ECE 321 - Quiz #2 - Name ____________________

Instrumentation Amplifiers, Calibration, Strain Sensors, Filters, Due midnight, April 23, 2020

Calculators, internet, Matlab permitted.

Please sign pledge if able (i.e. you did not work with anyone else)

No aid given, received, or observer: ________________________________

1) Instrumentation Amplifier: A strain gage has the temperature - resistance relationship of

$$R = 1000\ \exp\left(\frac{3905}{T+273} - \frac{3905}{298}\right)\ \Omega$$

where T is the temperature in degrees C. Design a circuit so that the output is

- -10V at -10C
- +10V at +10C
- Proportional for -10C < T < +10C

Let the resistor be 3000 Ohms

At -10C

- R = 5719.5 Ohms
- $X = \left(\frac{R}{R+3000}\right) 10V = 6.5594V$
- Y = -10V

At +10C

- R = 2002.82 Ohms
- $X = \left(\frac{R}{R+3000}\right) 10V = 4.0034V$
- Y = +10V

Y goes up as X goes down. Connect X to the minus input

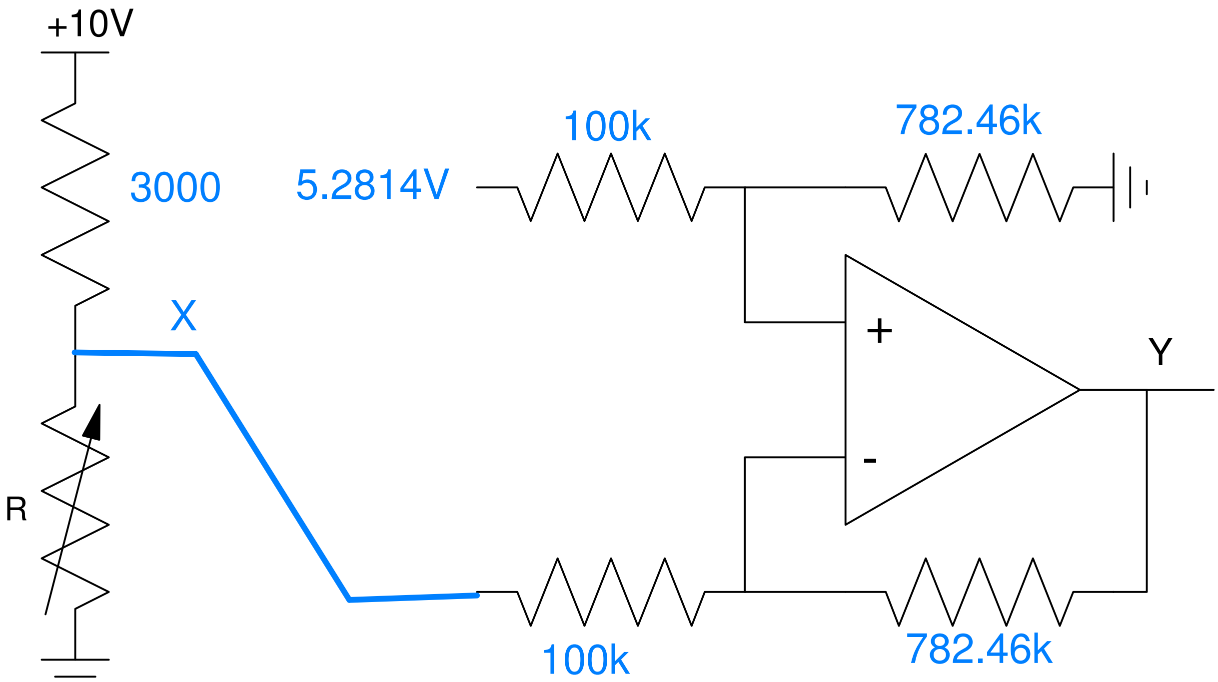
The gain required is

$$gain = \left(\frac{10V-(-10V)}{4.0034V-6.5594V}\right) = -7.8246$$

Pick the resistor ration to be 7.8246 : 1

The offset sets the output to 0V half way between -10V and +10V (half way between 6.5594V and 4.0034V)

$$offset = \left(\frac{6.5594+4.0034}{2}\right) = 5.2814V$$



+10V
3000
5.2814V
100k
782.46k
X
+
Y
-
R
100k
782.46k

2) Calibration. For your circuit of problem #1, determine a calibration funtion of the form

$$T = aV^2 + bV + c$$

where C is the temperature in degrees C over the range of -10C to +10C. Also determine the maximum error of your calibration scheme

a	b	c	mean error mean(T - B*A)	standard deviation std(T - B*A)
0.0022	**0.9910**	**-0.2167**	**0**	**0.0354**

```
T = [-10:0.1:10]';
R = 1000 * exp( 3905 ./ (T+273)   -  3905/298);
X = R ./ (R+3000) * 10;
Y = 7.8246 * (5.2814 - X);
B = [Y.^2, Y, Y.^0];
A = inv(B'*B)*B'*T

    0.0022
    0.9910
   -0.2167

plot(Y,T,'b',Y,B*A,'r')
xlim([-10,10])
xlabel('Temperature (C)');
ylabel('Volts');

x = mean(T - B*A)

x = -7.9980e-016

s = std(T - B*A)

s =    0.0354
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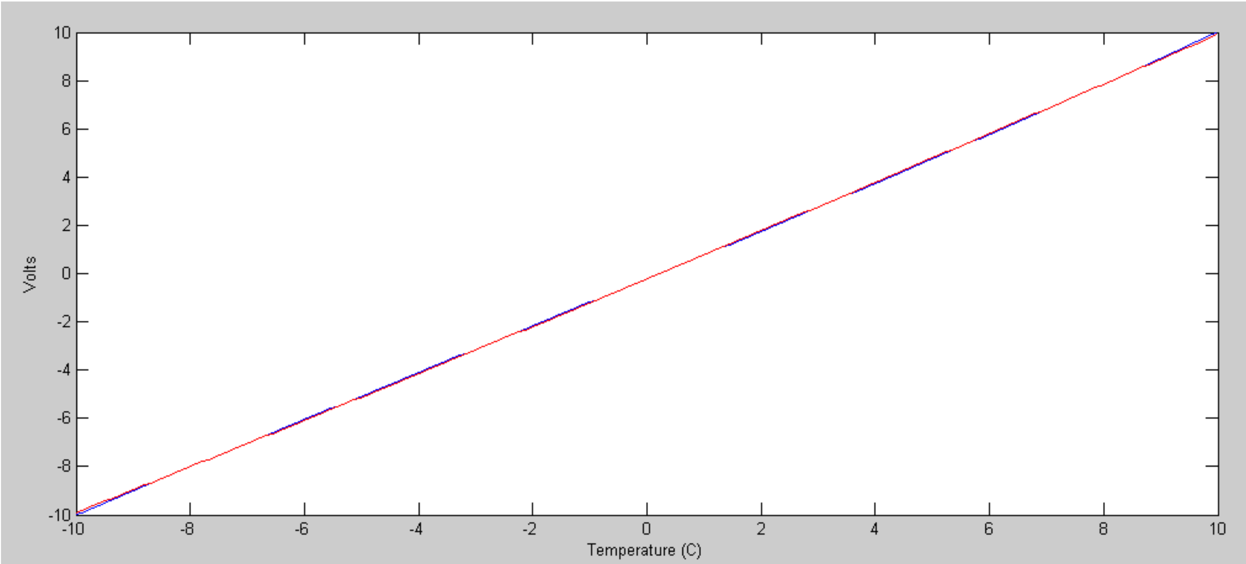
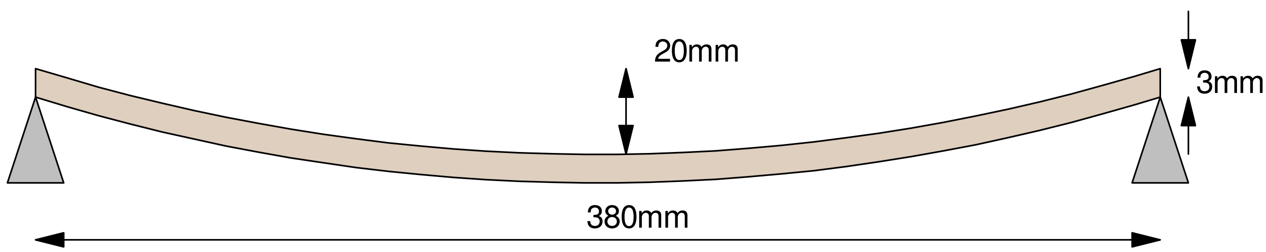


3) Strain Sensor: A metal beam deflects by 20mm when a force of 100N is applied to it. Determine the radius of curvature, the strain, and the resistance of a strain gage placed on the ourside edge

- Length of beam: 380mm
- Thickness of beam: 3mm
- Deflection: 20mm
- $R = 120(1 + 2.14\varepsilon)$

Radius of Curvature	Strain on Outside Edge	Resistance of Strain Gage
912.5mm	**0.00164**	**120.422 Ohms**



To determine the radius

$$R^2 = (R - 20)^2 + 190^2$$

$$R = 912.5mm$$

That's the radius to the center line. The radius to the outside edge is 1/2 of the thickness more

$$R_{outside} = 912.5mm + \tfrac{1}{2}(3mm) = 914mm$$

The strain on the outside edge is

$$\varepsilon = \left(\frac{914mm - 912.5mm}{912.5mm}\right) = +0.00164$$

The resistance is then

$$R = 120(1 + 2.14\varepsilon) = 120.422\Omega$$

4) Filter Analysis: Assume X and Y are related by the following transfer function

$$Y = \left(\frac{5000}{(s+10)(s+15)(s+20)} \right) X$$

4a) What is the differential equation relating X and Y?

Multiply out

$$(s^3 + 45s^2 + 650s + 3000)Y = 5000X$$

$$\boxed{\frac{d^3y}{dt^3} + 45\frac{d^2y}{dt^2} + 650\frac{dy}{dt} + 3000y = 5000x}$$

4b) Determine y(t) assuming

$$x(t) = 5 + 6\sin(7t)$$

Use superposition

x(t) = 5

s = 0

X = 5 + j0

$$Y = \left(\frac{5000}{(s+10)(s+15)(s+20)} \right)_{s=0} (5 + j0)$$

$$Y = 8.3333$$

$$y(t) = 8.3333$$

x(t) = 6 sin(7t)

s = j7

X = 0 - j6

$$Y = \left(\frac{5000}{(s+10)(s+15)(s+20)} \right)_{s=j7} (0 - j6)$$

$$Y = -6.885 - j1.301$$

$$y(t) = -6.885\cos(7t) + 1.301\sin(7t)$$

The total answer is then

$$\boxed{y(t) = 8.333 - 6.885\cos(7t) + 1.301\sin(7t)}$$

5) Filter Design using fminsearch: Design a filter of the form

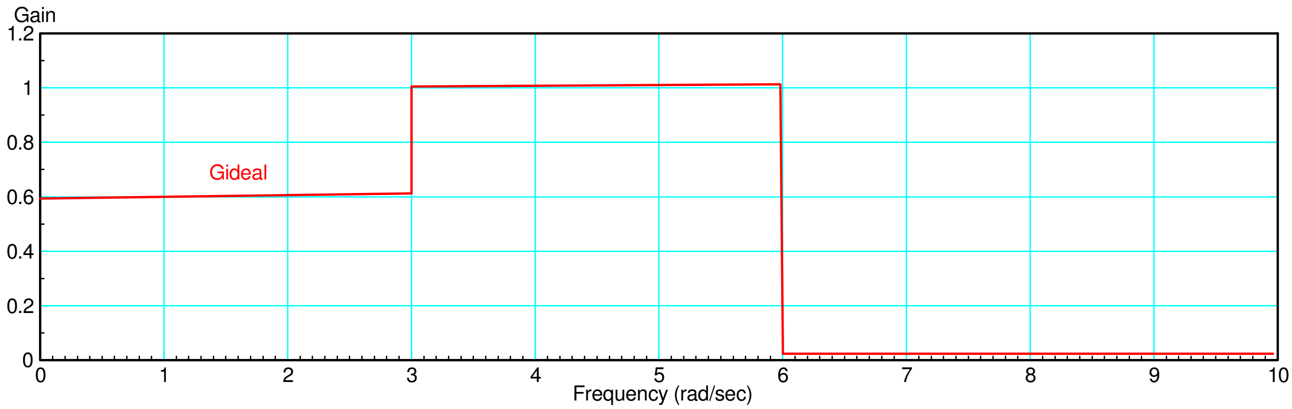
$$Y = G(s)X = \left(\frac{a}{(s+b)(s^2+cs+d)(s^2+es+f)} \right) X$$

to approximate

$$|G(jw)| = \begin{cases} 0.6 & 0 < w < 3 \\ 1.0 & 3 < w < 6 \\ 0 & 6 < w < 10 \end{cases}$$

over the rainge of 0 < w < 10. Give

- The resulting filter, G(s), and
- The resulting gain vs. frequency of G(s)



It helps if you give is an inital guess which is close. Place the poles at

- s = -1 passes DC
- s = -1 + j3 passes 3 rad/sec
- s = -1 + j6 passes 6 rad/sec
- DC gain is 1.0

Let Matlab iterate to improve the filter

$$G(s) = \left(\frac{370}{(s+1)(s^2+2s+10)(s^2+2s+37)} \right)$$

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>> [Z,e] = fminsearch('costF',[370,1,1,10,1,37])

Z =   428.2408    1.5093    1.5600    14.6579    0.6997    30.6578


e =     1.2400
```
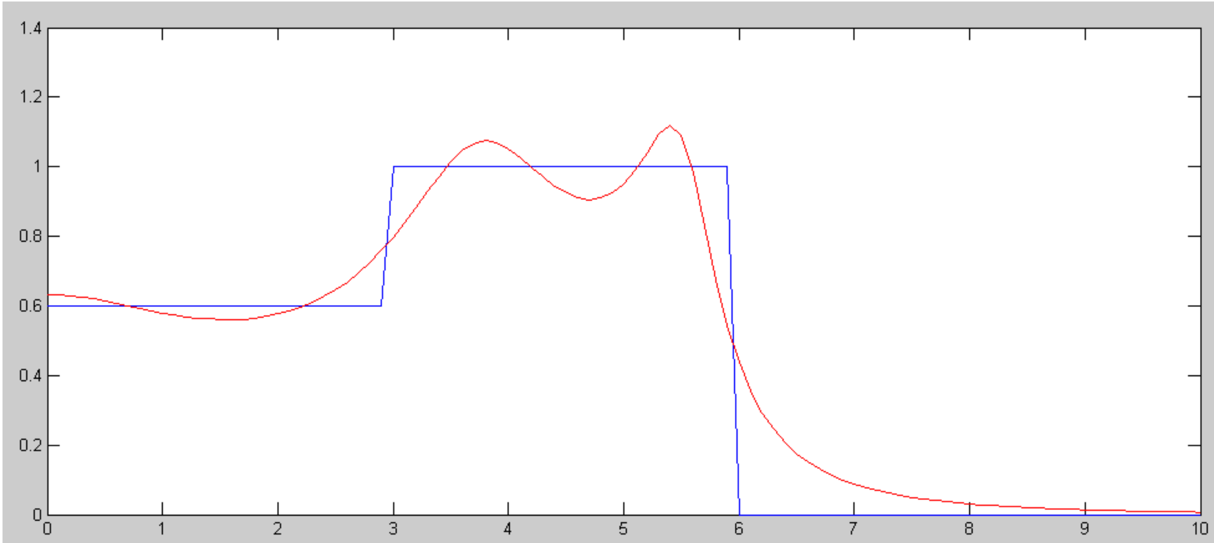

This results in

$$G(s) = \left(\frac{428.24}{(s+1.5093)(s^2+1.56s+14.65)(s^2+0.6997s+30.6578)} \right)$$

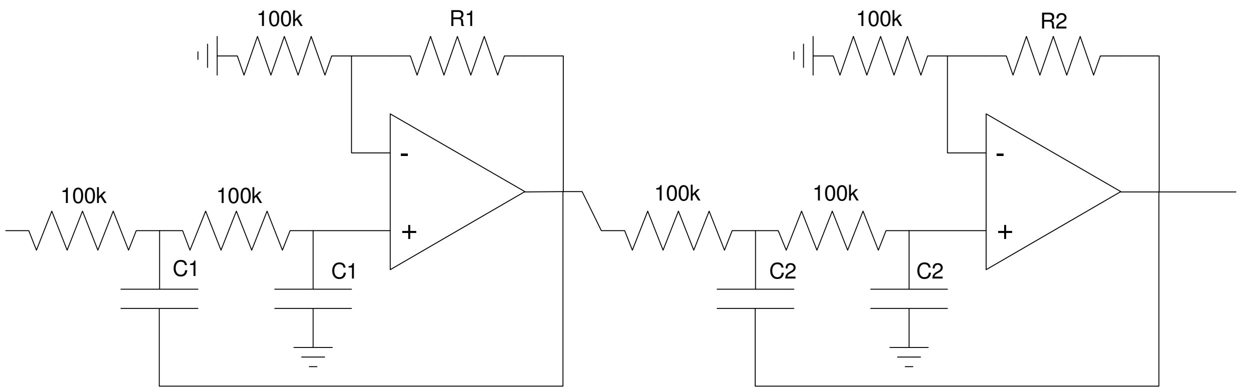


1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
9
10

6) Filter Implementation: Design an op-amp circuit to implement the following filter:

$$Y = \left(\frac{450}{(s^2+4s+15)(s^2+2s+30)} \right) X$$

C1	R1	C2	R2	Resulting DC gain
2.582 uF	**96.7k**	**1.857uF**	**163.4k**	**5.18**



Roots of $s^2 + 4s + 15$

$$s = -2 \pm j3.3166$$

$$s = -3.8730 \angle \pm 58.91^0$$

$$\left(\frac{1}{100k \cdot C_1} \right) = 3.8730$$

$$C_1 = 2.582 \mu F$$

$$3 - k = 2 \cos(58.91^0)$$

$$k = 1.967$$

$$R_1 = (k-1)100k$$

$$R_1 = 96.7k$$

roots of $s^2 + 2s + 30$

$$s = -1 \pm j5.3852$$

$$s = -5.4772 \angle \pm 79.48^0$$

$$\left(\frac{1}{100k \cdot C_2} \right) = 5.3853$$

$$C_2 = 1.857 \mu F$$

$$3 - k = 2 \cos(79.48^0)$$

$$k = 2.635$$

$$R_2 = 163.5k$$

Resulting DC gain

$$gain = (1.967)(2.6345) = 5.183$$