ECE 331 - Solution to Homework #2

Transformer Principles

Assume you are designing a power grid to deliver 240V, 60Hz, 10kVA 400km away. The generator produces 138kV.

1) Design a 2400V-240V 100kVA transformer at 60Hz. Assume silicon steel (B = 1.5T, μ_r =7000). Give a model for the transformer you designed including resistance of each winding and core losses.

$$B = \left(\frac{V}{377AN}\right) = \left(\frac{2400V}{377A \cdot 300}\right) = 1.5T$$

$$A = 0.0141m^2$$

Let the area be 12cm x 12cm

Let the length of the core be 144cm (12cm x 4 x 4 sides)

$$R = \frac{l}{\mu A} = \frac{1.44m}{7000 \cdot \mu_o \cdot 0.0141} = 11610$$

$$L = \frac{N^2}{R} = \frac{300^2}{11610} = 7.75H$$

$$jX_c = j377L = j2922\Omega$$

The hysteresis losses are:

$$P_h = k_h f v B_m^2$$

$$P_h = \left(350\frac{J}{m^3}\right)(60Hz)(0.0203m^3)(1.5T)^2 = 959W$$

Let

$$P_e = \tfrac{1}{2}P_h = 479W$$

The core resistance is then

$$R_c = \frac{V^2}{P} = \frac{(7200V)^2}{1438W} = 35k\Omega$$

Assume the line losses are 1% (1kW). Then,

$$I^2R = 1000W = (41.7A)(0.576\Omega)$$

On the high side, pick the wire so it has a resistance of 0.576 Ohms.

$$R_h = 0.576\Omega$$

Oh the low side, pick the wire so it has a the same resistance, transferred through the transformer:

$$R_l = \left(\frac{240V}{2400V}\right)^2 0.576\Omega = 0.00576\Omega$$

To get an idea of the wire gage you need,

$$R_h = \frac{\rho L}{A} = \frac{\left(1.75 \cdot 10^{-8}\Omega m\right)(0.12mx4x100)}{A} = 0.576\Omega$$

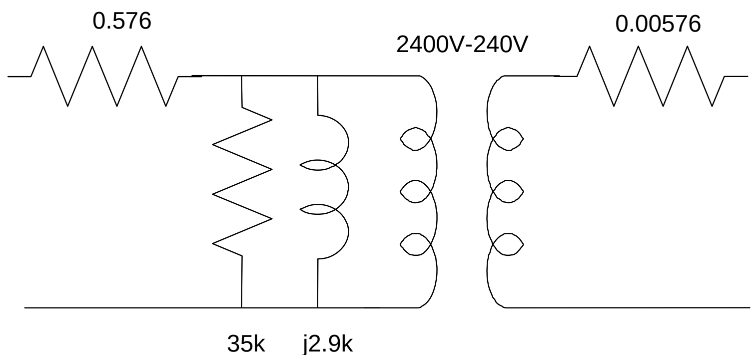
$$A = 0.00121^2$$

A wire with a diameter of 1.2mm is needed on the high side. On the low side:

$$R_l = \frac{\rho L}{A} = \frac{\left(1.75 \cdot 10^{-8}\Omega m\right)(0.12mx4x10)}{A} = 0.00576\Omega$$

$$A = 0.0038^2$$

The wire in the low side needs to have a diameter of 3.8mm.



2) What would the open-circuit and short-circuit V, I, P readings be for this transformer?

Open Circuit Model: As measured on the high side:

$$V = 2400V$$

$$I = \left(\frac{2400V}{35k||j2.9k}\right) = 0.83\angle -85^0$$

I = 0.83 Amps

P = real(VI)=VIcos(Q)=164W

Short-Circuit Model: As measured on the high-side:

V = 24V

$$I = \left(\frac{24V}{0.576+0.576}\right) = 20.8A$$

(note: you'll never get 240V on the low side on this test.)

P = 500W

3) Design a 13.8kV-2400V 1000kVA transformer at 60Hz. Assume silicon steel (B = 1.5T, =7000). Give a model for the transformer you designed including resistance of each winding and core losses.

Assume 600 turns

$$B = \left(\frac{V}{377AN}\right) = \left(\frac{13800}{377A \cdot 600}\right) = 1.5T$$

$$A = 0.0407m^2$$

The core could be 20cm x 20cm in cross section. Let each side of the core be 4 times this (0.8m) for a length of 3.2 meters:

$$R = \frac{l}{\mu A} = \frac{3.2m}{7000\mu_o 0.0407m^2} = 8938$$

$$L = \frac{N^2}{R} = \frac{600^2}{8938} = 40.2H$$

$$jX_c = j377L = j15.1k\Omega$$

The hysteresis losses are:

$$P_h = k_h f v B_m^2$$

$$P_h = \left(350\frac{J}{m^3}\right)(60Hz)(0.13m^3)(1.5T)^2 = 6.1kW$$

Assume the Eddy current losses are half this:

$$P_e \approx 3.1kW$$

The core resistance is then:

$$R_c = \frac{V^2}{P} = \frac{13800^2}{9.2kW} = 20.7k\Omega$$

Next, design the windings. Assume the copper losses equal to 1% of the rated load:

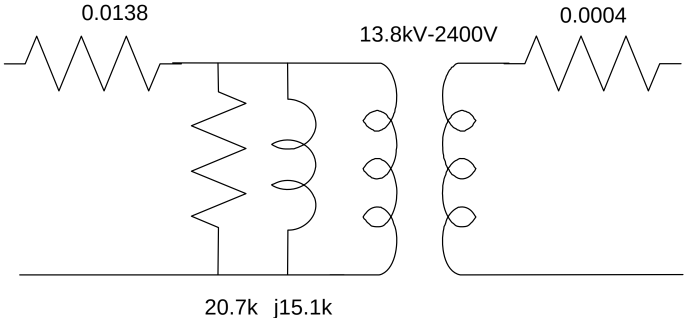
$$i_h = \frac{1000kVA}{13800V} = 72.4A$$

$$i_h^2 R_h = 10kW$$

$$R_h = 0.0138\Omega$$

On the low side:

$$R_l = \left(\frac{2400}{13800}\right)^2 R_h = 0.0004\Omega$$



4) What would the open-circuit and short-circuit V, I, P readings be for this transformer?

Open Circuit Test: As measured on the high side:

V = 13.8kV

$$I = \left(\frac{13.8kV}{20.7k\Omega}\right) + \left(\frac{13.8kV}{j15.1k\Omega}\right) = 1.13\angle -53.8^0$$

I = 1.13 Amps

$$P = VI\cos(\theta) = (13.8kV)(1.13A)(\cos(-53.8^0)) = 9.2kW$$

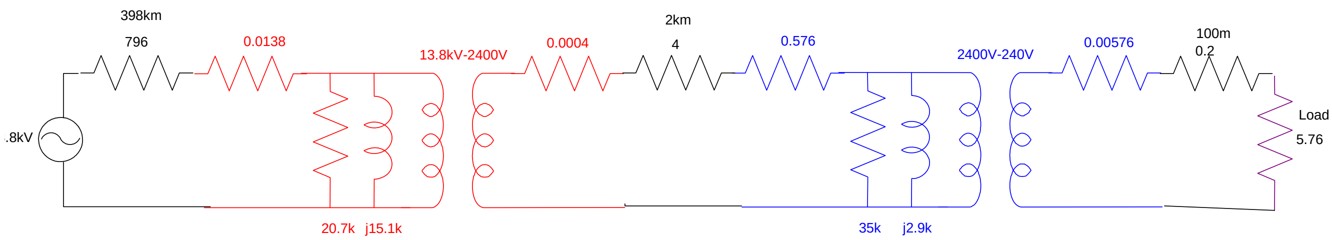
Short Circuit Test: As measured on the high side (short the low side):

V = 13.8V

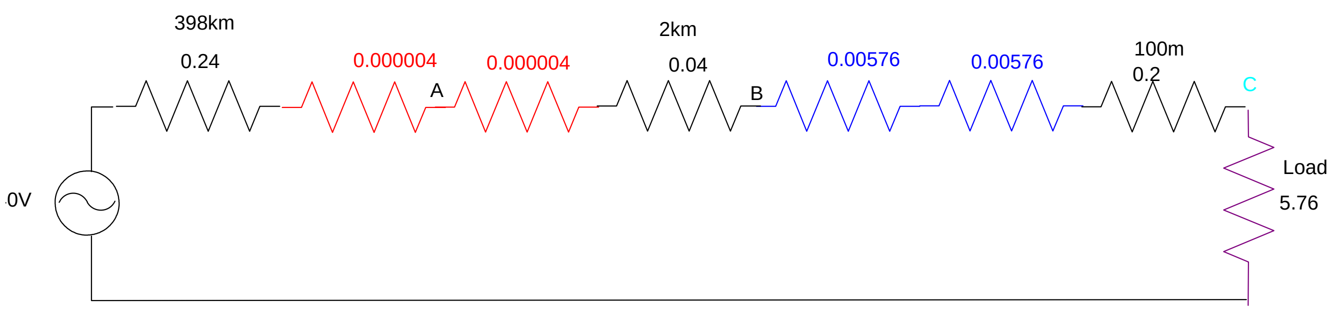
$$I = \left(\frac{13.8V}{0.0138\Omega + 0.0138\Omega}\right) = 500A$$

P = 6.9kW

5) Assume the load is 10kVA with a power factor of 1.0. Find the power delivered to the load, the efficiency ignoring the core losses, and the efficiency including the core losses.



Bring everything to the load side. Ignore the core losses for now:



By voltage division:

$$V_C = \left(\frac{5.76}{5.76+0.2+....}\right)240V = 221V$$

The efficiency is related to the resistance at the load over the total resistance (ignoring core losses)

$$I_L = \left(\frac{221V}{5.76\Omega}\right) = 38.39A$$

$$P_L = V_L I_L = 8.48kW$$

The copper losses are

$$P_{\text{copper}} = I_L^2(0.48\Omega) = 707W$$

The efficiency is

$$eff = \frac{8.48kW}{8.48kW+0.707kW} = 0.9208$$

If you includ core losses:

$$eff = \frac{8.48kW}{8.48kW+0.707kW+3.1kW+474W} = 0.66$$

You don't want to deliver power to a single customer 400km away.

6) Assume the load is 10kVA with a power factor of 0.8 lagging. Find the power delivered to the load, the efficiency ignoring the core losses, and the efficiency including the core losses.

$$I = \left(\frac{10000kVA}{240V}\right)\angle \arccos(0.8) = 41.67\angle - 36.9^0$$

$$Z_L = \frac{V}{I} = \frac{240V}{I} = 5.76\angle 36.9^0$$

The analysis is the same as before, just change the load:

$$V_L = \left(\frac{Z_L}{Z_{total}}\right)240V$$

$$V_L = \left(\frac{5.76\angle 36.9^0\Omega}{5.76\angle 36.9^0\Omega + 0.4915\Omega}\right)240V = 224.4\angle 2.74^0$$

$$I_L = \frac{V_L}{Z_L} = 38.96\angle - 34.15^0$$

The efficiency is

$$P_L = |V_L||I_L|\cos(36.9^0) = 760W$$

$$P_{\text{copper}} = I^2 R$$

$$P_{\text{copper}} = (38.96A)^2(0.4915\Omega) = 746W$$

$$eff = \frac{760W}{760W + 746W} = 0.505$$

The efficiency drops by a lot when the power factor moves away from one.

7) Find the rms voltage of the following:

- 7a) $v(t) = 200\sin(377t)$

```
-->t = [0:0.001:1]';
-->y1 = 200*sin(2*%pi*t);

-->rms1 = sqrt(mean(y1 .^ 2))
 rms1  =

    141.3507
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- 7b) $v(t) = 200$

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-->y2 = 200*(t > -1);

-->rms2 = sqrt(mean(y2 .^ 2))
 rms2  =

    200.
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- 7c) v(t) is a 60Hz triangle wave with a peak voltage of 200V.

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-->y3 = ((4*t-1) .* (t<0.5))+((3-4*t).*(t>=0.5)) * 200;

-->rms3 = sqrt(mean(y3 .^ 2))
 rms3  =

    81.854667
```

BONUS! What is the efficiency of a standard 60W incandescent light bulb?

ans: 2-3%.