ECE 331 - Homework #3

Maxwell's Equations, Magnetic Circuits, Inductors. Due Monday, February 3rd, 4PM

1) A rail gun is to accelerate a penny with an acceleration of 1000 m/s^2. Specify the current and magnetic field strength required.

From Wikipedia, a 2011 U.S. penny weight 2.5 grams. The force corresponding to 1000 m/s^2 is then

$$F = Mx'' = (0.0025kg)(1000m/s^2) = 2.5N$$

From Wikipedia (http://en.wikipedia.org/wiki/Magnetic_field)

$$f = BIl \sin\theta$$

Assume the penny is riding on a rail separated by 10mm, and the current is perpindicular to the B field:

$$2.5N = (B)(I)(0.01m)(1)$$

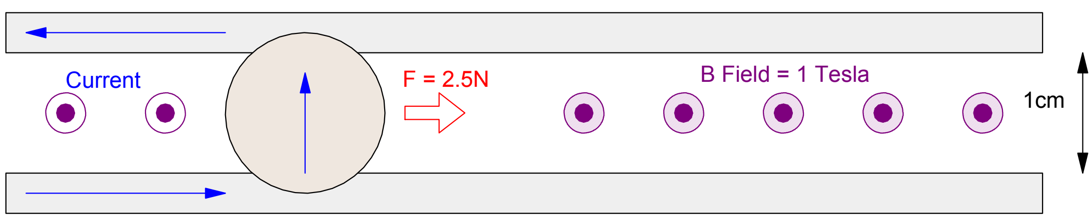
$$B \cdot I = 2500$$

Let

B = 1 Tesla (assume iron which saturates at 2.1 Tesla. Pick something less than this)

I = 2500 Amps

Note: If you assumed 1000N instead (typo in the assignment), you will need 1 million amps (!).



Reality Check:

The resistance of a penny is approximately $16\mu\Omega$. This means the I^2R heat is

- 100 Watts at 2500 Amps (the penny will be hot)
- 16MW at 1 million amps (the penny will vaporize)

If the rail gun is 2 meters long, the penny will be traveling at the end:

- 63 m/s if the acceleration is 1000 m/s^2
- 1,264 m/s if the force is 1000 N (ignoring the problem of vaporizing the penny)

In comparison, a bullet from a rifle can easily exceed 1300 m/s. Rail guns are not very practical.

2) Design a circuit (number of windings, current) to provide the magnetic field you spedified in problem #1. Assume an area of 1cm x 1cm.

First, do problem #3. The flux density is 0.000251 Teslas.

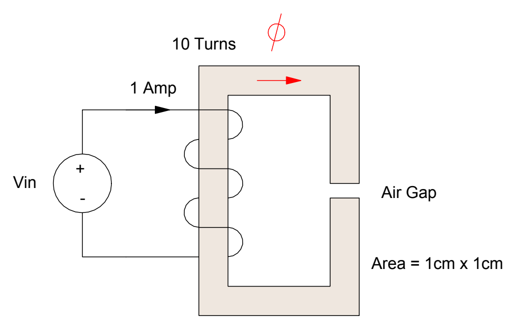
To increase this to 1 Tesla, increase the amp-turns by a factor of 3980

- Increase the number of turns to 3980 (x 398)
- Increase the current to 10A (x 10)

Now the B field is 1 Tesla.

3) The following circuit is used to generate a magnetic field across a 5mm air gap. Determine the flux density assuming

- The cross sectional area is 1cm x 1cm
- Each portion of the core has a length of 5cm
- The relative permiability of the core is 8,000



The permiability of the iron is

$$R = \frac{l}{\mu A}$$

$$R = \left(\frac{0.195m}{(8000)(4\pi\cdot 10^{-7})(0.01m)^2}\right)$$

$$R = 193,970\frac{Wb}{\text{amp turns}}$$

The permiability of the 5mm air gap is

$$R = \frac{l}{\mu A} = \left(\frac{0.005m}{(1)(4\pi\cdot 10^{-7})(0.01m)^2}\right)$$

$$R = 397,887,364$$

The flux for 1 Amp of current is then

$$F = (1A)(10turn) = 10 \text{ amp}\cdot\text{turns}$$

$$\phi = \frac{10\text{amp turns}}{(193,970)+(397,887,364)} = 25.1nWb$$

The flux density is

$$B = \frac{\phi}{A} = \frac{25.1nWb}{(0.01m)^2} = 0.000251\frac{Wb}{m^2} = 0.000251 \text{ Tesla}$$

4) Determine the flux if the air gap is redudec to 0.1mm

The permiability of iron is

$$R_{iron} = \left(\frac{0.1999m}{(8000)(4\pi \cdot 10^{-7})(0.01m)^2} \right)$$

$$R_{iron} = 198,843 \frac{Wb}{\text{amp turn}}$$

The permiability of the 1mm air gap is

$$R_{air} = \frac{l}{\mu A} = \left(\frac{0.0001m}{(1)(4\pi \cdot 10^{-7})(0.01m)^2} \right)$$

$$R_{air} = 7,957,747 \frac{Wb}{\text{amp turn}}$$

The flux is then

$$\phi = \frac{\text{10 amp turns}}{(198,843)+(7,957,747)} = 1.226\mu Wb$$

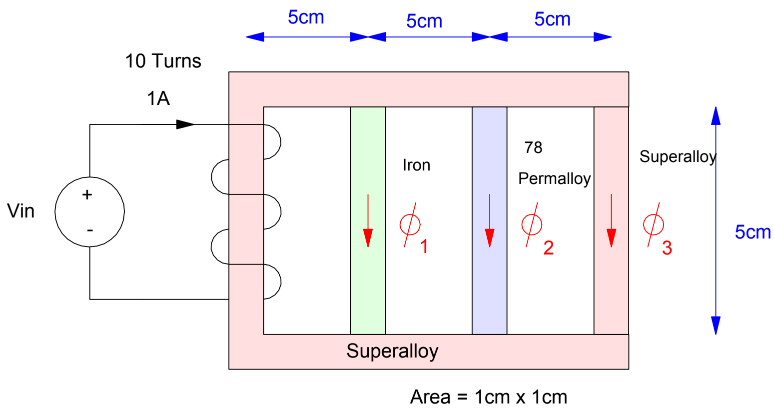
Note that by reducing the air gam from 5mm to 0.1mm, the flux has increased by a factor of 50x.

The air gap is what limits the magnetic flux.

- **If possible eliminate the air gap (transformers)**
- **If not possiblem make the air gap as small as possible (motors)**

5) Determine the flux $\phi_1 \ldots \phi_3$. Assume the relative permiabilities are

- Iron: 200
- 78 Permalloy: 8,000
- Superalloy 100,000



Determine the reluctance of each branch:

15cm of Superalloy:

$$R = \left(\frac{0.15m}{(100{,}000)(4\pi \cdot 10^{-7})(0.01m)^2} \right) = 11,936 \approx 12k$$

5cm of Superalloy:

$$R = \left(\frac{0.05m}{(100{,}000)(4\pi \cdot 10^{-7})(0.01m)^2} \right) = 3,978 \approx 4k$$

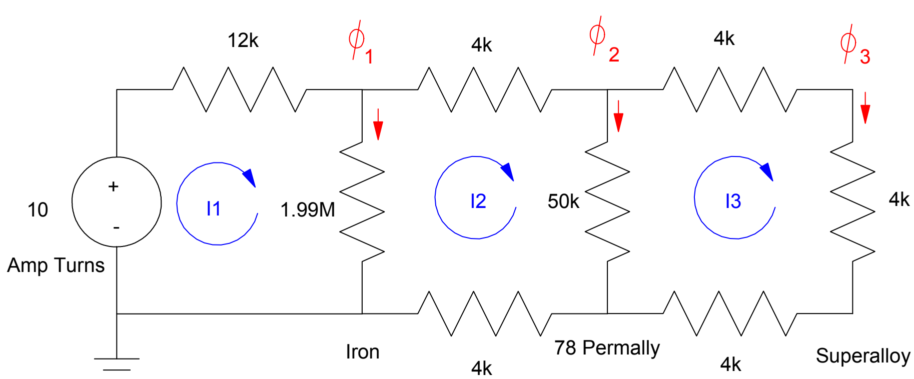
5cm of Iron

$$R = \left(\frac{0.05m}{(200)(4\pi \cdot 10^{-7})(0.01m)^2} \right) = 1,989,000 \approx 2M$$

5cm of 78 Permalloy:

$$R = \left(\frac{0.05m}{(8{,}000)(4\pi \cdot 10^{-7})(0.01m)^2} \right) = 49,725 \approx 50k$$

Draw the circuit model:



Write the loop equations

$$-10 + 12k(I_1) + 1.99M(I_1 - I_2) = 0$$

$$1.99M(I_2 - I_1) + 4k(I_2) + 50k(I_2 - I_3) + 4k(I_2) = 0$$

$$50k(I_3 - I_2) + 4k(I_3) + 4k(I_3) + 4k(I_3) = 0$$

Group terms

$$(1.99M + 12k)I_1 + (-1.99M)I_2 = 10$$

$$(-1.99M)I_1 + (1.99M + 4k + 50k + 4k)I_2 + (-50k)I_3 = 0$$

$$(-50k)I_2 + (50k + 4k + 4k + 4k)I_3 = 0$$

Solve in MATLAB

```
-->A=[1990+12,-1990,0;-1990,1990+4+50+4,-50;0,-50,62] * 1000
 A  (x 1000)

    2002.  - 1990.    0.
  - 1990.    2048.  - 50.
    0.     - 50.      62.

-->B = [10;0;0]
 B  =

    10.
    0.
    0.

-->I = inv(A)*B
 I  =

    0.0003387331
    0.0003357505
    0.0002707666

-->phi = [I(1)-I(2);I(2)-I(3);I(3)]

    0.0000029825     Wb   (Iron)
    0.0000649840     Wb   (78 permalloy)
    0.0002707666     Wb   (Superalloy)
```

The blux density is this divided by the area (0.0001m^2)

```
-->B = phi * 1e4

    0.029825    Tesla   (Iron)
    0.649840    Tesla   (78 permalloy)
    2.707666    Tesla   (Superalloy)
```

Note that most (80%) of the flux flows through the superalloy. In general, magnetic flux flows through the iron if possible, through the material with the highest relative permability.

Also note that this analysis is a little off. B is more than 2.1 Teslas, meaning the superalloy is saturated.

Problem 6-7) Determine a RL series and parallel model for a device with the following data:

Problem 6)

- Volts = 120V rms
- Current = 140mA rmx
- Power = 1 Watt
- Frequency = 60Hz

$$|Z| = \frac{120V}{140mA} = 857\Omega$$

$$pf = \frac{1Watt}{(120V)(0.14A)} = 0.0595$$

$$\arccos(0.0595) = 86.58^0$$

$$Z = 857\angle 86.58^0$$

$$Z = 51 + j855$$

Rs = 51 Ohms

jXs = j855 Ohms

$$\frac{1}{Z} = 0.0000696 - j0.001164$$

$$\frac{1}{Z} = \frac{1}{R_p} + \frac{1}{jX_p}$$

Rp = 1436 Ohms

jXp = j859 Ohms

Problem 7)

- Volts = 24V rms
- Current = 400 mA rms
- Current lags voltage by 70 degrees

$$Z = \left(\frac{24V}{400mA}\right)\angle 70^0$$

$$Z = (60\angle 70^0)\Omega$$

$$Z = 20.52 + j56.38$$

Rs = 20.52 Ohms

jXs = j56.38 Ohms

$$\frac{1}{Z} = 0.005700 - j0.015661$$

$$\frac{1}{Z} = \frac{1}{R_p} + \frac{1}{jX_p}$$

Rp = 175 Phms

jXp = j68.8 Ohms

Bonus! What percentage of the electricity produced in the U.S. comes from coal?

From http://www.eia.gov/tools/faqs/faq.cfm?id=427&t=3

Energy sources and percent share of total electricity generation in 2012 were:

- Coal 37%
- Natural Gas 30%
- Nuclear 19%
- Hydropower 7%
- Other Renewable 5%
- Biomass 1.42%
- Geothermal 0.41%
- Solar 0.11%
- Wind 3.46%
- Petroleum 1%
- Other Gases < 1%