## Transistor Amplifiers: DC Analysis

## Background:

Transistors can operate in three states:

- Off \& Saturated: Used when operating as a switch
- Active: Used when operating as an amplifier.

In active mode, the colelctor current is

$$
I_{C E}=\beta I_{B E}
$$

subject to the constraint

$$
V_{C E}>V_{C E: \min }
$$

or

$$
\beta I_{B E}<\max \left(I_{C E}\right) .
$$

For a transistor to act as an amplifier,

- You first have to bias the transistor to place it in the active region.
- Next, connect the base, collector, and emitter to the outside world with capacitors. This prevents the rest of the circuit from messing up the bias on the transistor.
- One of these three (base, emitter, collector) goes to the input, one goes to the output, and the third is grounded. This creates different types of amplifiers we'll discuss in a few days.

This lecture concentrates on biasing a transistor into the active region.

## NPN Transistor Amplifier Ciruit:

The basic amplifier we'll be using is as follows. The capacitors will be added in lecture 21.


To start, assume $\mathrm{Re}=0$. Taking the Thevenin equivalent of R1 and R2: (One theme in ECE 321 is the use of Thevenin equivalents. You don't have to do this, but if you get use to using Thevenin equivalents, problems in electronics become much simpler.)


The idea behind Thevenin equivalents is this is a linear circuit: meaning the VI characteristics are a straignt line. Any circuit which has the same VI characteristic behaves the same as far as the output is concerned.

The simplest circuit to give a linear VI charactistic is a single voltage source and a single resistor. So, let's replace the more complex circuit with the simpler one. (If they behave the same, why not use the simpler one for analysis?)

First, lets find Vth. For the circuit on the right, you find Vth by leaving Vb open and measuring the voltage to ground. Doing the same test on the circuit to the left results in Vth:

$$
V_{b}=\left(\frac{R_{2}}{R_{1}+R_{2}}\right) V_{c c}=V_{t h}
$$

Next, measure Rth. To do this, turn off the power supply (set Vth=0) and measure the resistance from Vb to ground. Doing this to the circuit to the left results in R1 and R2 being in parallel, both connecting Vb to ground:

$$
R_{t h}=R_{1} \| R_{2}=\left(\frac{R_{1} R_{2}}{R_{1}+R_{2}}\right)
$$

Now, lets go back to the transistor circuit and replace R1 and R2 with their Thevenin equivalent. Also, let's repalce the transistor with its model in the active mode: (sorry - my editor doesn't allow greek letters in figures. The current gain should be $\beta I_{b}$. Assume $\beta=200$ ).


Let's look at the load line for Ic and Vce.

- If Ic $=0 \mathrm{~V}$, Vce $=+12 \mathrm{~V}$.
- If Vce $=0 \mathrm{~V}$, Ic $=12 \mathrm{~mA}$.
- This is a linear circuit, so the VI characteristics map a line:


Depending upon what Ic is, you can place Vce anywhere you like on the load line.

- If you want the transistor to turn off, set $\mathrm{Ic}=0$ (meaning $\mathrm{Ib}=0$ ). The transistor is acting like a switch that's off.
- If you want the transistor to turn on, set Ic $>12 \mathrm{~mA}$ (meaning Ib $>6 \mathrm{uA}$ ). The transistor is acting like a switch that's on.
- If you want the transistor to be in the active mode and behave like an amplifier, pick your operating point (termed the Q point) somewhere between $\mathrm{Vce}=0 \mathrm{~V}$ and $\mathrm{Vce}=+12 \mathrm{~V}$.

Shortly, we'll be adding an AC signal to this amplifier. This results in the design point swinging up and down. To allow maximum swing, let's pick the Q point to be ( $6 \mathrm{~V}, 6 \mathrm{~mA}$ ).

Now, pick Vth and Rth to set the Q point to 6 V and 6 mA :

$$
\begin{aligned}
& \beta I_{b}=I_{c}=6 m A \\
& I_{b}=30 \mu A
\end{aligned}
$$

and

$$
I_{b}=\frac{V_{b}-0.7 V}{R_{b}}=30 \mu \mathrm{~A}
$$

Here you have one equation for two unknowns. This lets you set one of the unknowns at will. Let $\mathrm{Vb}=$ +12 V , resulting in

$$
\begin{aligned}
V_{b} & =12 \mathrm{~V} \\
R_{b} & =377 \mathrm{k} \Omega
\end{aligned}
$$

Converting back to R1 and R2:

$$
\begin{aligned}
& V_{b}=\left(\frac{R_{2}}{R_{1}+R_{2}}\right) 12 \mathrm{~V}=12 \mathrm{~V} \\
& R_{2}=\infty \\
& R_{b}=R_{1} \| R_{2}=377 \mathrm{k} \Omega \\
& R_{1}=377 \mathrm{k} \Omega
\end{aligned}
$$

so your circuit is:


## Q Point Stabilization

This circuit isn't very good to to variance in the gain, $\beta$. Transistors typically have a wide variance in gain. A 2 n 222 transistor, for example has a gain of $200 \pm 100$. This results in the Q point being between: If you compute the Q point as a funciton of $\beta$ :

$$
\begin{aligned}
& I_{b}=\left(\frac{12 V-0.7 V}{377 k}\right)=30 \mu A \\
& I_{c}=\beta I_{b} \\
& V_{c}=12-1000 I_{c}
\end{aligned}
$$

or Vc is somewhere between $9 \mathrm{~V}(\beta=100)$ and $3 \mathrm{~V}(\beta=300)$.
ou can do a Monte-Carlo simulation of this circuit as well. For a Monte-Carlo simulation, you select random values for each component and compute the results for those values. Assuming $1 \%$ tolerance on the resistors and $\pm 100$ for $\beta$ results in:

```
for i=1:100
    Rb = 377000 * (1 + (rand()*2-1)*0.01);
    Rc}=1000*(1+(rand()*2-1)*0.01)
    Beta = 200 + 100* (rand()*2-1);
    Ib = (12-0.7)/Rb;
    Ic = Beta*Ib;
    Vc = 12-Ic*Rc;
    plot(Vc,Ic*1000,'.');
    end
```

A scattergram of the resulting Q point is shown below. Note that there is a lot of variation in where you are operating.


To compensate for variations in $\beta$, add Re.


Writing the loop equations:

$$
\begin{aligned}
& I_{c}=\beta I_{b} \\
& -V_{b}+R_{b} I_{b}+0.7+R_{e}\left(I_{b}+I_{c}\right)=0
\end{aligned}
$$

or

$$
I_{b}=\left(\frac{V_{b}-0.7}{R_{b}+(1+\beta) R_{e}}\right)
$$

To stabilze the voltage at Vc :

$$
\begin{aligned}
& V_{c}=V_{c c}-R_{c} I_{c} \\
& V_{c}=V_{c c}-\left(\frac{\beta R_{c}}{R_{b}+(1+\beta) R_{e}}\right)\left(V_{b}-0.7\right)
\end{aligned}
$$

If

then

$$
V_{c} \approx V_{c c}-\left(\frac{\beta R_{c}}{(1+\beta) R_{e}}\right)\left(V_{b}-0.7\right)
$$

For $\beta$ large,

$$
\left(\frac{\beta}{1+\beta}\right) \approx 1
$$

so the voltage, Vc , is no longer affected by variations in $\beta$.

To stabilize the Q point, pick Re such that

$$
(1+\beta) R_{e} \gg R_{b}
$$

Going back to our original design, let's design for $\mathrm{Vc}=6 \mathrm{~V}$. Ideally, I'd like $\mathrm{Re}=0$ so that Vc can swing from 0 V to +12 V . $\mathrm{Re}=0$ results in a Q point that's sensirive to variations in $\beta$, however. So, instead, let Re be $1 / 10$ th of Rc (small to allow a large voltage swing but non-zero.)

$$
R_{e}=100 \Omega
$$

To stabize the Q point

$$
\begin{aligned}
& (1+\beta) R_{e} \gg R_{b} \\
& 20,100 \Omega \gg R_{b}
\end{aligned}
$$

Let

$$
R_{b}=2 k \Omega
$$

From before,

$$
\begin{aligned}
I_{c} & =6 m A \\
I_{b} & =30 \mu A
\end{aligned}
$$

so

$$
\begin{aligned}
I_{b} & =\left(\frac{V_{b}-0.7}{R_{b}+(1+\beta) R_{e}}\right)=30 \mu A \\
V_{b} & =1.363 \mathrm{~V}
\end{aligned}
$$

This corresponds to R1 and R2 being:

$$
\begin{aligned}
& \left(\frac{R_{1} R_{2}}{R_{1}+R_{2}}\right)=2000 \\
& \left(\frac{R_{2}}{R_{1}+R_{2}}\right) 12 \mathrm{~V}=1.363 \mathrm{~V}
\end{aligned}
$$

and

$$
\begin{aligned}
& R_{1}=17.6 \mathrm{k} \Omega \\
& R_{2}=2256 \Omega
\end{aligned}
$$

So your circuit is as shown to the right. If the gain, $\beta$, varies, the Q point varies as:

```
- }\beta=100: Vc=6.52V
- }\beta=200: Vc=6.00\textrm{V
- }\beta=300: Vc=5.80V
for i=1:100
    R1 = 17600* (1 + (rand()*2-1)*0.01);
    R2 = 2256 * (1 + (rand()*2-1)*0.01);
    Rc = 1000 * (1 + (rand()*2-1)*0.01);
    Re = 100 * (1 + (rand()*2-1)*0.01);
    Beta = 200 + 100*(rand()*2-1);
    Vb = 12*(R2 / (R1+R2));
    Rb = 1/(1/R1 + 1/R2);
    Ib = (Vb-0.7) / (Rb + (1+Beta)*Re);
    Ic = Beta*Ib;
    Vc = 12 - Rc*Ic;
    plot(Vc,Ic*1000,'.');
    end
```



The Q point can still vary a bit - bit it's a lot better than before.

