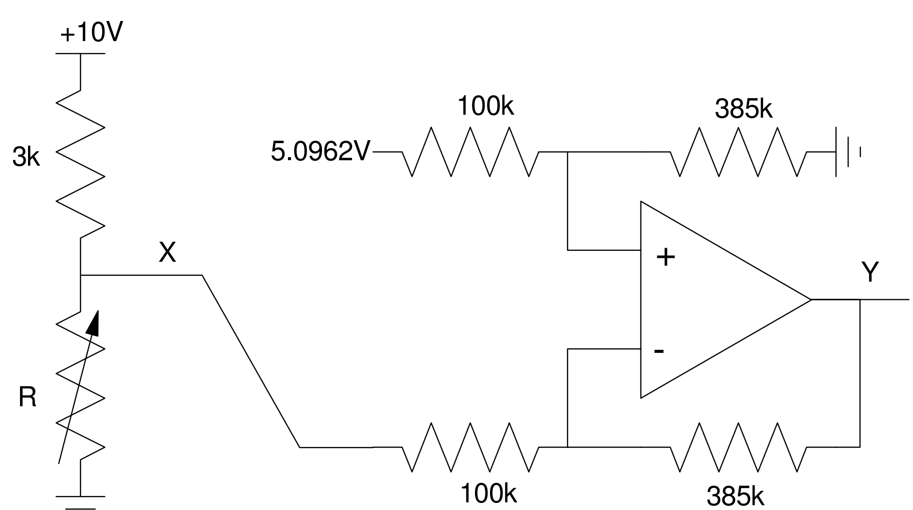
The gain you need is

$$gain = \left(\frac{10V-(-10V)}{2.500V-7.6923V}\right) = -3.8519$$

The offset you need makes the voltage in the middle equal to zero

$$A = \left(\frac{7.7923+2.500}{2}\right)$$

$$A = 5.0962V$$



2) Determine a calibration funciton which determines Lux based upon the output voltage for problem #1

$$Lux = f(V)$$

```
Lux = [10:0.1:100]';
R = 100000 ./ Lux;
X = R ./ (3000 + R) * 10;
Y = 3.8519 * ( 5.0962 - X );
plot(Y, Lux);
xlabel('Y (Volts)');
ylabel('Lux');
```

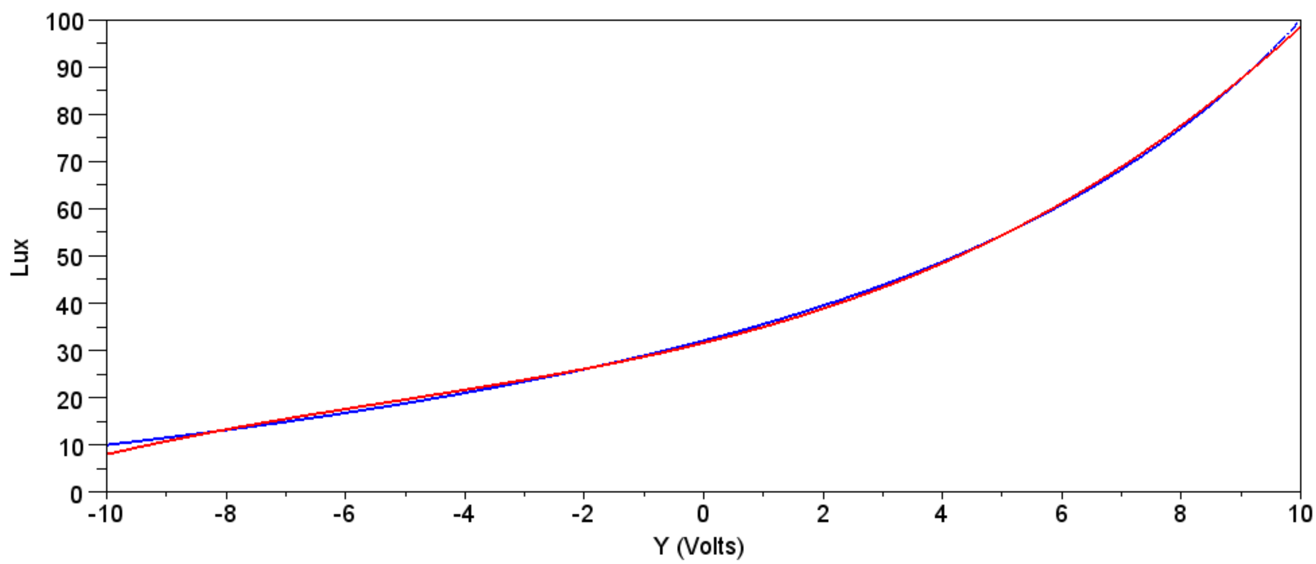
Approximate this with a cubic funciton

$$Lux \approx a + bY + cY^2 + dY^3$$

```
B = [Y.^0, Y, Y.^2, Y.^3];

A = inv(B'*B)*B'*Lux
```

a 31.598096
b 3.1319854
c 0.2167463
d 0.0139799

```
plot(Y, Lux, Y, B*A);
xlabel('Y (Volts)');
ylabel('Lux');
```



Light Level (blue) and cubic approximation (red)

Sidelight: For data which follows a 1/x relationship, plotting the log(Lux) works better

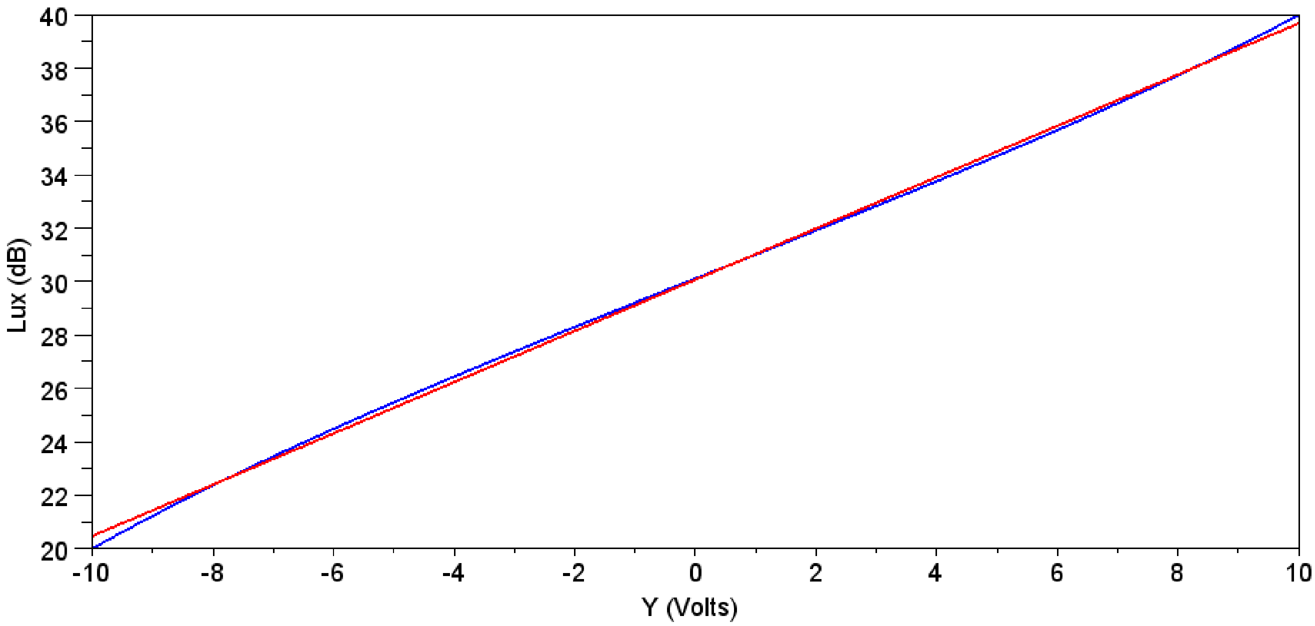
$$dB = 20 \cdot \log_{10}(Lux)$$

$$dB(Lux) \approx a + bY$$

```
B = [Y.^0, Y];
A = inv(B'*B)*B'*20*log10(Lux)
```

```
a    30.082157
b    0.9606058
```

```
plot(Y, 20*log10(Lux), Y, B*A);
```



Lus (blue) and it's linear approxiamtion (red)

3) Assume X and Y are related by the following filter

$$Y = \left(\frac{100}{(s+5)(s+10)}\right)X$$

a) What is the differential equation relating X and Y?

Cross multiply

$$((s+5)(s+10))Y = 100X$$

$$(s^2+15s+50)Y = 100X$$

'sY' means 'the derivative of Y'

$$\frac{d^2y}{dt^2} + 15\frac{dy}{dt} + 50y = 100x$$

or using prime notation

$$\boxed{y'' + 15y' + 50y = 100x}$$

b) Determine y(t) assuming

$$x(t) = 3 + 4\cos(6t)$$

Use superposition and phasor analysis

x(t) = 3

$$X = 3$$

$$s = 0$$

$$Y = \left(\frac{100}{(s+5)(s+10)}\right)X$$

$$Y = \left(\frac{100}{(s+5)(s+10)}\right)_{s=0} \cdot 3$$

$$Y = (2) \cdot 3$$

$$Y = 6$$

x(t) = 4 cos(6t)

$$X = 4 + j0$$

$$s = j6$$

$$Y = \left(\frac{100}{(s+5)(s+10)}\right)_{s=j6} \cdot 4$$

$$Y = (0.1688 - j1.0849) \cdot 4$$

$$Y = 0.6750 - j4.3394$$

meaning

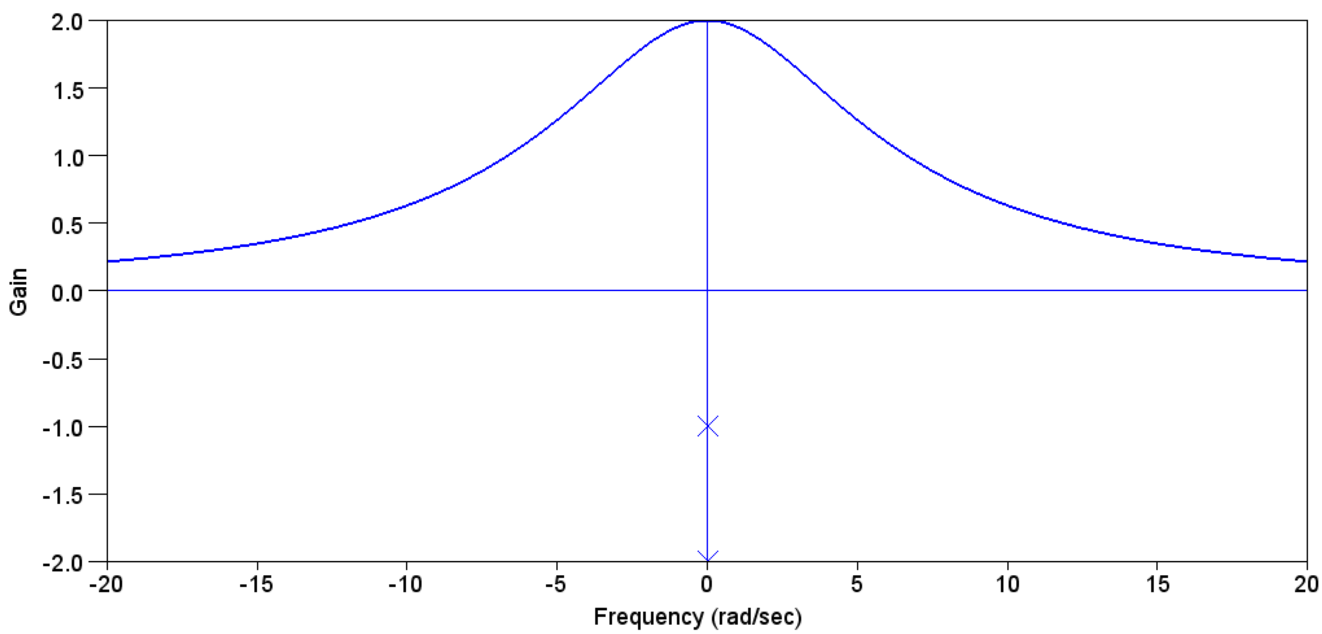
$$y = 0.6750\cos(6t) + 4.3394\sin(6t)$$

The total answer is the DC + AC part

$$y(t) = 6 + 0.6750\cos(6t) + 4.3394\sin(6t)$$

c) Plot the gain vs. frequency of this filter from 0 to 20 rad/sec.

```
w = [0:0.1:20]';
s = j*w;

G = 100 . /( (s+5) .* (s+10) );

plot(w,abs(G))
xlabel('Frequency (rad/sec)');
ylabel('Gain');
```



Gain vs. Frequency along with pole location (-5 + j0, -10 + j0)

4) Use Matlab and fminsearch to find a filter to approximate

$$G(s) = \begin{cases} 1 & 0 < s < 5 \\ 0 & otherwise \end{cases}$$

Assume G(s) is in the form of

$$G(s) = \left(\frac{ace}{(s+a)\left(s^2+bs+c\right)\left(s^2+ds+e\right)} \right)$$

Plot the gain vs. frequency of your resulting filter along with it's pole locations.

Cost Function (SciLab funciton)

```
function J = cost(z)
// low pass filter

   a = z(1);
   b = z(2);
   c = z(3);
   d = z(4);
   e = z(5);

   j = sqrt(-1);
   w = [0:0.01:10]';
   s = j*w;

   Gideal = 1 * (w < 5);
   G = a*c*e ./ ( (s+a) .* (s.^2 + b*s + c) .* (s.^2 + d*s + e));

   E = abs(Gideal) - abs(G);

   J = sum(E .^ 2);

endfunction
```

Optimal Filter:

SciLab

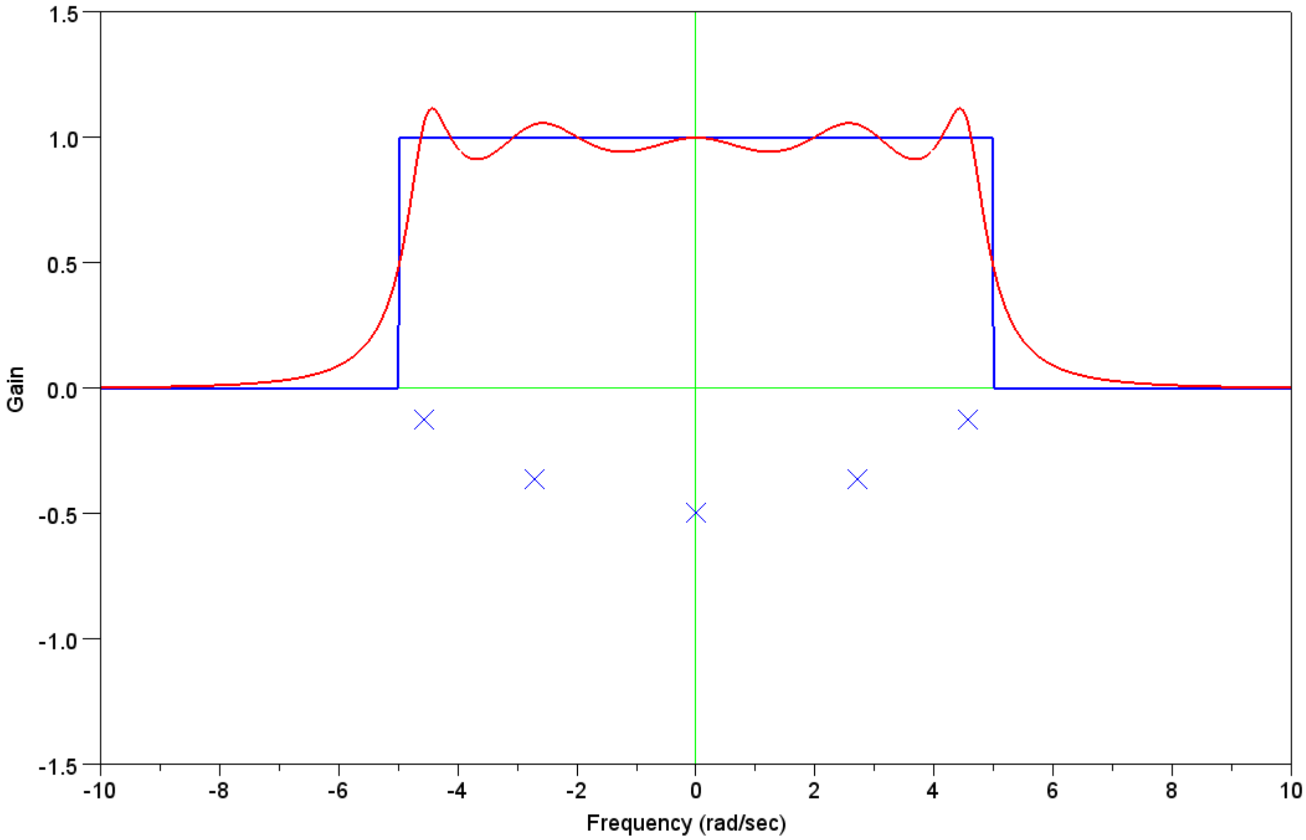
```
[e,z] = leastsq(cost,[1,2,3,4,5])
```

	a	b	c	d	e
z =	**1.4945112**	**0.7362332**	**20.993731**	**2.1655715**	**8.6011974**

```
 e  =    125.20971
```

MatLab

```
[z,e] = fminsearch('cost',[1,2,3,4,5])

 z  =    1.4945112    0.7362332    20.993731    2.1655715    8.6011974

 e  =    125.20971
```



Gain vs. Frequency for a Low-Pass Filter. Pole locations marked with a 'x'

Note that the location of the poles is along an arc centered on the bandwidth of the filter

Also note that

- As you get close to a pole, you get a resonance (the gain goes up)
- As you move away from all 5 poles the gain drops

5) Use Matlab and fminsearch to find a filter to approximate

$$G(s) = \begin{cases} s/5 & 0 < s < 5 \\ 0 & otherwise \end{cases}$$

Assume G(s) is in the form of

$$G(s) = \left(\frac{as}{\left(s^2+bs+c\right)\left(s^2+ds+e\right)} \right)$$

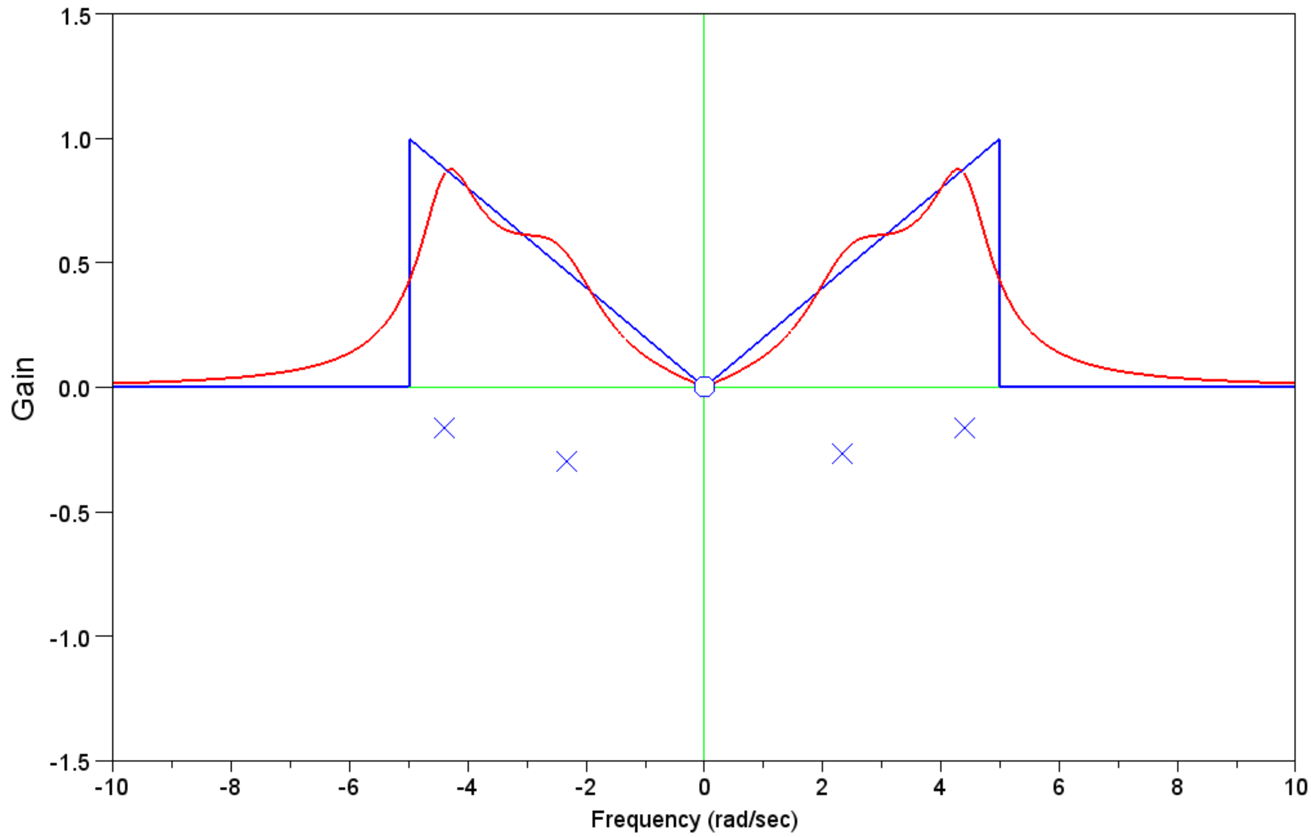
Plot the gain vs. frequency of your resulting filter along with it's pole locations.

The cost function

```
function J = cost(z)
// low pass filter

   a = z(1);
   b = z(2);
   c = z(3);
   d = z(4);
   e = z(5);

   j = sqrt(-1);
   w = [0:0.01:10]';
   s = j*w;

   Gideal = 0.2*w .* (w < 5);
   G = a*s ./ ( (s.^2 + b*s + c) .* (s.^2 + d*s + e));

   plot(w,abs(Gideal), w, abs(G));
   E = abs(Gideal) - abs(G);

   J = sum(E .^ 2);

endfunction
```

Minimizing the cost function (filter design)

```
[e,z] = leastsq(cost,[625,10,25,10,25])

             a            b            c            d            e
 z  =    12.640092    0.9824992    19.679872    1.6015253    6.1387986

 e  =    251.5432
```



Gain vs. Frequency for a Low-Pass Filter. Pole locations marked with an 'x'

Note that the location of the poles is along an arc centered on the bandwidth of the filter

Also note that

- As you get close to a zero, the gain goes to zero (s = 0)
- As you get close to a pole, you get a resonance (the gain goes up)
- As you move away from all 5 poles the gain drops