ECE 321 - Homework #4

Butterworth & Chebychev filters, Analog Computers. Due Monday, April 27th

Please make the subject "ECE 321 HW#4" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

Butterworth and Chebychev Filters

1) Design a filter, G(s), to meet the following specifications:

Input:

- 20 - 1 kHz audio signal
- 10Vpp, capable of driving 10mA @ 5V

Output:

- 20 - 1 kHz audio signal
- Capable of driving 10mA @ 5V

Relationship:

- 0.9 < Gain < 1.1   0 < w < 250 Hz
- Gain < 0.1   w > 400 Hz

First, determine the number of poles you need:

$$\left(\frac{250hz}{400hz}\right)^2 < 0.1$$

$$n > 4.899$$

Let n = 5 (kind of pushing it)

Assume a Chebychev filter. Pick the corner to be 250Hz (500 pi rad/sec). For a corner at 1 rad/sec

$$G(s) = \left(\frac{k}{(s+0.48)\left(s+0.76\angle\pm59.3^0\right)\left(s+1.06\angle\pm82.0^0\right)}\right):$$

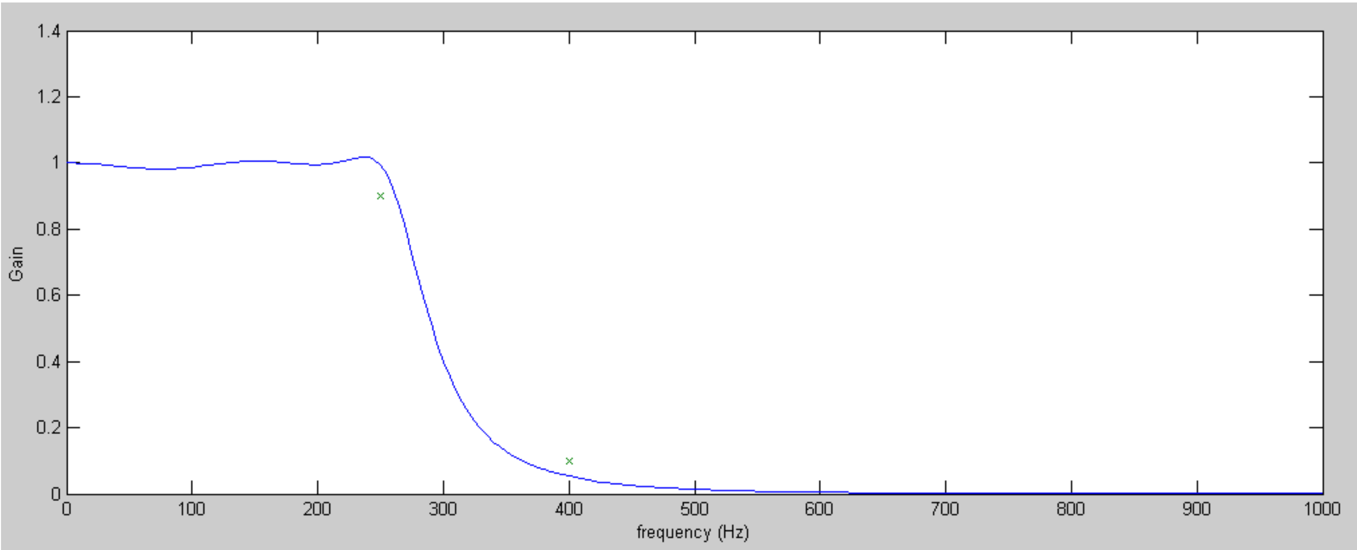
For a corner at 500 pi

$$G(s) = \left(\frac{k}{(s+754)\left(s+1194\angle\pm59.3^0\right)\left(s+1665\angle\pm82.0^0\right)}\right)$$

Pick k to make the gain at DC equal to 1.000. Checking this filter in Matlab

```
f = [0:5:1000]';
w = 2*pi*f;
s = j*w;
p1 = 754;
p2 = 1194*exp(j*59.3*pi/180);
p3 = conj(p2);
p4 = 1665*exp(j*82*pi/180);
p5 = conj(p4);
num = abs(p1*p2*p3*p4*p5);
Gs = num ./ ( (s+p1).*(s+p2).*(s+p3).*(s+p4).*(s+p5) );
plot(f,abs(Gs),[250,400],[0.9,0.1],'x');
xlabel('frequency (Hz)');
ylabel('Gain');
```

Plot your filter's gain vs. frequency using Matlab (or similar program)



Kind of got lucky: this meets the specs with the first guess.

So...

$$G(s) = \left(\frac{k}{(s+754)\left(s+1194\angle\pm59.3^0\right)\left(s+1665\angle\pm82.0^0\right)} \right)$$

2) Design an op-amp circuit to implement your filter from problem #1

$$G(s) = \left(\frac{k}{(s+754)\left(s+1194\angle\pm59.3^0\right)\left(s+1665\angle\pm82.0^0\right)} \right)$$

Assume 10k - 100k - 100k resisitors.

Stage 1: RC filter

Pole = 754

$$\left(\frac{1}{RC}\right) = 754$$

R = 10k

C = 132nF

Stage 2: Poles = $1194\angle\pm59.3^0$

$$\left(\frac{1}{RC}\right) = 1194$$

R = 100k

C = 8.37nF

$$3 - k = 2\cos(59.3^0)$$

$$k = 1.98$$

R1 = 100k, R2 = 98k

Stage 3: Poles = $1665\angle\pm82.0^0$

$$\left(\frac{1}{RC}\right) = 1665$$

R = 100k

C = 6.01nF

$$3 - k = 2\cos(82^0)$$

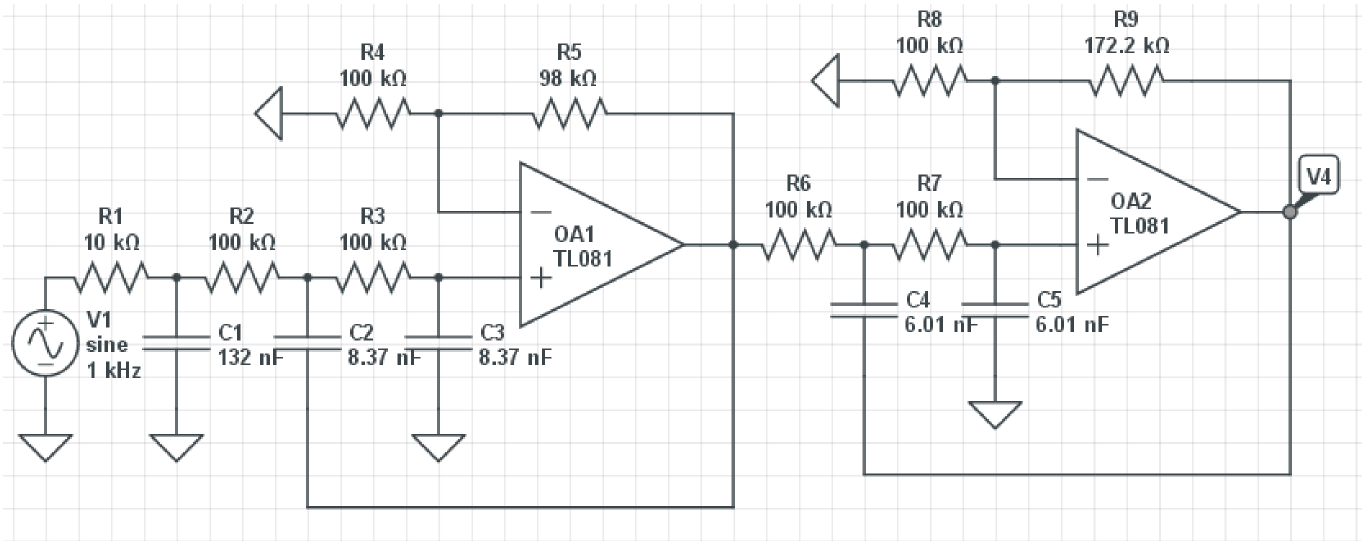
172.6k

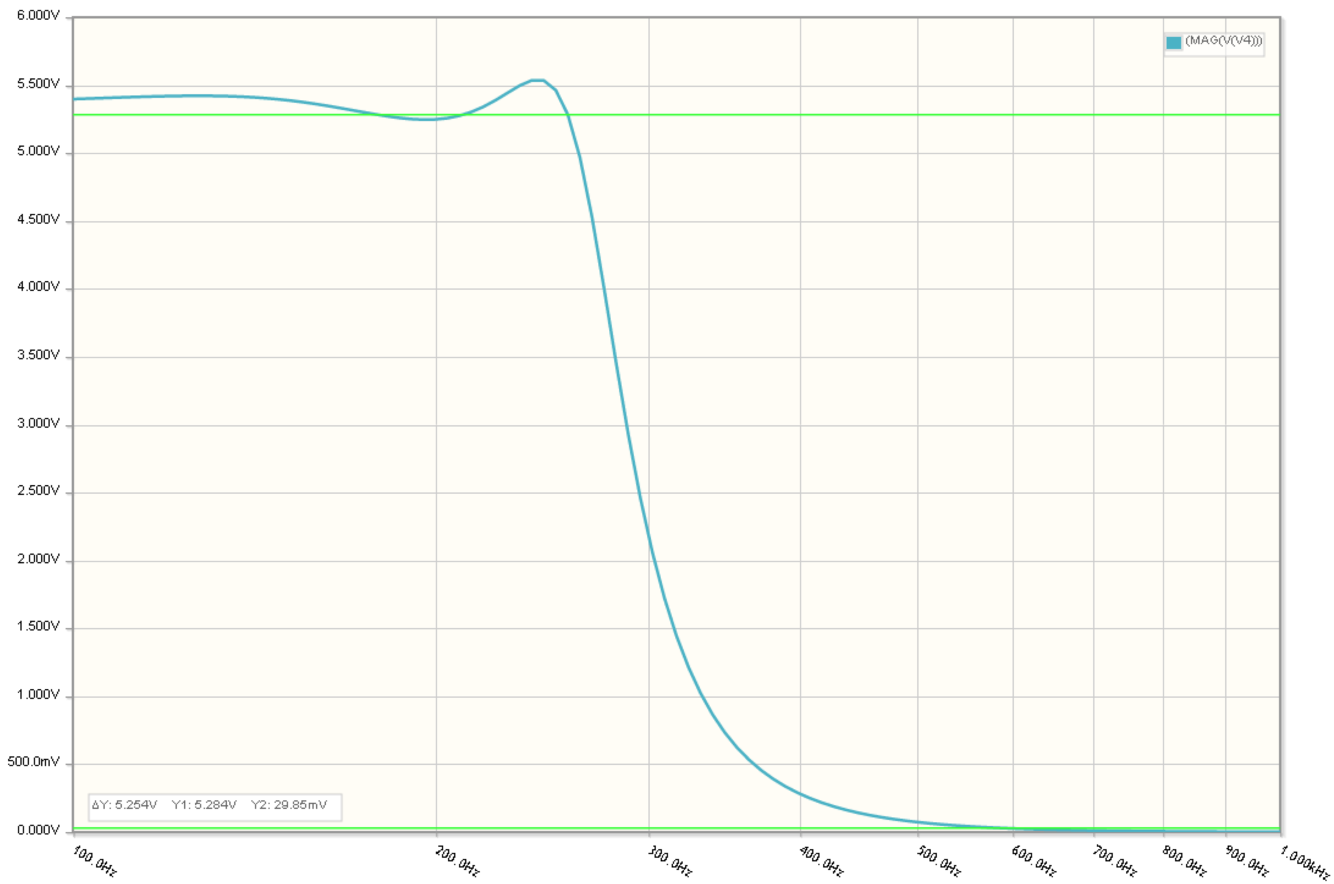
k = 2.7216

R1 = 100k, R2 = 172.16k

3) Verify your design in CircuitLab

- Check the gain at the design points (250Hz and 400Hz)
- Check the gain at two other points (100Hz and 1000Hz)





Analog Computers

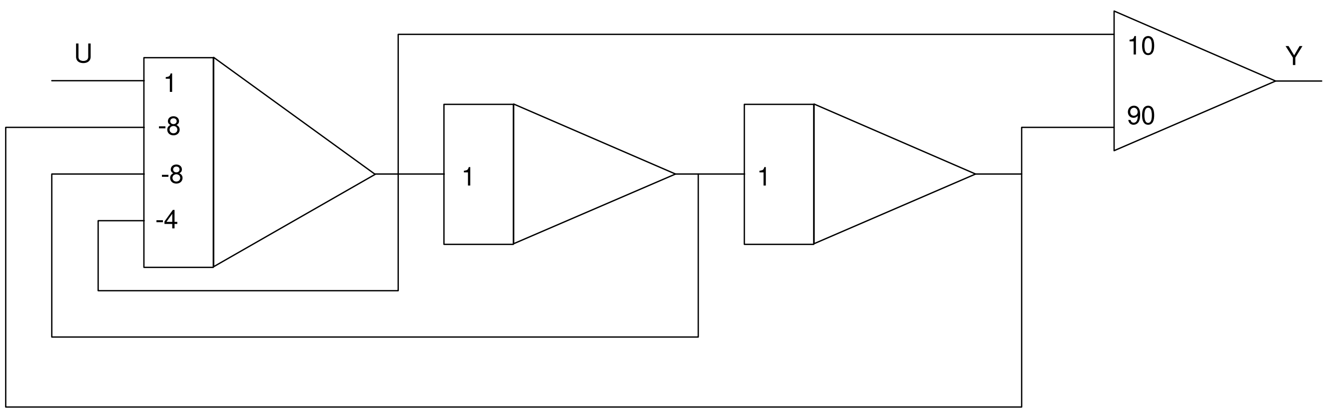
4) Design an analog computer to implement

$$Y = \left(\frac{10(s^2+9)}{(s+2)\left(s^2+2s+4\right)} \right) X$$

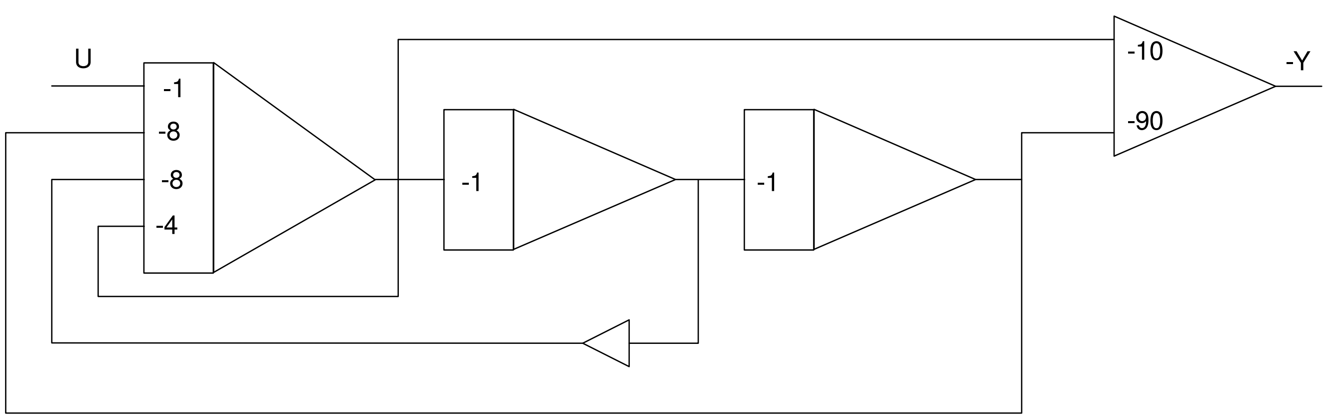
Multiply out

$$Y = \left(\frac{10s^2+90}{s^3+4s^2+8s+8} \right) X$$

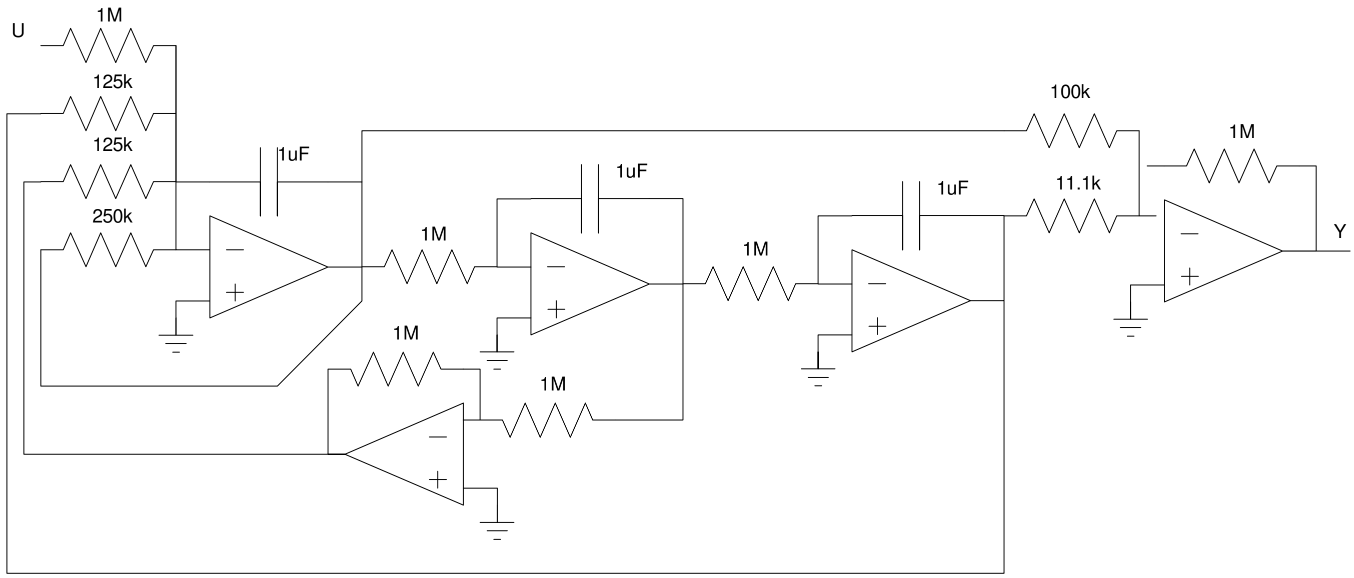
Using analog computer notation



Make all gains negative



Replace with an op-amp circuit


U
1M
125k
125k
250k
1uF
1M
1uF
1M
1uF
1M
1M
100k
11.1k
1M
Y