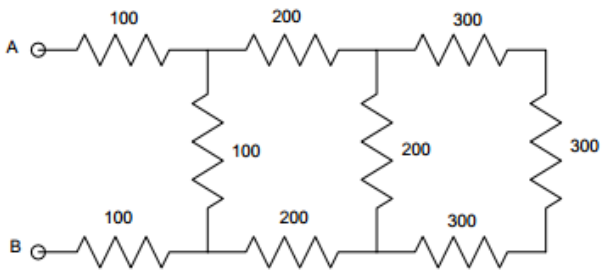
ECE 331 - Solution to Homework #1

Kirchoff's Laws, Complex Numbers. Due Wednesday, January 22nd, 4PM

1) Determine the resistance of the following circuit:



Starting from the right

300 + 300 + 300 = 1200 (series)

1200 || 200 = 171

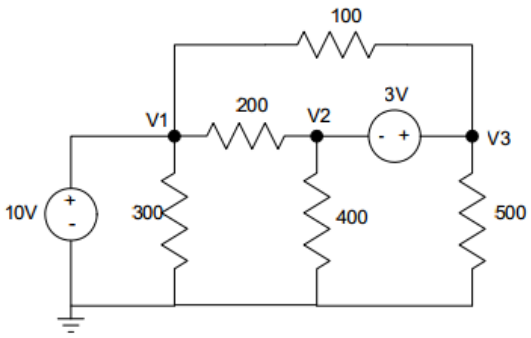
200 + 171 + 200 = 571

571 || 100 = 85

100 + 85 + 100 = 285

answer: 285 Ohms

2) For the following circuit, write N equations to solve for N unknown voltage nodes (KVN)



You need three equations for three unknown voltages. Start with the voltage sources:

$$V_1 = 10$$

$$V_3 - V_2 = 3$$

You need one more equation. Sum the currents to zero flowing away from the 3V source (super-node V2 and V3)

$$\left(\frac{V_2 - V_1}{200}\right) + \left(\frac{V_2}{400}\right) + \left(\frac{V_3 - V_1}{100}\right) + \left(\frac{V_3}{500}\right) = 0$$

Problem 3) Solve these N equations to find the voltages V1, V2, and V3

Group these three equations into common terms:

$$\left(\frac{-1}{200} + \frac{-1}{100}\right) V_1 + \left(\frac{1}{200} + \frac{1}{400}\right) V_2 + \left(\frac{1}{100} + \frac{1}{500}\right) V_3 = 0$$

Place in matrix form:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ \left(\frac{-1}{200} + \frac{-1}{100}\right) & \left(\frac{1}{200} + \frac{1}{400}\right) & \left(\frac{1}{100} + \frac{1}{500}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \\ 0 \end{bmatrix}$$

Solve in MATLAB:

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-->A = [1,0,0;0,-1,1;-1/100-1/200,1/200+1/400,1/100+1/500]

    1.        0.        0.
    0.      - 1.        1.
  - 0.015     0.0075    0.012

-->B = [10;3;0]
-->V = inv(A)*B

V1 =     10.
V2 =      5.8461538
V3 =      8.8461538
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Complex Numbers: Determine the value of Y - showing the steps to find t

4) $Y = \left(\frac{10(s+3)}{(s+1)(s+5)}\right)_{s=j4}$

$Y = \left(\frac{10(j4+3)}{(j4+1)(j4+5)}\right)$

$$\left(\frac{10(s+3)}{(s+1)(s+5)}\right)_{s=j4} = 0.9039 - j1.6643$$
$$= 1.8939\angle -61.49^0$$

Either answer is correct.

5) $e^{(2+j3)} = (e^2)(e^{j3})$

$7.3891\angle 3\text{rad} = 7.3891\angle 171.88^0$ (polar form)

$$e^{(2+j3)} = 7.3891\angle 171.88^0$$
$$= -7.3151 + j1.0427$$

6) $Y = \ln(2 + j3)$

$Y = \ln(3.6056\angle 56.31^0)$

$Y = \ln(e^{\ln(3.6056)}\angle 1.2825 \text{ rad})$

$Y = \ln(e^{0.9828} \cdot e^{j1.2825})$

$Y = \ln(e^{0.9828+j1.2825})$

$$\ln(2 + j3) = 0.9828 + j1.2825$$
$$= 1.6157\angle 37.46^0$$

7) $(2 + j3)^{(4+j5)}$

$= (e^{(0.9828+j1.2825)})^{(4+j5)}$

$= e^{(0.9828+j1.2825)(4+j5)}$

$= e^{0.2159+j10.3435}$

$= (e^{0.2159})\angle 10.34 \text{ rad}$

$$(2 + j3)^{(4+j5)} = -0.7530 - j0.9864$$
$$= 1.2410\angle 127.35^0$$