ECE 331 - Solution to Homework #9

DC Shunt Motors

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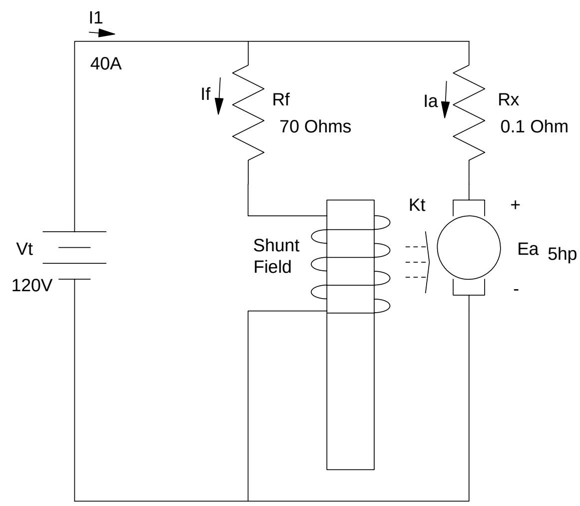
1) A 5hp, 120V, 1200 rpm shunt motor has an armature resistance of 0.1 Ohm and field resistance of 70 Ohms. When delivering 5hp, the motor draws 40A.

First, find the parameters for the motor under rated load:




```
-->Vt = 120;
-->w = 1200/60*2*%pi
    125.66371

-->Ra = 0.1;
-->Rf = 70;
-->If = Vt/Rf
    1.7142857

-->Ia = 40 - If
    38.285714

-->P = 5*746
    3730.

-->T = P / w
    29.682397

-->Ea = Vt - Ia*Ra
    116.17143

-->Kt = Ea/w
    0.9244629
```

OK - so this motor has a torque constant of 0.9244

a) Find the motor speed and power out when the motor develops a torque of 10 Nm

```
-->T = 10;
-->Ia = T / Kt
    10.817092

-->Ea = Vt - Ia*Ra
    118.91829

-->w = Ea / Kt
    128.63501

-->Po = Ia*Ea
    1286.3501
```

b) Find the motor speed and power out when the motor develops a torque of 10Nm if the field resistance is increases to 100 Ohms.

If you increase the field resistance to 100 Ohms, the current (and torque constant) drop proportionally:

```
-->Kt = 70/100 * Kt
    0.6471
```

So now, Kt = 0.5177. At 10Nm torque, find the new speed and power:

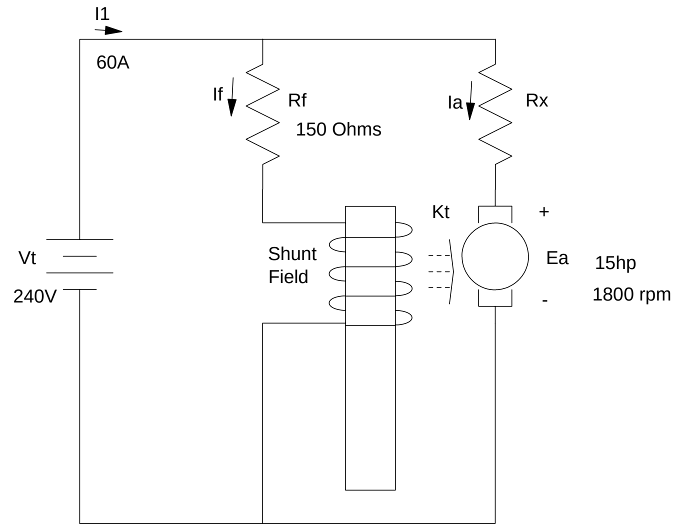
```
-->T = 10;
-->Ia = T / Kt
    15.4530

-->Ea = Vt - Ia*Ra
    118.4547
```

-->w = Ea / Kt
183.05 rad/sec

-->Po = Ia*Ea
1830.48 Watts

2) A 15hp 240V, 1800 rpm shunt motor draws 60A from the line at rated output (15hp). ~~The resistance of the armature circuit is 0.2 Ohms,~~ the field resistance is 150 Ohms.

a) Find the no-load speed of the motor.



Like most DC motor problems, first find the torque constant:

```
-->Rf = 150;
-->Vt = 240;
-->If = Vt/Rf
    1.6

-->Ia = 56 - If
    54.4

-->Po = 15*746
    11190.

-->w = 1800 / 60 * 2*%pi
    188.49556

-->T = Po / w
    59.364794
```

-->Kt = T/Ia
1.0912646

This tells you the no-load speed of the motor:

-->w = Vt / Kt
219.92833   The no - load speed of the motor

You can also calculate the armature resistance if you really want to....

```
-->Ea = Kt*w
    205.69853

-->Rx = (Vt - Ea) / Ia
    0.6305417        The armature resistance for the voltages to match up
```

b) Determine how much the field resistance must be increased to keep the speed constant when the load is increased to 15hp.

```
-->Po = 15 * 746
    11190.

-->T = Po / w
    59.364794
```

This gives two equations for two unknowns:

$$P_o = E_a I_a$$

$$V_t = I_a R_x + E_a$$

or

$$11,190W = E_a I_a$$

$$240V = I_a \cdot 0.6305\Omega + E_a$$

This has the solution

Ia = 54.477 Amps

Ea = 205.407V

which means that

$$E_a = K_t \omega$$

$$K_t = \frac{E_a}{\omega} = \frac{205.407V}{219.92 rad/\sec} = 0.9340 \frac{V}{rad/\sec}$$

To keep the speed the same as the no-load speed (219.9 rad/sec), the torque constant needs to be decreased to 0.9003 Nm/A when delivering 15hp. This can be accomplished by increasing the field resistance proportionally:

```
-->Rf = 150 * (1.0912646 / 0.9340)
    175.2
```

$$\boxed{R_f = 175\Omega}$$

DC Series - Shunt Motor:

3) A 5hp, 120V, 1200 rpm shunt motor has an armature resistance of 0.3 Ohms and a field resistance of 100 Ohms.

First, determine the torque constant. Similar to problem #1...

```
-->Vt = 120;
-->w = 1200/60*2*%pi
    125.66371

-->Ra = 0.3;
-->Rf = 100;

-->If = Vt / Rf
    1.2

-->Po = 5*746
    3730.
```

This is...

$$P_o = 3730W = E_a I_a = T\omega$$

```
-->T = Po / w
    29.682397
```

You also know

$$V_t = I_a R_x + E_a$$

$$120V = 0.3\Omega \cdot I_a + E_a$$

$$3730 = I_a E_a$$

This gives two equations and two unknowns. Solving....

Ia = 33.8679A

Ea = 109.80V

This tells you the torque constant:

$$K_t = \frac{T}{I_a} = \frac{E_a}{\omega} = 0.8738\frac{Nm}{A}$$

The no-load speed is then:

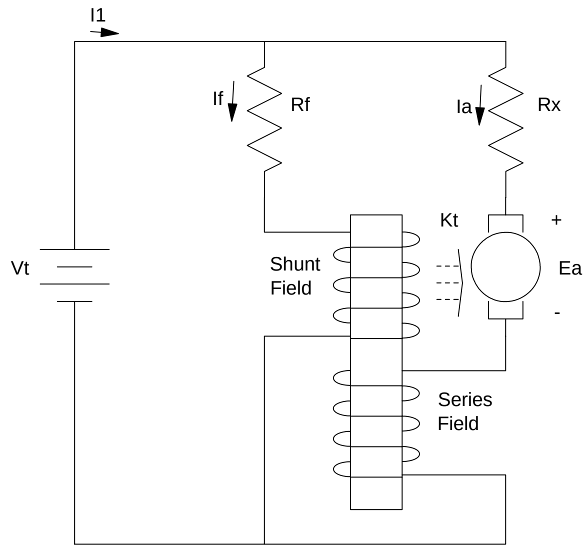
$$\omega_0 = \frac{V_t}{K_t} = 137.325\frac{rad}{\sec}$$

() No shunt field: At a load of 30Nm, the speed drops to

$$I_a = \frac{T}{K_t} = 34.3313A$$

$$E_a = V_t - I_a R_x = 109.7V$$

$$\omega = \frac{E_a}{K_t} = 125.53\frac{rad}{\sec}$$



a) Shunt field reduced by 0.1% / Amp

$$K_t = 0.8738 \cdot (1 - 0.001 I_a)$$

This gives the following equtations:

$$T = 30Nm = K_t I_a = 0.8738 \cdot (1 - 0.001 I_a) \cdot I_a$$

$$I_a = 35.6A$$

$$K_t = 0.8427\frac{Nm}{A}$$

$$E_a = V_t - I_a R_x = 109.32V$$

$$\boxed{\omega = \frac{E_a}{K_t} = 129.72\frac{rad}{\sec}}$$

This is unstable - it's going faster under load. In theory, it runs at 129.72 rad/sec however....

b) 1% / Amp

$$K_t = 0.8738 \cdot (1 - 0.01 I_a)$$

This gives the following equtations:

$$T = 30Nm = K_t I_a = 0.8738 \cdot (1 - 0.01 I_a) \cdot I_a$$

$$I_a = 50 \pm j30.54A$$

There is no solution.

c) 10% / Amp

$$K_t = 0.8738 \cdot (1 - 0.1 I_a)$$

This gives the following equtations:

$$T = 30Nm = K_t I_a = 0.8738 \cdot (1 - 0.1 I_a) \cdot I_a$$

$$I_a = 5 \pm j17$$

There is no solution.

Sidelight - The no-load speed is

$$\omega_{T=0} = 137.325\frac{rad}{\sec}$$

If you weaken the field by 0.25%/A:

$$K_t = 0.8738 \cdot (1 - 0.0025 I_a)$$

then

$$T = 30Nm = K_t I_a = 0.8738 \cdot (1 - 0.0025 I_a) \cdot I_a$$

$$I_a = 378.929A$$

$$K_t = 0.7909\frac{Nm}{A}$$

$$E_a = V_t - I_a R_x = 108.62V$$

$$\omega = \frac{E_a}{K_t} = 137.33\frac{rad}{\sec}$$

Weakening the field by 0.25%/A keeps the speed almost constant under load.

4) A 15hp 240V, 1800 rpm shunt motor draws 60A from the line at rated output (15hp). The resistance of the armature circuit is 0.2 Ohms, the field resistance is 150 Ohms.

Determine how much the field needs to be weakened with the armature current so that the motor maintains speed as the load goes from 0 to 15hp.

At 15hp, the motor is drawing:

If = 240V / 150 Ohms = 1.6A

Ia = 60A - 1.6A = 58.4A

Ea = 240V - (58.4A)(0.2 Ohms) = 228.32V

$$K_t = \frac{E_a}{\omega} = \frac{228.32V}{188.49 rad/\sec} = 1.2113 \frac{Nm}{A}$$

The mis-match in power must be the rotational losses

$$P_{rot} = E_a I_a - 15hp$$

$$P_{rot} = 2,143W$$

At no load, you want the speed to be the same...

If = 240V / 150 Ohms = 1.6A

Ea Ia = 2143W (to cover the rotational losses)

$$240 - 0.2 I_a = E_a$$

so

$$(240 - 0.2 I_a) I_a = 2143W$$

Ia = 8.9966 A

Ea = 238.2V

$$K_t = \frac{E_a}{\omega} = \frac{238.2V}{188.49 rad/\sec} = 1.2637 \frac{Nm}{A}$$

This gives two points for (Kt, Ia):

- When Ia = 58.4A, Kt = 1.2113Nm/A,
- When Ia = 8.9966A, Kt = 1.2637Nm/A

$$K_t = a \cdot I_a + b$$

$$K_t = -0.0011 \cdot I_a + 1.2732$$

$$K_t = 1.2732(1 - 0.0009 I_a) \frac{Nm}{A}$$

The torque constant should be reduced by 0.09%/A

DC Series Motor

Problem 5) A 200hp, 600V, 500 rpm DC series motor draws a line current of 300A at rated load. ~~The resistance of the series winding is 0.15 Ohms.~~ Determine both the speed and torque when th load is changed to that the motor draws 200A.

Again, the problem is over specified. One of the numbers given is off.

Assume resistance is incorrect. Find the torque constant:

```
-->Vt = 600;
-->w = 500/60*2*%pi
    52.359878

-->Po = 200*746
    149200.

-->Ia = 300;

-->T = Po / w
    2849.5101

-->Kt = T / Ia
    9.498367

-->Ea = Kt*w
    497.33333

-->Ra = (Vt - Ea) / Ia
    0.3422222
```

To be consistent, Ra = 0.342 Ohms.

The torque constant is 9.49 when Ia = 300A. When Ia is reduced to 200A, the torque constant reduced by 200/300:

```
-->Ia = 200;
-->Kt = Kt*200/300
    6.3322447

-->Ea = Vt - Ia*Ra
    531.55556

-->w = Ea / Kt
    83.944254

-->T = Kt*Ia
    1266.4489
```

If you reduce the load torque from 2849Nm to 1266Nm,

- The current drops from 300A to 200A
- The speed increases from 52 rad/sec to 83 rad/sec

Problem 6) A 200-hp 600V, 500rpm DC series motor draws 300A at rated output. ~~The resistance of the armature and series field windings are 0.6 Ohm and 0.3 Ohm respectively.~~ The torque is reduced to 1/4th of rated value. Find the new operating speed.

Again, the problem is overspecified. Either current is whatever it takes to produce 200hp, or the resistance isn't 1.1 Ohms (total). Let's assume you know the current is 300A:

```
-->Po = 200*746
    149200.

-->w = 500/60*2*%pi
    52.359878

-->Ia = 300;

-->T = Po / w
    2849.5101
```

-->Kt = T/Ia
9.498367

```
-->Ea = Kt*w
    497.33333

-->Vt = 600;

-->Ra = (Vt - Ea) / Ia
    0.3422222
```

The resistance needs to be 0.342 Ohms (total) to be consistent with the rest of the problem statement. If the torque is dropped by 75%:

```
 -->T = 0.25*T
    712.37753
```

The torque constant is $Kt = K_{t1} I_a$. At 300A, Kt = 9.49, meaning

```
-->Kt1 = Kt / Ia
    0.0316612
```

From

$$T = K_t I_a = (K_{t1} I_a) I_a$$

```
-->Ia = sqrt(T / Kt1)
    150.

-->Kt = Kt1 * Ia
    4.7491835

-->Ea = Vt - Ia*Ra
    548.66667
```

-->w = Ea / Kt
115.52863

When you decrease the load by 75%,

- The current drops by half (300A to 150A)
- The speed increases from 52 rad/sec to 115 rad/sec