## ECE 321 - Homework \#3

Calibration, Filter Circuits, and Frequency Response. Due Monday, April 20th

Please make the subject "ECE $321 \mathrm{HW} \# 3$ " if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

## Calibration

Problem $1 \& 2$ ) Assume you are using a thermistor where the temperature - resistance relationship is

$$
R=1000 \exp \left(\frac{3905}{T}-\frac{3905}{298}\right) \Omega
$$

along with a voltage divider ( 10 V source, 2 k resistor:

$$
V=\left(\frac{R}{R+2000}\right) \cdot 10 V
$$

1) Determine a calibration function of the form

$$
T \approx a V+b
$$

to estimate temperature over the range of $(0 \mathrm{C},+30 \mathrm{C})$. What is the maximum error in this calibration function?

```
T = [0:0.1:30]';
R = 1000 * exp(3905 ./ (T + 273) - 3905/298);
V = R ./ (R + 2000) * 10;
B = [V, V.^0];
A = inv(B'*B)*B'*T
    - 8.7698745
    54.106726
max(T - B*A)
    ans = 1.0738336 maximum error on the high side
min(T - B*A)
    ans = - 0.4660400 maximum error on the low side
```


2) Determine a calibration function of the form

$$
T \approx a V^{3}+b V^{2}+c V+d
$$

to estimate temperature over the range of $(0 C,+30 C)$. What is the maximum error in this calibration function?

```
B = [V.^3, V.^2, V, V.^0];
A = inv(B'*B)*B'*T
    - 0.1489113
        2.4763568
    - 21.769242
        75.579249
max(T - B*A)
    ans=0.0353359
min(T - B*A)
    ans = - 0.0151326
```



## Filters

3) Assume $X$ and $Y$ are related by the following transfer function:

$$
Y=\left(\frac{30}{(s+3)(s+8)}\right) X
$$

a) What is the differential equation relating x and y ?

$$
\begin{aligned}
& \left(s^{2}+11 s+24\right) Y=30 X \\
& \frac{d^{2} y}{d y^{2}}+11 \frac{d y}{d t}+24 y=30 x
\end{aligned}
$$

b) Determine $y(t)$ assuming

$$
x(t)=5+6 \cos (4 t)
$$

Use superposition

$$
\begin{aligned}
\mathrm{x}(\mathrm{t})= & 5 \\
& \mathrm{~s}=0 \\
\mathrm{X} & =5+\mathrm{j} 0 \\
& Y=\left(\frac{30}{(s+3)(s+8)}\right)_{s=0}(5+j 0) \\
& Y=6.25 \\
& y(t)=6.25
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{x}(\mathrm{t})= 6 \cos (4 \mathrm{t}) \\
& \mathrm{s}=\mathrm{j} 4 \\
& \mathrm{X}=6+\mathrm{j} 0 \\
& Y=\left(\frac{30}{(s+3)(s+8)}\right)_{s=j 4}(6+j 0) \\
& Y=0.720-j 3.960 \\
& y(t)=0.720 \cos (4 t)+3.960 \sin (4 t)
\end{aligned}
$$

The total answer is then

$$
y(t)=6.25+0.720 \cos (4 t)+3.960 \sin (4 t)
$$

## Filter Design using fminsearch()

4) Design a filter of the form

$$
Y=\left(\frac{a}{\left(s^{2}+b s+c\right)\left(s^{2}+d s+e\right)}\right) X
$$

to give a gain vs. frequency as close to $\mathrm{Gd}(\mathrm{s})$ as possible over the range of $(0,10) \mathrm{rad} / \mathrm{sec}$.

$$
G_{d}(j \omega)= \begin{cases}1 & 0<\omega<6 \\ 0 & \text { otherwise }\end{cases}
$$

Plot your filter's actual frequency response vs. it's ideal response (given by Gd ).

```
Z,e] = fminsearch('costF',[130,2,5,2,26])
Z = 181.1933 3.2716 7.0623 1.2414 27.4180
e = 1.9369
```

meaning

$$
G(s)=\left(\frac{181.1933}{\left(s^{2}+3.2716 s+7.0623\right)\left(s^{2}+1.2414 s+27.4180\right)}\right)
$$



Code:

```
function [ J ] = costF( z )
    a = z(1);
    b = z(2);
    c = z(3);
    d = z(4);
    e = z(5);
    w = [0:0.1:10]';
    s = j*W;
    Gd = 1 .* (w < 6);
    num = a;
    den = (s.^2 + b*s + c).*(s.^2 + d*s + e);
    Gs = num ./ den;
    e = abs(Gd) - abs(Gs);
    J = sum(e.^2);
    plot(w,abs(Gd),'b',w,abs(Gs),'r');
    pause(0.01);
    end
```

5) Design circuit to implement the filter you designed in problem \#4

$$
G(s)=\left(\frac{181.1933}{\left(s^{2}+3.2716 s+7.0623\right)\left(s^{2}+1.2414 s+27.4180\right)}\right)
$$

Put in polar form

$$
G(s)=\left(\frac{181.1933}{\left(s+2.657 \angle \pm 52.01^{0}\right)\left(s+5.2362 \angle \pm 83.19^{0}\right)}\right.
$$

Stage 1:

$$
\begin{array}{ll}
\left(\frac{1}{R C}\right)=2.657 & \left(\frac{1}{R C}\right)=5.2362 \\
C=1 \mu F & C=1 \mu F \\
R=376 k \Omega & R=191 k \Omega \\
3-k=2 \cos \left(52.01^{0}\right) & 3-k=2 \cos \left(83.19^{0}\right) \\
k=1.769 & k=2.763
\end{array}
$$



Note: The DC gain of this filter is 4.58 .

- Reduce the gain (add a voltage divider) to make the DC gain match, or
- Label the output as 4.58 Y

The latter is better: you're probably going to add gain somewhere. This filter provides a DC gain of 4.58. The rest of the circuit adds the rest.
6) Check your filter using CircuitLab



