ECE 321 - Quiz #3 - Name ____________________

Filter Design, Butterworth & Chebychev Filters, Analog Computers. Due midnight, April 30, 2020

Calculators, internet, Matlab, circuit lab, tarot cards permitted. Just not someone else.

Please sign pledge if able (i.e. you did not work with anyone else)

No aid given, received, or observer: ______________________________

1) Filter Design: Design a filter to meet the following requirements. (Only the transfer function is needed - you don't need to design the op-amp circuit. That will come with problems 3-6)

0.9 < gain < 1.1	w < 100 Hz
gain < 0.1	w > 400 Hz

Filter: G(s)	Gain at 100Hz	Gain at 400Hz
$G(s) = \left(\frac{533 \cdot 761^2}{(s+533)\left(s+761\angle \pm 69.50^0\right)} \right)$	**0.9800**	**0.0204**

$$\left(\frac{100}{400}\right)^n < 0.1$$

$$n > 1.66$$

Let n = 3 since a 3rd-order Chebychev is almost as easy to build as a 2nd-order one

Choose a Chebychev filter (a Butterworth could also work)

Choose the corner to be 100 Hz (628 rad/sec) (a guess). For a corner at 1 rad/sec

$$G(s) = \left(\frac{k}{(s+0.8494)\left(s+1.2130\angle \pm 69.50^0\right)} \right)$$

For a corner at 100 rad/sec

$$G(s) = \left(\frac{533 \cdot 761^2}{(s+533)\left(s+761\angle \pm 69.50^0\right)} \right)$$

Checking at 100Hz and 400Hz

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p1 = 533;
p2 = 761 * exp(j*69.5*pi/180);
p3 = conj(p2);
num = p1*p2*p3

num =   308671493

f = [100;400]
w = 2*pi*f;
s = j*w;
G = num ./ ( (s+p1).*(s+p2).*(s+p3) );
abs(G)

    0.9800
    0.0204
```

2) Filter Design: Design a filter to meet the following requirements. (Only the transfer function is needed - you don't need to design the op-amp circuit. That will come with problems 3-6)

0.9 < gain < 1.1	w < 100 Hz
gain < 0.1	w > 130 Hz

Filter: G(s)	
$G(s) = G(s) = \left(\frac{k}{(s+163.1)\left(s+269.3\angle\pm55.36^0\right)\left(s+435.4\angle\pm73.32^0\right)\left(s+567.8\angle\pm81.74^0\right)\left(s+639.6\angle\pm87.46^0\right)} \right)$	
Gain at 100Hz	Gain at 130Hz
0.9735	**0.01**

$$\left(\frac{100}{130}\right)^n < 0.1$$

$$n > 8.77$$

Let n = 9. For a corner at 1 rad/sec

$$G(s) = \left(\frac{k}{(s+0.25977)\left(s+0.4298\angle\pm55.36^0\right)\left(s+0.6933\angle\pm73.32^0\right)\left(s+0.9041\angle\pm81.74^0\right)\left(s+1.0185\angle\pm87.46^0\right)} \right)$$

For a corner at 628 rad/sec

$$G(s) = \left(\frac{k}{(s+163.1)\left(s+269.3\angle\pm55.36^0\right)\left(s+435.4\angle\pm73.32^0\right)\left(s+567.8\angle\pm81.74^0\right)\left(s+639.6\angle\pm87.46^0\right)} \right)$$

Checking:

```
p1 = 161.3;
p2 = 269.3*exp(j*55.36*pi/180);
p3 = conj(p2);
p4 = 435.4*exp(j*73.72*pi/180);
p5 = conj(p4);
p6 = 567.8*exp(j*81.74*pi/180);
p7 = conj(p6);
p8 = 639.6*exp(j*87.46*pi/180);
p9 = conj(p8);

f = [100, 130];
s = j * 2 * pi * f;

num = p1*p2*p3*p4*p5*p6*p7*p8*p9;
G = num ./ (
(s+p1).*(s+p2).*(s+p3).*(s+p4).*(s+p5).*(s+p6).*(s+p7).*(s+p8).*(s+p9));
abs(G)

ans =    0.9735    0.0109

>>
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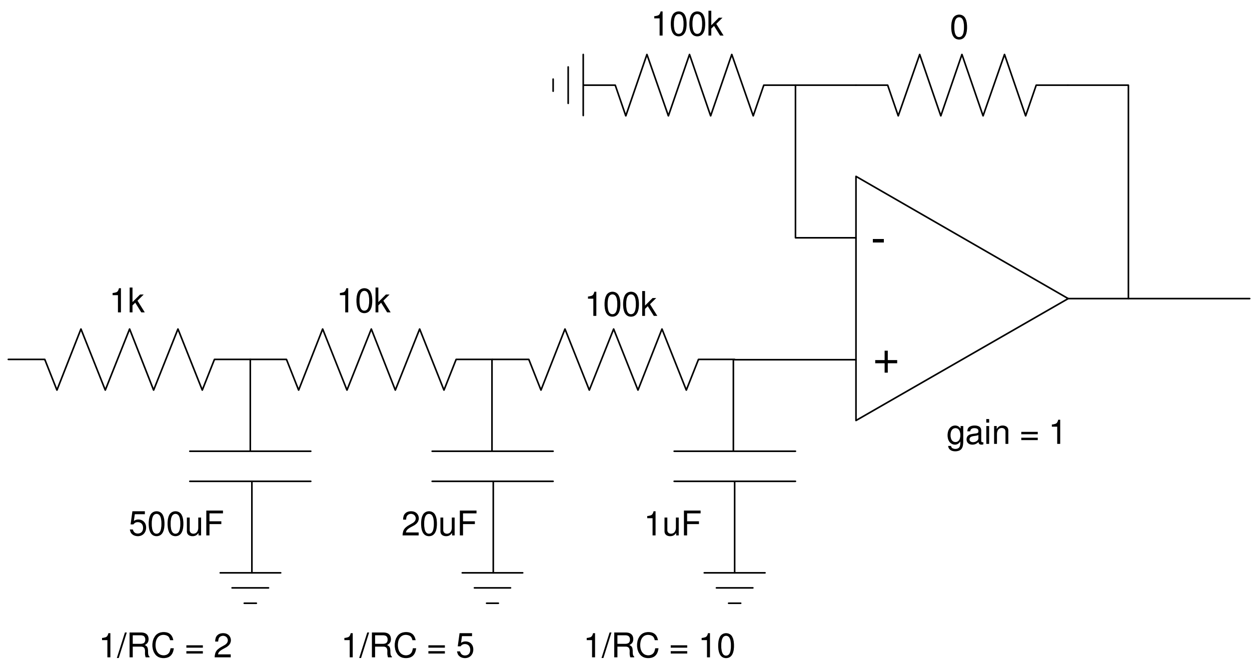
3) Filter Implementation: Design an op-amp circuit to implement the following filter:

$$Y = \left(\frac{100}{(s+2)(s+5)(s+10)} \right) X$$

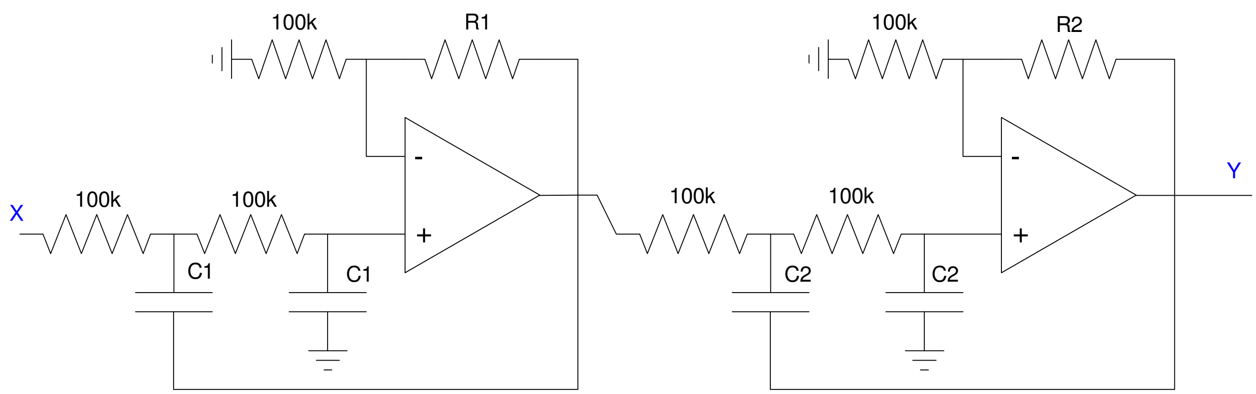
There are several answers. This is one of them



4) Filter Implementation: Design a circuit to implement the following Butterworth filter:

$$Y = \left(\frac{k}{(s+30\angle\pm 22.5^0)(s+30\angle\pm 67.5^0)} \right) X$$

C1	R1	C2	R2	Resulting DC gain
333nF	**15.2k**	**333nF**	**123.5k**	**2.575**



$$\left(\frac{1}{RC_1}\right) = 30$$

$$C_1 = 333nF$$

$$\left(\frac{1}{RC_2}\right) = 30$$

$$C_2 = 333nF$$

$$3 - k = 2\cos(22.5^0)$$

$$k = 1.152$$

$$k = 1 + \frac{R_1}{100k}$$

$$R_1 = 15.2k$$

$$3 - k = 2\cos(67.5^0)$$

$$k = 2.235$$

$$R_2 = 123.5k$$

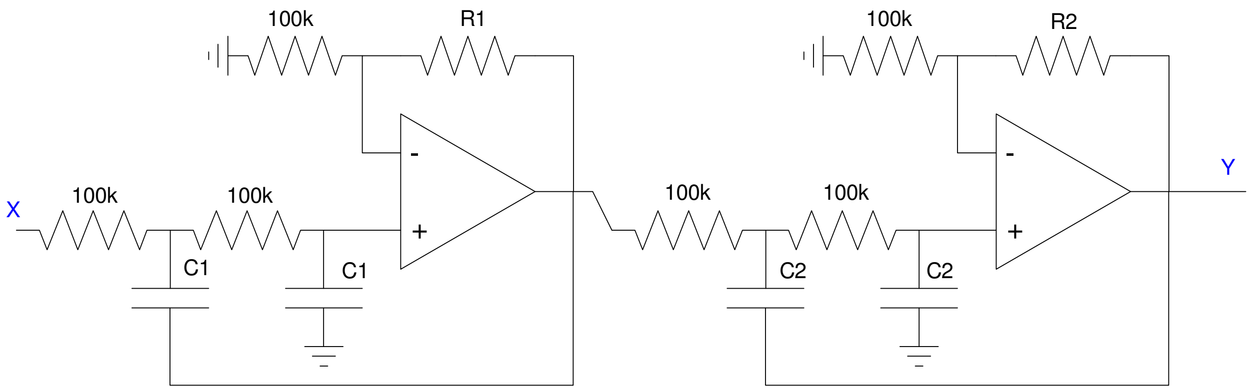
Resulting DC gain

$$DC = k_1\ k_2 = 1.152 \cdot 2.235 = 2.575$$

5) Filter Implementation: Design a circuit to implement the following Chebychev filter:

$$Y = \left(\frac{k}{\left(s+20\angle\pm38.5^0\right)\left(s+33.3\angle\pm77.8^0\right)} \right) X$$

C1	R1	C2	R2	Resulting DC gain
500nF	**43.5k**	**300nF**	**157.7k**	**3.698**



$$\left(\frac{1}{RC_1}\right) = 20$$

$$C_1 = 500nF$$

$$\left(\frac{1}{RC_2}\right) = 33.3$$

$$C_2 = 300nF$$

$$3 - k = 2\cos(38.5^0)$$

$$k = 1.435$$

$$R_1 = 43.5k$$

$$3 - k = 2\cos(77.8^0)$$

$$k = 2.577$$

$$R_2 = 157.7k$$

Resulting DC gain = 1.435 * 2.577 = 3.698

6) Design an analog computer to implement

$$Y = \left(\frac{3s+50}{(s+2)(s+4)(s+6)}\right)X = \left(\frac{3s+50}{s^3+12s+44s+48}\right)X$$