ECE 331 - Solution to Homework #1

Magnetics, Generated Voltage, Energy Conversion

1) An inductor has a reluctance of 1000 A-T/Wb. The inductor has 500 turns of copper wire and draws 5A when connected to +30VDC. Determine a) the core flux, b) the resistance of the wire.

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The resistance of the wire is
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$$R = \left(\frac{30V}{5A}\right) = 6\Omega$$

The core flux is

$$F = \phi R$$

$$(5A)(500\text{ turns})=\phi(100At/Wb)$$

$$\phi = 25.0Wb$$

2) A magnetic circuit has a length of 2m and a cross sectional area of $0.1m^2$. Excitation is provided by a 100-turn 50 Ohm coil. Determine the voltage required to establish a flux density of 2T. The reluctance of the magnetic circuit is 800 A-T/Wb.

$$B = 2T = 2\frac{Wb}{m^2}$$

$$\phi = \left(2\frac{Wb}{m^2}\right)(0.1m^2) = 0.2Wb$$

$$F = \phi R$$

$$(I)(100t) = (0.2Wb)(800At/Wb)$$

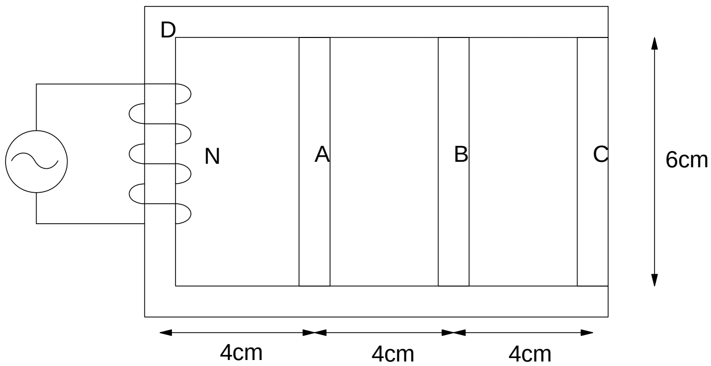
$$I = 1.6A$$

$$V = IR = (1.6A)(50\Omega) = 80V$$

3) A coil with 100 turns draws 3A. The material of the core has a cross sectional area of 3cm x 3cm for each part. Four different materials are used:

- A: Aluminum: permeability = 1
- B: Purified Iron: permeability = 180,000
- C: 4% Silicon Iron: permeability = 7,000
- D: 78-Permalloy: permeability = 800,000

The length of A/B/C are 4cm and the length of



Determine the flux in each section of the core.

The reluctance of each segment is:

RA: $R = \left(\frac{l}{\mu A}\right) = \left(\frac{0.06m}{(1)\left(4\pi\cdot 10^{-7}\right)(0.03m)^2}\right) = 53.05M$

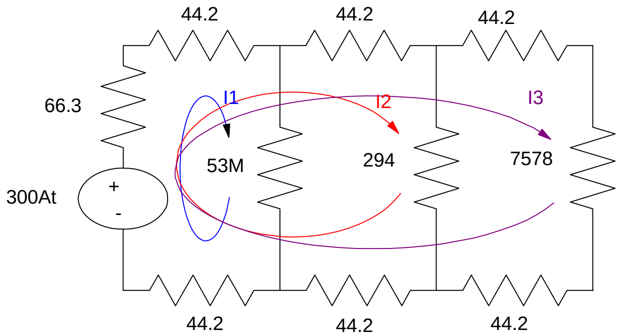
RB: $R = \left(\frac{l}{\mu A}\right) = \left(\frac{0.06m}{(180{,}000)\left(4\pi\cdot 10^{-7}\right)(0.03m)^2}\right) = 294$

RC: $R = \left(\frac{l}{\mu A}\right) = \left(\frac{0.06m}{(7000)\left(4\pi\cdot 10^{-7}\right)(0.03m)^2}\right) = 7578$

RD $R = \left(\frac{l}{\mu A}\right) = \left(\frac{0.06m}{(800{,}000)\left(4\pi\cdot 10^{-7}\right)(0.03m)^2}\right) = 66.3$

RDA $R = \left(\frac{l}{\mu A}\right) = \left(\frac{0.04m}{(800{,}000)\left(4\pi\cdot 10^{-7}\right)(0.03m)^2}\right) = 44.2$

So the circuit model is:



Solving:

$$(53M)I_1 + (154)I_2 + (154)I_3 = 300$$

$$(154)I_1 + (537)I_2 + (243)I_3 = 300$$

$$(154)I_1 + (243)I_2 + (7909)I_3 = 300$$

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-->A=[53e6,154,154;154,537,243;154,243,7909]
 A  =
    53000000.    154.    154.
    154.         537.    243.
    154.         243.    7909.

-->B=[300;300;300]
 B  =

    300.
    300.
    300.

-->I = inv(A)*B
 I  =
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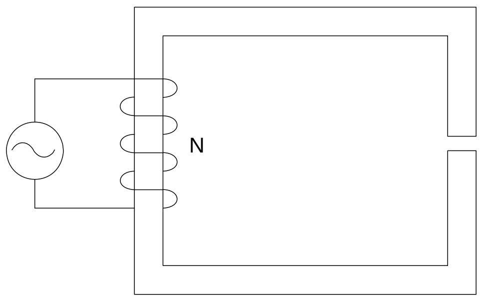
ϕ_A 0.0000040

ϕ_B 0.5491283

ϕ_C 0.0210597

4) The magnetic circuit below has a cross sectional area of 3cm x 3cm and a length of 20cm. The coil has a resistance of 60 Ohms and has 100 turns. The air gap is 1mm. Determine the battery voltage required to obtain flux density of 2T in the air gap.



$$B = 2T = 2\frac{Wb}{m^2}$$

$$\phi = \left(2\frac{Wb}{m^2}\right)(0.03m)^2 = 0.0018Wb$$

$$R = \frac{l}{\mu A} = \left(\frac{0.001m}{\left(4\pi\cdot10^{-7}\right)(0.03m)^2}\right) = 884,194$$

$$(I)(100t) = (0.0018Wb)(884194At/Wb)$$

$$I = 15.9A$$

$$V = IR = (15.9A)(60\Omega) = 954V$$

5) Assume the air gap is removed. The input is 240VDC, the resistance is 60 Ohms, and there are 100 turns. Determine a) the magnetic field intensity, b) the core flux density, c) the relative permeability of the core, and d) the reluctance of the magnetic circuit.

Assume Silicon iron (permiability = 7000).

$$I = \left(\frac{240V}{60\Omega}\right) = 4A$$

$$F = (4A)(100t) = 400At$$

$$R = \frac{l}{\mu A} = \frac{(0.2m)}{(7000)\left(4\pi\cdot10^{-7}\right)(0.03m)^2} = 25,262$$

$$\phi = \left(\frac{400At}{25262}\right) = 0.0158Wb$$

$$B = \left(\frac{0.0158Wb}{(0.03m)^2}\right) = 17.6\frac{Wb}{m^2}$$

6) Find the inductance of this device. Assume N = 100 turns, R = 60 Ohms, the cross sectional area is 3cm x 3cm, the length of the iron is 20cm, and the material is Silicon-steel.

$$L = \frac{N^2}{R} = \frac{(100t)^2}{25262} = 396mH$$

7) Find the hysteresis losses at a) 60Hz, b) 180 Hz, and c) 600Hz. Assume n=2. Note: Ke = 350/m^3 according to the CRC Handbook of Physics. Ke = 0.001 must be in cgs units or something like that...

a) 60Hz. First find the current.

$$i = \frac{240V}{R+j\omega L} = \frac{240}{60+j149} = 1.49\angle -68^0$$

The B field is then scaled down from 4A in problem #5:

$$B_m = \frac{NI}{RA} = \left(\frac{1.49A}{4A}\right)17.6T = 6.55T$$

The hysteresis losses are

$$P_h = vfK_hB_m^n$$

$$P_h = (0.00018m^3)(60Hz)\left(350\frac{J}{m^3}\right)(6.55T)^2 = 162W$$

b) 180Hz:

$$i = \frac{240V}{R+j\omega L} = \frac{240}{60+j447} = 0.53\angle -82^0$$

$$B_m = \left(\frac{0.53A}{4A}\right)17.6T = 2.34T$$

$$P_h = (0.00018m^3)(180Hz)\left(350\frac{J}{m^3}\right)(2.34T)^2 = 62W$$

c) 600Hz:

$$i = \frac{240V}{R+j\omega L} = \frac{240}{60+j1492} = 0.16\angle -87^0$$

$$B_m = \left(\frac{0.16A}{4A}\right)17.6T = 0.70T$$

$$P_h = (0.00018m^3)(600Hz)\left(350\frac{J}{m^3}\right)(0.70T)^2 = 18.7W$$

note:

- The flux density (B) drops as frequency goes up. That's why aircraft run at 400Hz - higher frequencies require less iron.
- These numbers are probably off. The core will be saturated for B > 1.5T, and these models won't work anymore.
- If you used Ke = 0.001, the losses are on the order of nW. If so, you don't need to worry about hysteresis losses and this section is of no practical importance. With Ke = 350 J/m^3, you get about half of the losses in I2R losses, half in Eddy current and hysteresis losses.

8) Find the eddy current losses at a) 60Hz, b) 180Hz, and c) 600Hz. Assume the lamination thickness is 1mm, and Ke = 0.001.

Using the equation given:

$$P_e = K_e f^2 B_m^2 \tau^2 v^2$$

60Hz: $P_e = (0.001)(60Hz)^2(6.55T)^2(0.001m)^2(0.00018m^3)^2 = 5.00pW$

180Hz: $P_e = (0.001)(180Hz)^2(2.34T)^2(0.001m)^2(0.00018m^3)^2 = 5.74pW$

600Hz: $P_e = (0.001)(600Hz)^2(0.70T)^2(0.001m)^2(0.00018m^3)^2 = 5.71pW$

I personally don't believe these numbers: if Eddy-current losses were really that small there would be no need to laminate the core. Probably the Ke given in the notes is for cgs units or something like that.

As a guess, let's assume the Eddy current losses at 60Hz are 1/2 of the hysteresis losses at 60Hz. That would give

$$Ke = 16.2 \cdot 10^9$$

Then, using this 'guess':

60Hz: $P_e \stackrel{?}{=} 81W$

180Hz: $P_e \stackrel{?}{=} 93W$

600Hz: $P_e \stackrel{?}{=} 92.5W$

9) Give a model for this inductor operating at a) 60Hz, b) 180Hz, and c) 600Hz.

Assume you have an inductor in series with 60 Ohms wire resistance. In addition, you have some more losses due to hysteresis and core losses. These act like a resistor in parallel:

60Hz:

$$\frac{V^2}{R} = 162W + 82W$$

$$R_c = 236\Omega$$

$$Z_{net} = R_c || (R + j\omega L)$$

$$Z_{net} = 236\Omega || (60 + j149)$$

$Z_{net} = 85.9 + j75.5$ series model

$Z_{net} = 152 || j173$ parallel model

180Hz:

$$\frac{V^2}{R} = 62W + 93W$$

$$R = 371\Omega$$

$$Z_{net} = 371\Omega || (60 + j447)\Omega$$

$Z_{net} = 217 + j159\Omega$ series model

$Z_{net} = 335\Omega || j455\Omega$ parallel model

600Hz:

$$\frac{V^2}{R} = 18.7W + 92.5W$$

$$R = 518\Omega$$

$$Z_{net} = 518\Omega || (60 + j1490)\Omega$$

$Z_{net} = 457 + j156\Omega$ series model

$Z_{net} = 510\Omega || j1494\Omega$ parallel model

Note:

- These numbers are probably off.
- The real part of Znet should be zero for an ideal inductor. Note that it's getting more resistive as frequency goes up.
- At 60Hz, the losses are 134W (I^2R), 162W (hysteresis), 82W (Eddy). That's probably reasonable: if one were really dominant, you'd adjust that part of the design to reduce it's losses until something else started to matter.

10) A coil with 100 turns and a cross sectional area of 10cm x 10cm is rotating in a magnetic field of 0.5T. Determine the voltage produced if this coil is rotating at a) 60Hz, b) 400Hz.

$$e = N\frac{d\phi}{dt}$$

$$\phi_m = \left(0.5\frac{Wb}{m^2}\right)(0.1m)^2 = 0.005Wb$$

$$\phi(t) = 0.005\sin(377t)$$

$$e = (100)(1.88\cos(377t))$$

The peak is 188V

$$\phi(t) = 0.005\sin(2513t)$$

$$e = (100)\cdot 12.56\cos(377t)$$

The peak is 1256V

Bonus! What is the peak energy usage of the U.S. in kW? (from www.wikipedia.com): $3.35\text{x}10^{12}$ Watts. You'd need 3,350 x 1000MW Nuclear plants to supply the entire US energy consumtion, which would cost about 7 trillion dollars. At that point we would be energy independent, fix the balance of trade, and emit virtually no green-house gasses. In comparison, the war in Iraq costs about $2 trillion.

Fuel type	2004 US consumption in TW[10]	2004 World consumption in TW[11]
Oil	1.34	5.6
Gas	0.77	3.5
Coal	0.77	3.8
Hydroelectric	0.09	0.9
Nuclear	0.27	0.9
Geothermal, wind, solar, wood	0.11	0.13
Total	**3.35**	**15**

U.S, Primary Energy Consumption by Source and Sector in 2008 is tabled as following:

Consumption Summary[12]

Supply Sources	Percent of Source	Demand Sectors	Percent of Sector
Petroleum 37.1%	71% Transportation 23% Industrial 5% Residential and Commercial 1% Electric Power	**Transportation 27.8%**	95% Petroleum 2% Natural Gas 3% Renewable Energy
Natural Gas 23.8%	3% Transportation 34% Industrial 34% Residential and Commercial 29% Electric Power	**Industrial 20.6%**	42% Petroleum 40% Natural Gas 9% Coal 10% Renewable Energy
Coal 22.5%	8% Industrial <1% Residential and Commercial 91% Electric Power	**Residential and Commercial 10.8%**	16% Petroleum 76% Natural Gas 1% Coal 1% Renewable Energy
Renewable Energy 7.3%	11% Transportation 28% Industrial 10% Residential and Commercial 51% Electric Power	**Electric Power 40.1%**	1% Petroleum 17% Natural Gas 51% Coal 9% Renewable Energy 21% Nuclear Electric Power
Nuclear Electric Power 8.5%	100% Electric Power		

Note: Sum of components may not equal 100 percent due to independent rounding.