

# ECE 331 - Solution to Homework #4 - Lab

## AC Induction Motors

1) Using MATLAB or SciLab, show that a 3-phase stator produces a rotating magnetic field.

2-3) A three-phase, two-pole, 30hp, 120V, 60Hz Y connected induction motor draws a current of 30A from the line source at a power factor of 0.9. At this condition, the motor losses are:

- Stator copper losses =  $P_{cu1} = 500W$
- Rotor copper losses =  $P_{cu2} = 300W$
- Stator core losses =  $P_c = 240W$
- Rotation losses =  $P_{rot} = 200W$

Determine

a) the power transferred across the air gap

Analyze this on a per-phase basis. The total answer will be 3 times the result per phase.

The line-to-neutral voltage is 120V (if you interpret it as  $120V_{LN}$ ) or 69.28V (if you interpret it as  $120V_{LL}$ ). Assume  $120V_{LN}$ .

The electric power input and power across the gap are

$$P_{elec} = 3 \cdot 120V \cdot 30A \cdot 0.90 = 9,720W$$

$$P_g = P_{elec} - P_{Cu1} - P_c$$

$$P_g = 9,720W - 500W - 240W = 8,980W$$

b) The internally developed torque in Nm

$$n_s = 2\pi \cdot 60Hz = 377 \text{ rad/sec}$$

$$T = \frac{8980W}{377 \text{ rad/sec}} = 23.82 \text{ Nm}$$

c) the slip expressed in per unit and in rpm

$$P_{Cu2} = s \cdot P_g$$

$$s = \frac{P_{Cu2}}{P_g} = \frac{300W}{8980W} = 0.0334$$

$$s \cdot n_s = 12.59 \text{ rad/sec} = 120.27 \text{ rpm}$$

d) the mechanical power developed in watts

$$P_m = P_g - P_{Cu2} = 8,980W - 300 = 8,680W$$

e) the horsepower output

$$P_o = P_m - P_{rot} = 8,680W - 200W = 8,480W = 11.63 \text{ hp}$$

f) the motor speed in rpm and radians/second

$$n = (1 - s)n_s = 364.4 \text{ rad/sec} = 3,480 \text{ rpm}$$

g) the torque at the output shaft

$$T_o = \frac{P_o}{n} = 2.4368 \text{ Nm}$$

h) the torque needed to overcome rotational losses

$$T_{rot} = \frac{P_{rot}}{n} = 0.5488 \text{ Nm}$$

i) the efficiency of the operation in the stated conditions

$$\eta = \frac{P_o}{P_e} = \frac{8480W}{9720W} = 87.2\%$$

4-5) A three-phase, two pole, 20hp, 120V, 60Hz, Y connected induction motor has the following parameters per phase:

$$r_1 = 0.1 \text{ Ohm}, \quad x_1 = 0.35 \text{ Ohms}$$

$$r_2 = 0.12 \text{ Ohms} \quad x_2 = 0.40 \text{ Ohms}$$

The stator core losses are 500W and the rotational losses are 400W. At no-load, the motor draws 10A with a power factor of 0.1 lagging.

When the motor operates at a slip of 2%, find

Again, it isn't specified if this is 120V line-to-line or line-to-neutral. Assume 120V line-to-neutral. Your answers will differ if you assumed 120V line-to-line.

a) the input line current and power factor

$$V_1 = 120V$$

$$I_1 = I_m + I_2$$

$$I_m = 10 \angle -84.26^\circ$$

$$R_m = r_2 \left( \frac{1-s}{s} \right) = 5.88 \Omega$$

$$I_2 = \frac{120V}{5.88 \Omega + (0.22 + j0.75) \Omega} = 19.525 \angle -7.01^\circ$$

so

$$I_1 = I_m + I_2 = 23.82 \angle -31.18^\circ$$

$$\text{power factor} = 0.855 \text{ lagging}$$

b) the developed electromagnetic torque in Nm

$$n_s = 2\pi \cdot 60 \text{ Hz} = 377 \text{ rad/sec}$$

$$n = (1-s)n_s = 369.46 \text{ rad/sec}$$

$$P_m = 3 \cdot |I_2|^2 R_m = 6724.82 \text{ W}$$

$$T_m = \frac{P_m}{n} = 18.20 \text{ Nm}$$

c) the horsepower output

$$P_o = P_m - P_{rot}$$

$$P_o = 6724.82 \text{ W} - 400 \text{ W} = 6324.82 \text{ W}$$

$$P_o = \frac{6324.8 \text{ W}}{746 \text{ W/hp}} = 8.47 \text{ hp}$$

d) the efficiency

$$P_e = 3 \cdot \text{real}(V_1 I_1) = 7336.47 \text{ W}$$

$$\eta = \frac{P_o}{P_e} = \frac{6324.82 \text{ W}}{7336.47 \text{ W}} = 0.8621$$

6) Plot the torque-slip speed relationship for the motor in problem 4-5.

Modify the previous program for this motor's parameters. Check at a slip of 0.02 to make sure the computations are correct:

```
-->[T,P,eff] = slip(0.02)
    eff
    0.8631982
    P
    =
    6332.9017
    T
    =
    17.141371
```

They're close - I probably had rounding errors in the previous problem.

```
-->s = [0.001:0.001:0.999]';

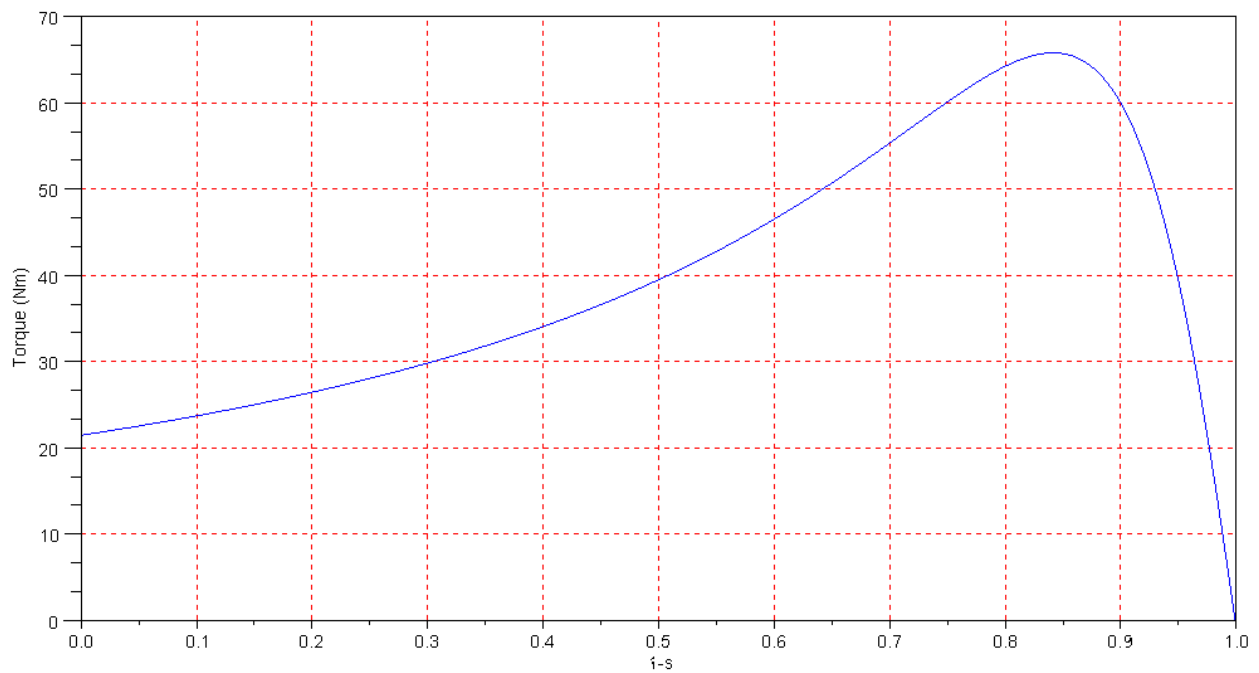
-->for i=1:length(s)
-->    [T(i),P(i),eff(i)] = slip(s(i));
-->    end

-->plot(1-s,T)

-->xlabel('1-s')

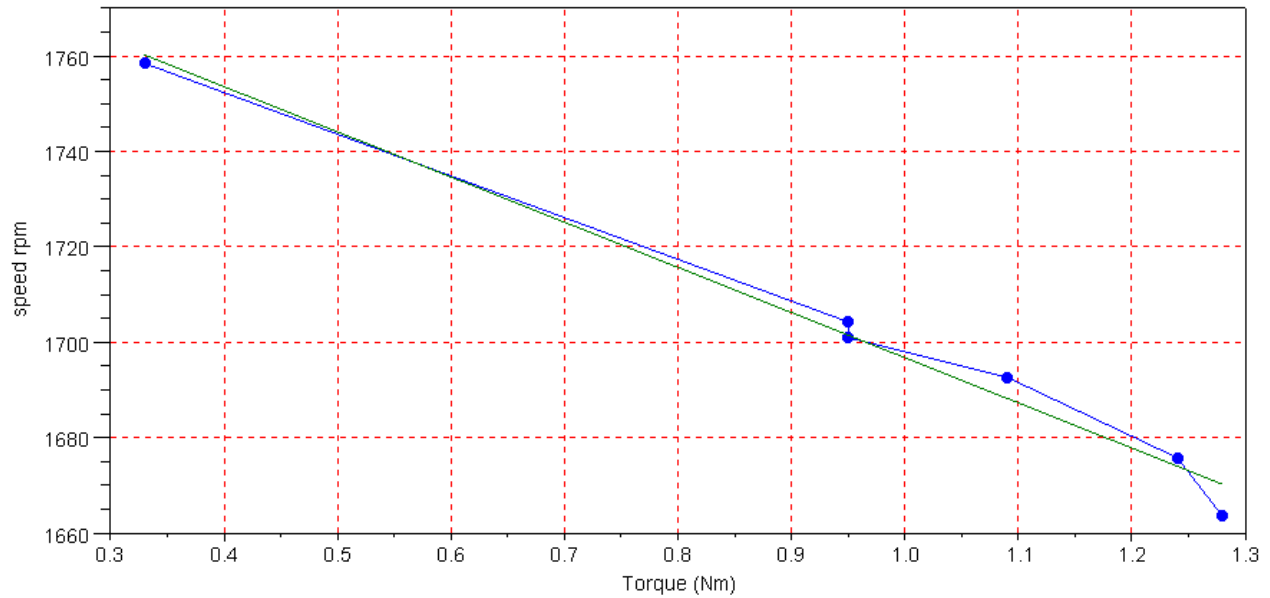
-->ylabel('Torque (Nm)')

-->xgrid(5)
```



Give the data you collected in lab (table print out is OK)

7a) Plot the torque vs. speed for the AC induction machine



note: As torque increases, the speed drops.

7b) Give an equation relating speed as a function of torque.

```
-->X = [T,T.^0];
```

```
-->A = inv(X'*X)*X'*N
```

```
A =
```

```
- 94.689928  
 1791.4432
```

8a) Plot current vs. voltage for a fixed load.

V	I
204.66	1.
180.34	1.08
158.07	1.25
136.04	1.69
113.39	2.36

Do a least squares curve fit for a parabola:

```
-->X = [V1.^2, V1, V1.^0];
```

```
-->A = inv(X'*X)*X'*I1
```

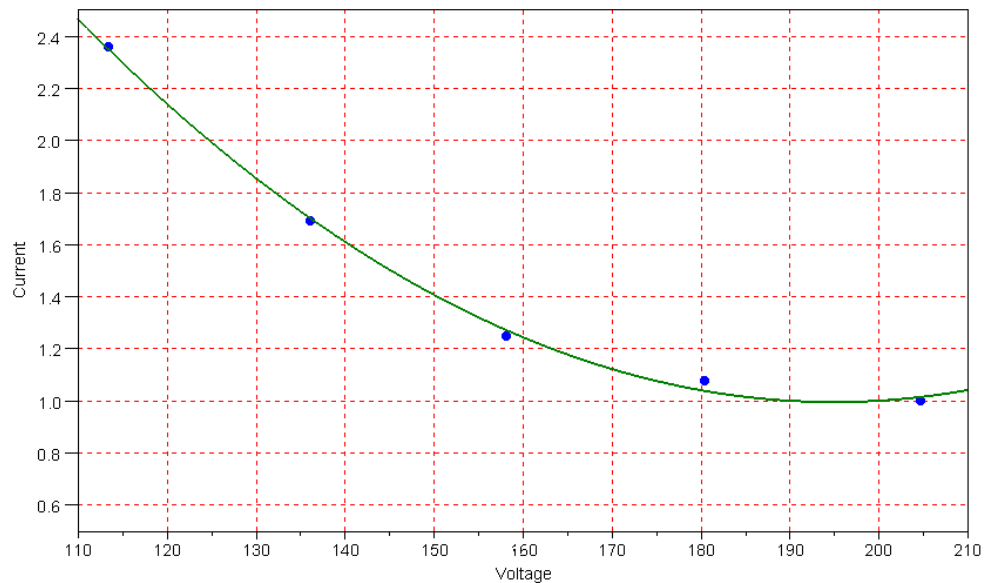
```
0.0002038  
- 0.0794346  
8.7370505
```

So

$$I \approx 0.0002038V^2 - 0.0794346V + 8.7370505$$

Plot this with more points to give a smooth curve

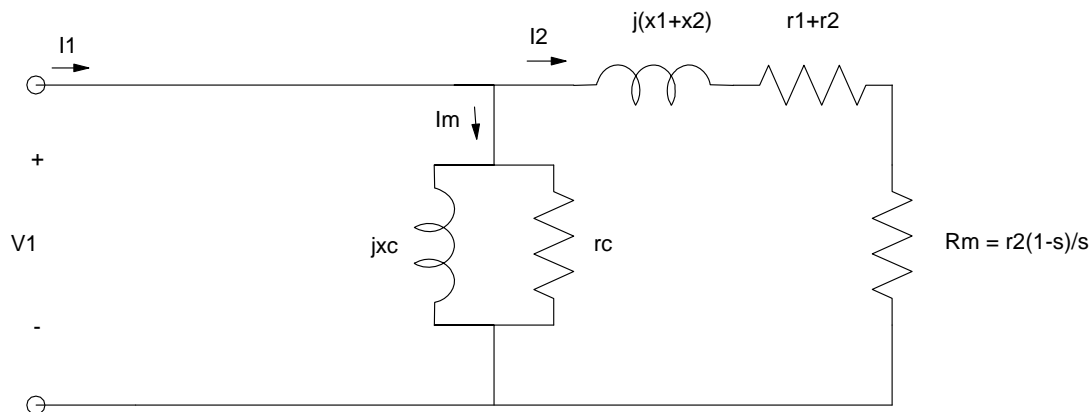
```
-->V2 = [110:210]';  
-->X2 = [V2.^2, V2, V2.^0];  
-->plot(V1,I1,'.',V2,X2*A)  
-->xlabel('Voltage');  
-->ylabel('Current');  
-->xgrid(5);
```



8b) Discuss what the increasing current for a lower voltage implies. What happens to AC induction motors if the voltage is lower than what it's supposed to be?

The current shoots up quickly as the voltage drops. This can cause an over-current, burning out the motor.

9) Compute a model of the motor using the no-load, stuck rotor, and DC measurements from lab.



	E1	I1	P1+P2	PF
Short-Circuit	58.86	1.02	64.06	0.6
Open-Circuit	205.36	0.6	39.06	0.17
DC Test	26.76	1.07	28.16	0.71

Short-circuit test:

$$r_1 + r_2 + j(x_1 + x_2) = \frac{58.86V}{1.02A} \angle \arccos(0.6)$$

$$r_1 + r_2 + j(x_1 + x_2) = 57.71 \angle 53.13^\circ = 34.62 + j46.16 \Omega$$

Open-circuit test:

$$r_c \parallel jx_c = \frac{205.36V}{0.6V} \angle \arccos(0.17)$$

$$r_c \parallel jx_c = 342.27 \angle 80.12^\circ$$

$$\frac{1}{r_c} + \frac{1}{jx_c} = 0.0005 - j0.0029$$

$$r_c = 2013 \Omega$$

$$jx_c = j347.3 \Omega$$

DC test

$$r_1 = \frac{26.76V}{1.07A} = 25.0 \Omega$$

$$r_2 = 34.62 \Omega - 25.0 \Omega = 9.61 \Omega$$