ECE 321 - Quiz #3 - Name ______________________

Filters - Spring 2023

1) Assume X and Y are related by the following transfer function

$$Y = \left(\frac{200}{(s+2)(s+7)}\right)X$$

Find y(t) assuming

$$x(t) = 10 + 5\cos(5t) + 14\sin(5t)$$

DC

s = 0

X = 10

$$Y = \left(\frac{200}{(s+2)(s+7)}\right)_{s=0} \cdot 10$$

$$Y = 142.857$$

AC

s = j5

X = 5 - j14

$$Y = \left(\frac{200}{(s+2)(s+7)}\right)_{s=j5} \cdot (5 - j14)$$

$$Y = -63.840 - j6.617$$

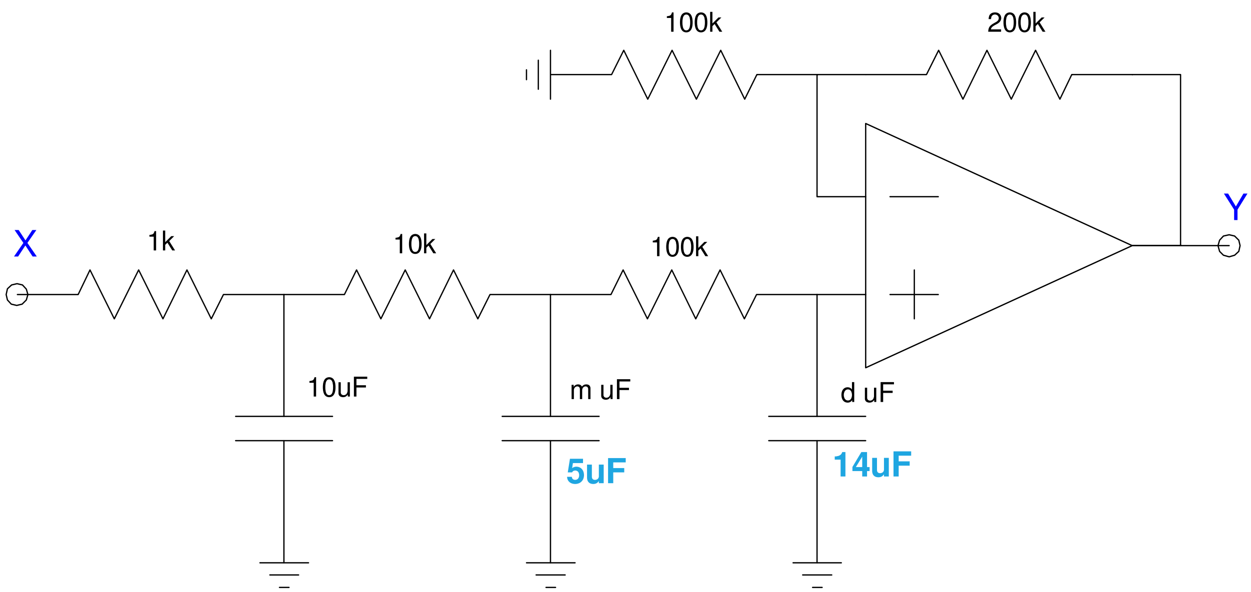
$$y(t) = -63.840\cos(5t) + 6.617\sin(5t)$$

The total answer is DC + AC

$$\boxed{y(t) = 142.857 - 63.840\cos(5t) + 6.617\sin(5t)}$$

2) Determine the transfer function for the following filter. Assume

- m is your birth month (1..12) (C2 = 1..12 uF)
- d is your birth date (1..31) (C3 = .. 31 uF)



pole 1

$$\frac{1}{R_1 C_1} = \frac{1}{(1k)(10\mu)} = 100$$

pole 2

$$\frac{1}{R_2 C_2} = \frac{1}{(10k)(5\mu)} = 20$$

pole 3

$$\frac{1}{R_3 C_3} = \frac{1}{(100k)(14\mu)} = 0.714$$

DC gain

$$DC = 1 + \frac{200k}{100k} = 3$$

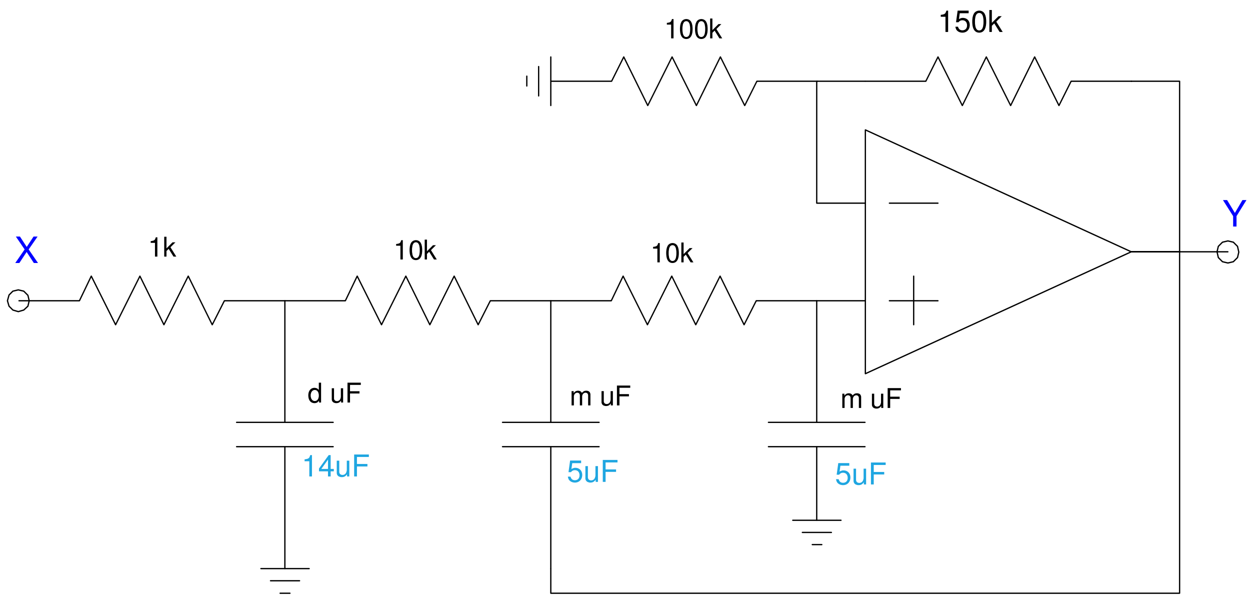
The transfer function is then

$$Y = \left(\frac{3 \cdot 1000 \cdot 20 \cdot 0.714}{(s+1000)(s+20)(s+0.714)} \right) X$$ *format used in controls*

$$Y = \left(\frac{3}{\left(\frac{s}{1000}+1\right)\left(\frac{s}{20}+1\right)\left(\frac{s}{0.714}+1\right)} \right) X$$ *format used in communications*

3) Determine the transfer function for the following filter. Assume

- m is your birth month (1..12) (C2 = C3 = 1..12 uF)
- d is your birth date (1..31) (C1 = 1..31 uF)



1st pole (real)

$$\frac{1}{R_1C_1} = \frac{1}{(1k)(14\mu F)} = 71.429$$

2nd pole (complex)

$$|pole| = \frac{1}{R_2C_2} = \frac{1}{(10k)(5\mu F)} = 20$$

$$k = 1 + \frac{150k}{100k} = 2.5$$

$$3 - k = 2\cos\theta$$

$$\theta = 75.52^0$$

$$Y = \left(\frac{2.5(71.429)(20)(20)}{(s+71.429)\left(s+20\angle 75.52^0\right)\left(s+20\angle -75.52^0\right)} \right)$$

4) Give the transfer function for a filter which meets the following requirements

- 0.9 < gain < 1.1 for frequencies below 80 rad/sec
- gain < 0.2 for frequencies above 110 rad/sec

Number of poles needed:

$$\left(\frac{80}{110}\right)^n = 0.2$$

$$n = 5.054$$

Let n=6

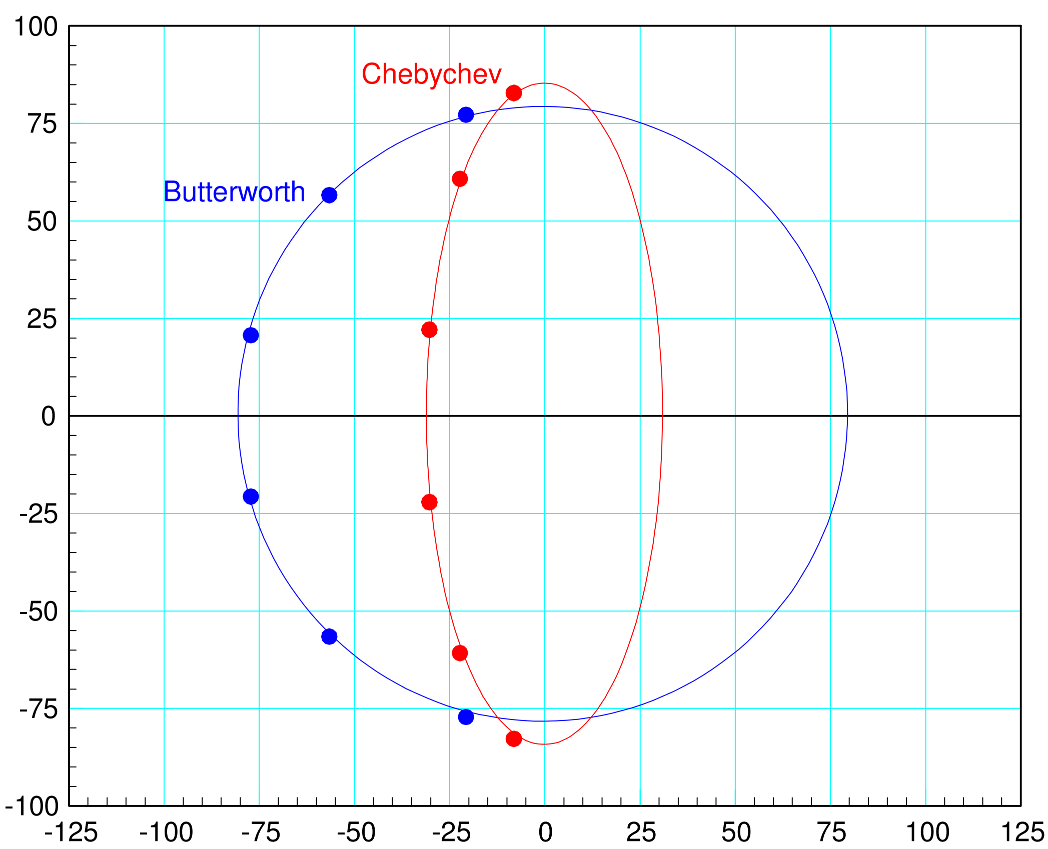
Assume a Butterworth filter with a corner at 80 rad/sec

$$G(s) = \left(\frac{k}{\left(s+80\angle\pm15^0\right)\left(s+80\angle\pm45^0\right)\left(s+80\angle\pm75^0\right)}\right)$$

Assume a Chebychev filter with a corner at 80 rad/sec

$$G(s) = \left(\frac{k}{\left(s+37.6\angle\pm36.1^0\right)\left(s+64.8\angle\pm69.8^0\right)\left(s+83.2\angle\pm84.4^0\right)}\right)$$

Either solution is OK. Given time and Matlab, you can adjust the poles



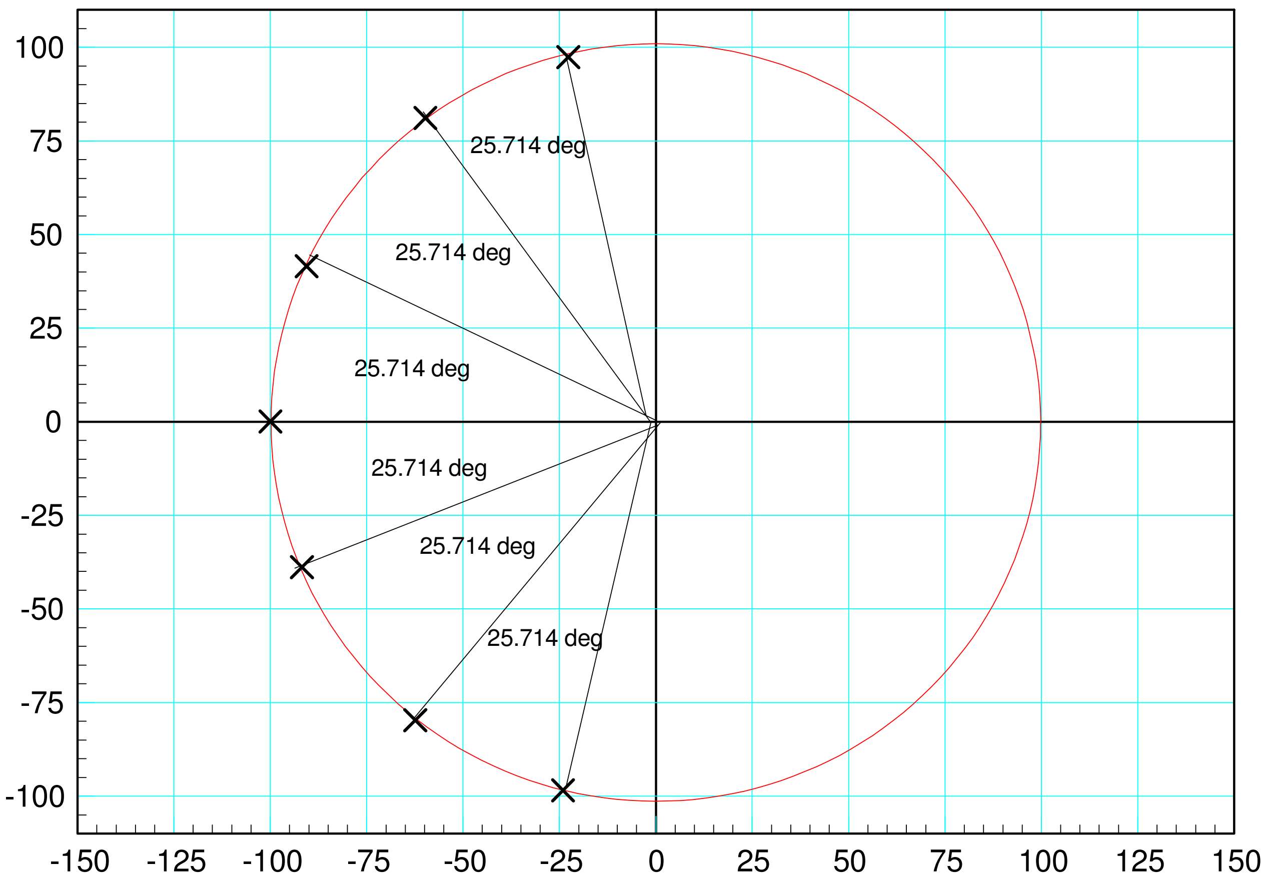
5) Give the transfer function for a 7th-order Butterworth low-pass filter with a corner at 100 rad/sec

Angle of poles is

$$\theta = \frac{180^0}{7} = 25.714^0$$

so

$$G(s) = \left(\frac{100^7}{(s+100)\left(s+100\angle\pm25.71^0\right)\left(s+100\angle\pm51.43^0\right)\left(s+100\angle\pm77.14^0\right)} \right)$$

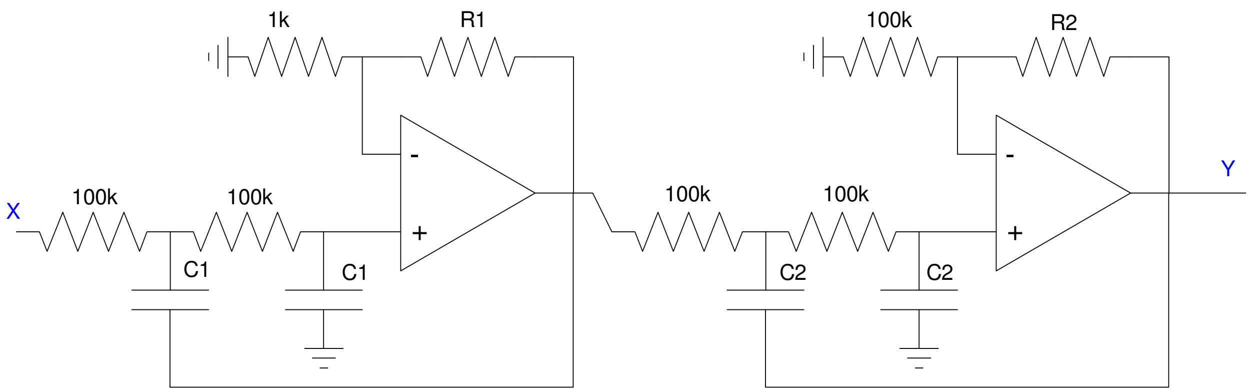


6) The transfer function for a 4th-order Chebychev filter with a corner at 100 rad/sec is

$$G(s) = \left(\frac{k}{\left(\left(s+72\angle\pm38.5^0\right)\left(s+111\angle\pm77.8^0\right)\right)} \right)$$

Find the R's and C's to implement this filter as well as the resulting DC gain

C1	R1	C2	R2	DC Gain
138.9nF	**435** or 43.5k with a 100k	**90.09nF**	**157.7k** or 1577 with a 1k	**3.698** 1.435 * 2.577



$$\frac{1}{RC_1} = 72$$

R = 100k

C1 = 138.9nF

$$3 - k = 2\cos\theta$$

$$\theta_1 = 38.5^0$$

$$k = 1.435$$

$$R_1 = 43.5k$$

$$\frac{1}{RC_2} = 111$$

R = 100k

C2 = 90.09nF

$$3 - k = 2\cos\theta$$

$$\theta = 77.8^0$$

$$k = 2.577$$

$$R_2 = 157.7k$$