# Analog Computers ECE 321: Electronics II 

## Lecture \#11

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

## Analog Computers

- Electric circuit to implement a differential equation
- Allows you to duplicate the dynamics of an expensive system using an inexpensive circuit

https://images.easytechjunkie.com/1949-electronic-analog-computer.jpg


## Analog Computer Application

## Jet Engine

https://www.airlineratings.com/did-you-know/how-is-a-jet-engine-tested/

- $\$ 10,000$ / minute to test
- \$4,000,000 if you go unstable



## Analog Computer

http://www.analogmuseum.org/english/collection/eai/180/1_small.jpg

- 10 cents / kWh to operated
- Red LED turns on if you go unstable



## Analog Computers

Design a circuit to implement a generic proper transfer function

$$
Y=\left(\frac{b_{n-1} s^{n-1}+b_{n-2} s^{n-2}+\ldots+b_{1} s+b_{0}}{s^{n}+a_{n-1} s^{s-1}+a_{n-2} s^{n-2}+\ldots+a_{1} s+a_{0}}\right) U
$$

Solution:
There are many. This is one way to do it. Just to make it more manageable, assume a 3rd-order system

$$
Y=\left(\frac{b_{2} s^{2}+b_{1} s+b_{0}}{s^{3}+a_{2} s^{2}+a_{1} s+a_{0}}\right) U
$$

Step 1: Change the problem. Create a dummy state, X

$$
\begin{aligned}
& X=\left(\frac{1}{s^{3}+a_{2} s^{2}+a_{1} s+a_{0}}\right) U \\
& Y=\left(b_{2} s^{2}+b_{1} s+b_{0}\right) X
\end{aligned}
$$

Step 2: Cross multiply and solve for the highest derivative of X:

$$
\begin{aligned}
& X=\left(\frac{1}{s^{3}+a_{2} s^{2}+a_{1} s+a_{00}}\right) U \\
& \left(s^{3}+a_{2} s^{2}+a_{1} s+a_{0}\right) X=U \\
& s^{3} X=-\left(a_{2} s^{2}+a_{1} s+a_{0}\right) X+U
\end{aligned}
$$

Step 3: Given $\mathrm{s}^{\mathrm{n}} \mathrm{X}$, solve for X by integrating n times (notation: $\mathrm{X}^{\prime}$ means dx/dt)


Step 4: Create X "' using the differential equation from step 2:

$$
s^{3} X=-\left(a_{2} s^{2}+a_{1} s+a_{0}\right) X+U
$$



Step 5: Now that you know X and its derivatives, create Y :

$$
Y=\left(b_{2} s^{2}+b_{1} s+b_{0}\right) X
$$



Step 6) Convert to analog computer notation. Here, a triangle means an amplifier:

whereas a triangle with a box means integrator:


$$
Y=(1 / s)(-3 A-4 B)
$$

## Applying this to the above block diagram:



## Make all gains negative.

- Keep the sign for all paths from U to Y
- Keep the sign for all loops
- You may have to add inverters


Now convert to an op-amp circuit.


Final Op-Amp Circuit: All units M Ohms and uF.

## Sidelight

This technique works well if the poles are close to 1.000
If the poles are not close to 1 ,

- Scale the poles so that they are close to 1.000
- Design the analog computer using the previous techniques
- Scale the circuit by making C larger (slower) or smaller (faster) to return to the original pole locations.


## Modern Analog Computers

Anything you can do in hardware you can do in software
Analog computers are now implemented in software

- LabView, Simulink, VisSim


