ECE 331 - Solution to Homework #4

Ideal Transformers, Transformer Testing, Transformer Design, Auto-Transformers
Due Wednesday, February 19th, 4PM

1) The following circuit uses a transformer to buffer a transistor amplifier to an 8 Ohm speaker.

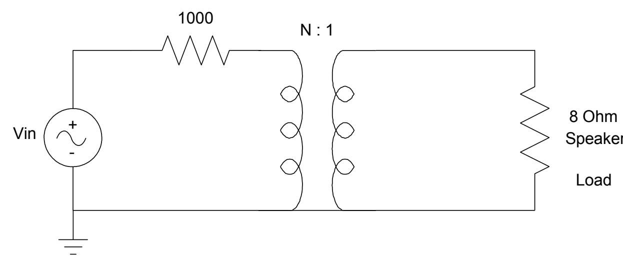
1a) Determine the turns ratio for this amplifier to be 90% efficient.

The efficiency is

$$\eta = \frac{P_{out}}{P_{in}}$$

$$0.9 = \frac{I^2 \cdot 8\Omega}{I^2 \cdot (8\Omega + R)}$$

R = 0.8888



The 1000 Ohm resistor needs to look like an 0.8888 Ohm resistor at the load. To do this, you need a turns ratio of

$$0.8888\Omega = \left(\frac{1}{N}\right)^2 1000\Omega$$

N = 33.55

1b) For the turn ration of part a), determine the voltage, Vin, to drive the speaker at 10 Watts.

For 10 Watts at the load,

$$P = I^2 R$$

$$10W = I^2 \cdot 8\Omega$$

$$I = 1.118A$$

The voltage source is then

$$V_{in} = I \cdot (0.888 + 8) = 9.938V$$

This is on the right side of the transformer. Bringing it left by the turns ratio

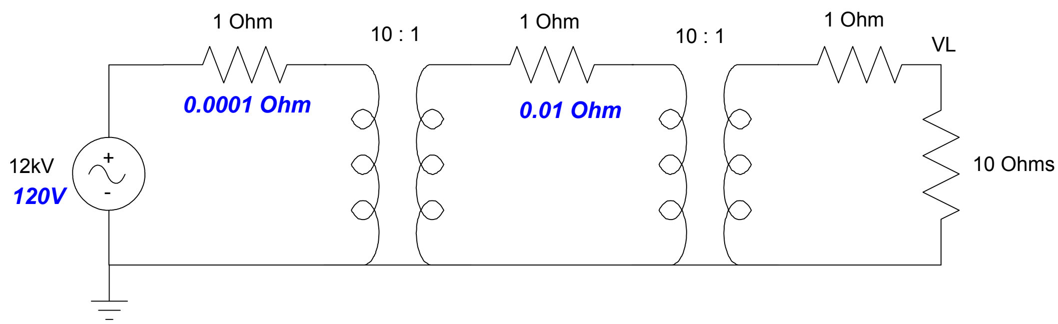
$$V_{in} = (33.55)(9.938V)$$

$$V_{in} = 333V$$

You need fairly high voltages to drive an 8-Ohm speaker through a transformer.

2) The following circuit uses ideal transformers.

- Determine the voltage at the 10 Ohm load (VL), and
- Determine the efficiency of this system.



Bringing everything to the load side (right) results in the voltages and impedances shown.

The voltage at the load by voltage division is then

$$V_L = \left(\frac{10}{10+1+0.01+0.0001}\right) 120V$$

$$V_L = 108.99V$$

The efficiency is

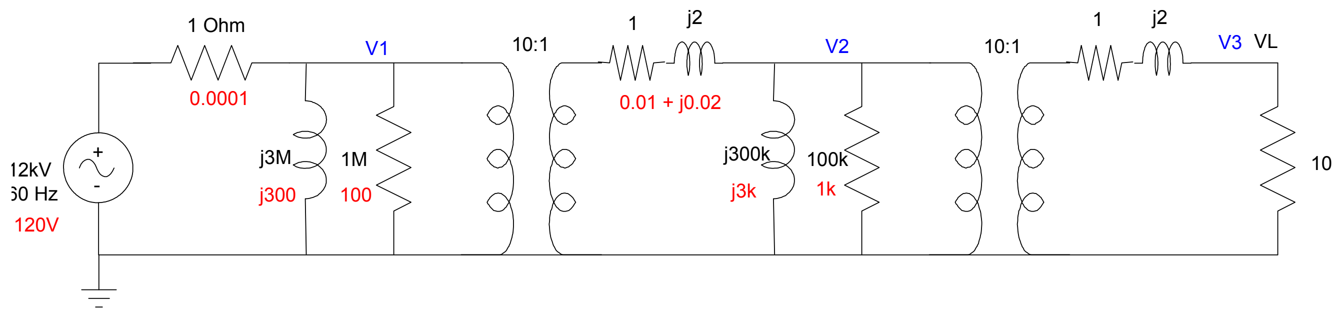
$$\eta = \frac{P_{out}}{P_{total}} = \frac{I^2 \cdot 10\Omega}{I^2 \cdot (10+1+0.01+0.001)}$$

$$\eta = 0.9083$$

3) The following circuit uses a more accurate model of a transformer:

- Determine the voltage at the 10 Ohm load (VL), and
- Determine the efficiency of this system.

Bring everything to the load side (right). This uses the values shown in red.



Writing 3 equations for 3 unknowns

$$\left(\frac{V_1-120}{0.0001}\right)+\left(\frac{V_1}{j300}\right)+\left(\frac{V_1}{100}\right)+\left(\frac{V_1-V_2}{0.01+j0.02}\right)=0$$

$$\left(\frac{V_2-V_1}{0.01+j0.02}\right)+\left(\frac{V_2}{j3000}\right)+\left(\frac{V_2}{1000}\right)+\left(\frac{V_2-V_3}{1+j2}\right)=0$$

$$\left(\frac{V_3-V_2}{1+j2}\right)+\left(\frac{V_3}{10}\right)=0$$

Putting this in MATLAB

```
a1 = [1/0.0001+1/(j*300)+1/100+1/(0.01+j*0.02),-1/(0.01+j*0.02),0]
a2 = [-1/(0.01+j*0.02),1/(0.01+j*0.02)+1/(j*3000)+1/1000+1/(1+j*2),-1/(1+j*2)]
a3 = [0,-1/(1+j*2),1/(1+j*2)+1/10]


-->A = [a1;a2;a3]

    10020.01 - 40.003333i  - 20. + 40.i               0
  - 20. + 40.i               20.201 - 40.400333i   - 0.2 + 0.4i
    0                      - 0.2 + 0.4i              0.3 - 0.4i

-->B = [120/0.0001;0;0]

    1200000.
    0.
    0.

-->V = inv(A)*B

    119.99881 + 0.0002375i
    119.85268 - 0.1932892i
    105.43943 - 19.346523i
```

The voltage at the load is

$$\boxed{V_L = 107.2V\angle -10^0}$$

To find the efficiency, find the power out and power in. Power out is the power across the 10 Ohm resistor:

```
-->Pout = ( abs(V(3)) ) ^2 / 10

    1149.1762
```

Power (or total power) is a little harder to find. One way is to compute V*I at the source:

```
-->Iin = (120 - V(1))/0.0001
 Iin  =

    11.86372 - 2.3747902i

-->Pin = real(120 * Iin)            actually you need the conjugate:
```

$P = V \cdot I^*$

```
 Pin  =

    1423.6465

-->eff = Pout / Pin
 eff  =

    0.8072062
```

The efficiency is 80.7%

This isn't very high - but you have only one customer. The efficiency goes up to 90.8% as the number of customers becomes large (meaning you can discount the core losses to zero).

4) A 10kVA transformer has the following open-circuit and short-circuit test results:

Open-Circuit Test

- Vin = 12kV
- Iin = 1A
- Pin = 1 Watt

The core impedance is

$$Z_{core} = \left(\frac{12kV}{1A}\right) = 12k\Omega$$

$$P_{in} = \frac{V^2}{R}$$

$$1W = \frac{(12kV)^2}{R}$$

$$R_p = 144M\Omega$$

$$jX_p = j12k\Omega$$

Short-Circuit Test:

- Vin = 24V
- Iin = 2A
- Pin = 40W

Determine a model for this transformer.

$$Z_{Cu} = \left(\frac{24V}{2A}\right) = 12\Omega$$

$$pf = \frac{40W}{2A \cdot 24V} = 0.8333$$

$$Z_{Cu} = 12\angle 33.5573^0$$

$$Z_{Cu} = 10 + j6.63$$

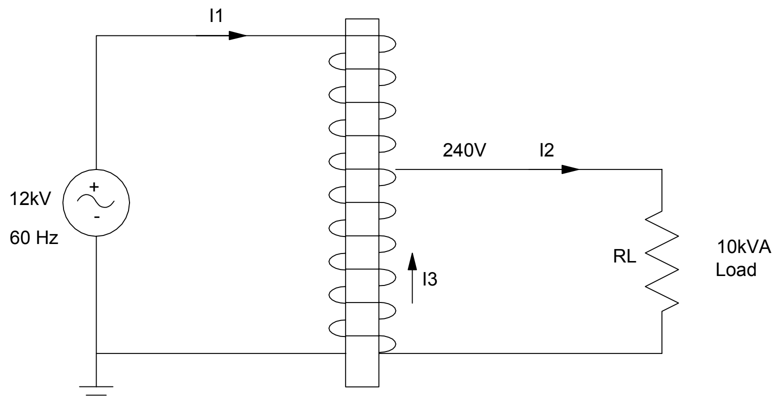
5) For problem #4, what current would you expect if you ran the short-circuit test at 12kV?

Assuming the short-circuit test was done on the high side, if you raist the voltage by a factor of 500, power goes up as 500 squared:

Pin = 10MW

You'll fry the transformer.

6) The following auto-transformer steps 12kV down to 240V. Determine the currents I1, I2, and I3



Power has to balance (and be 10kVA). At the left side

$$10kVA = (12kV)(I_1)$$

$$I_1 = 0.8333A$$

On the right side:

$$10kVA = (240V)(I_2)$$

$$I_2 = 41.6667A$$

Current has to balance:

$$I_1 + I_3 = I_2$$

$$I_3 = 40.83A$$