Butterworth, Elliptic, & Chebychev Filters ECE 321: Electronics II Lecture #9:

Please visit Bison Academy for corresponding lecture notes, homework sets, and solutions

Butterworth, Elliptic, Chebychev Filters

Objective:

- Know what each filter tries to optimize
- Know how these filters compare

Filter Types:

- RC
- Butterworth
- Chebychev (Type-1)
- Elliptic (Type-2 Chebychev)

RC Filter

Closest approximation to an ideal low pass filter subject to

- There are N poles
- All poles are real
- There are no zeros
- The maximum gain is 1.000

An n-pole RC filter has all n-poles on the real axis.

- It's advantage is you can build it with a passive RC filter (good)
- It's problem is it's a pretty poor filter.

RC Filter Example:

$$G(s) = \left(\frac{10}{s+10}\right)^n$$

As n increases...

- The high-frequency gain gets smaller and smaller (good)
- The gain below 5 rad/sec starts to droop more an more (bad)



5th-Order RC Filter

- Hardware Design
- 1/RC = 10
- R increases 10x each stage to reduce loading



CircuitLab Simulation: 5th-Order RC Filter

- Frequency Sweep
- Not a very good filter



Butterworth Filter:

Closest approximation to an ideal low pass filter subject to

- There are N poles
- There are no zeros
- The maximum gain cannot exceed 1.0000

Solution:

- Magnitude of poles = corner frequency
- Angle between poles: $\phi = \frac{180^{\circ}}{N}$

Buttorworth Polo Locations: Corpor - 1 rad/soc					
	N=2 N=3 N=4 N=5 N=6				
	N=2	N=3	N=4	N=5	N=6
zeros	none	none	none	none	none
poles	$-1.0 \angle \pm 45^{\circ}$	-1.0	$-1.0 \angle \pm 22.5^{\circ}$	-1.0	$-1.0 \angle \pm 15^{0}$
		$-1.0 \angle \pm 60^{0}$	$-1.0 \angle \pm 67.5^{\circ}$	$-1.0 \angle \pm 36^{\circ}$	$-1.0 \angle \pm 45^{\circ}$
				$-1.0 \angle \pm 72^{0}$	$-1.4 \angle \pm 75^{\circ}$



The resulting gain vs. frequency is then



Circuit Implementation

- $\left(\frac{1}{RC}\right) = 10$
- $3-k=2\cos{(\theta)}$



CircuitLab Results

• Corner = 10 rad/sec (1.59 Hz)



Type-1 Chebychev Filter

Closest approximation to an ideal low pass filter subject to

- There are N poles
- There are no zeros
- The maximum gain is $(1 + \varepsilon)$. (Some ripple is permitted).

Type-1 Chebychev Filter Pole Locations: Corner = 1 rad/sec					
	N=2	N=3	N=4	N=5	N=6
zeros	none	none	none	none	none
poles	$-1.60 \angle \pm 50.7^{\circ}$	-0.85	$-0.72 \angle \pm 38.5^{\circ}$	-0.48	$-0.47 \angle \pm 36.1^{\circ}$
		$-1.21 \angle \pm 69.5^{\circ}$	$-1.11 \angle \pm 77.8^{\circ}$	$-0.76 \angle \pm 59.3^{\circ}$	$-0.81 \angle \pm 69.8^{\circ}$
				$-1.06 \angle \pm 82.0^{\circ}$	$-1.04 \angle \pm 84.4^{\circ}$

Example: Design a 5th-order Chebuchev filter, Corner = 10 rad/sec

$$G(s) = \left(\frac{4.8 \cdot 7.6^2 \cdot 10.6^2}{(s+4.8)(s+7.6 \neq \pm 59.3^0)(s+10.6 \neq \pm 82^0)}\right)$$





CircuitLab Implementation

- $\left(\frac{1}{RC}\right) = pole$
- $3-k=2\cos\theta$



CircuitLab: Frequency Sweep

• Gain is closer to 1.00 out to 1.59 rad/sec



Elliptic Filters: (Type-2 Chebychev)

Closest approximation to an ideal low pass filter subject to

- There are N poles
- There are N zeros
- The maximum gain cannot exceed $(1 + \varepsilon_1)$. (Some ripple is permitted).
- The maximum gain the band reject region cannot exceed ϵ_2

Pass	W1 = 0 to 1	W1 = 0 to 1	W1 = 0 to 1	W1 = 0 to 1
Reject	W2 = 9 to infinity	W2 = 3 to infinity	W2 = 1.7 to infinity	W2 = 1.3 to infinity
zeros	j 9.919	j3.246	j1.805	j1.316
		j7.705	j2.423	j1.524
				j2.491
poles	-0.391 + j1.242	-0.572 + j0.467	-0.807	-0.437
	-0.942	-0.221 + j1.076	-0.529 + j0.784	-0.331 + j0.583
			-0.151 + j1.095	-0.159 + j0.907
				-0.040 + j1.024



CircuitLab Implementation

• Can't implement complex zeros with circuits given

Frequency Scaling

The previous circuits had a corner at 10 rad/sec

• Redesign so that the corner is 100Hz

Change the capacitors

- Capacitors are integrators
- If you make the capacitor 10x smaller,
 - They integrate 10x faster
 - The corner frequency becomes 10x larger

To make the corner 100Hz

• Scale the capacitors by x 10/628

5th-Order Chebychev filter with a corner at 100Hz

• Capacitors scaled by 1 / 62.8



5th-Order Chebychev Low-Pass Filter

• CircuitLab simulation



Digikey

- Filters are really common and useful
- Digikey likewise sells 1001 filters on a chip



Results: 1,001

earch Within Results			
Filter Type	Frequency - Cutoff or Center	Number of Filters	Filter Order
	* 2kHz *] [1	2nd
Antialiasing	5kHz	2	2nd, 3rd
Band Pass	7kHz	3	4th
Band Pass, Low Pass	7.5kHz	4	5th
Bessel, Low Pass Switched Capacitor	10kHz	5	6th
Butterworth Low Pass	12kHz	6	7th
Butterworth or Bessel Low Pass	14kHz/20kHz	7	8th
Butterworth	15kHz		9th
Butterworth, Low Pass Switched Capacitor	20kHz		10th
Cauer, Low Pass Switched Capacitor	25kHz		
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Example: LTC 1164

- 8th-order Elliptic filter
- Corner frequency is set by a clock input
- Good for senior design (easy to implement)
- Not so good for Electronics II (doesn't demonstrate knowledge of electronics)



TYPICAL APPLICATION

Handout:

Find R and C to implement a Chebychev filter with a corner at 1 rad/sec

$$G(s) = \left(\underbrace{\frac{0.639}{(s+0.72 \neq \pm 38.5^{\circ})(s+1.11 \neq \pm 77.8^{\circ})}}_{(s+0.72 \neq \pm 38.5^{\circ})(s+1.11 \neq \pm 77.8^{\circ})} \right)$$

Solution

• To make the corner 628 rad/sec (100Hz), scale each C by 1/628



Summary

- If you are willing to use complex poles, you can do much better than an RC filter
- A Butterworth filter is one option:
 - Maximum gain at DC
 - Fairly straight forward design
- A Chebychev filter is another option
 - Slight resonance
 - Much better gain vs. frequency (closer to ideal)
 - Need to use a table (or Wikipedia) to design
- Changing the corner frequency is really easy
 - Scale the capacitors