ECE 331 - Homework #7

3 Phase Induction Motors and Generators. Due March 24th 4PM
Assume all units are rms.

1) A three-phase, two-pole, 30hp, $120V_{LN}$, 60Hz Y connected induction motor draws a current of 30A from the line source at a power factor of 0.9. At this condition, the motor losses are:

- Stator copper losses = Pcul = 400W
- Rotor copper losses = Pcu2 = 200W
- Stator core losses = Pc = 140W
- Rotation losses = Prot = 100W

Calculations:

a) The power transferred across the air gap

Analyze this on a per-phase basis. The total answer will be 3 times the result per phase.

The line-to-neutral voltatage is $120V_{LN}$.

The electric power input and power across the gap are

Pelec = 3 · 120V · 30A · 0.90 = 9,720W

Pg = Pelec - PCu1 - Pc

Pg = 9,720W - 400W- 140W = 9,180W

b) The internally developed torque in Nm

n_s = 2p · 60Hz = 377 rad/sec

$$T = \left(\frac{9180W}{377 \text{ rad/sec}}\right) = 24.35Nm$$

c) the slip expressed in per unit and in rpm

$$P_{Cu2} = s \cdot P_g$$

$$s = \left(\frac{P_{Cu2}}{P_g}\right) = \left(\frac{200W}{9180W}\right) = 0.0218$$

d) the mechanical power developed in watts

$P_m = P_g - P_{Cu2}$ = 9180W- 200 = 8,980W

e) the horsepower output

$P_o = P_m - P_{rot}$ = 8,980W- 100W = 8,880W = 12.00 hp

f) the motor speed in rpm and radiands/second

$$n = (1 - s)n_s = (1 - 0.0218)377 \text{ rad/aec}$$

n = 368.78 rad/sec = 3,521 rpm

g) the torque at the output shaft

$$T_o = \frac{P_o}{n} = \frac{8880W}{368.78 \text{ rad/sec}} = 24.08Nm$$

h) the torque needed to overcome rotational losses

$$T_{rot} = \frac{P_{rot}}{n} = \frac{100W}{377 \text{ rad/sec}}$$

$$T_{rot} = 0.2653\ Nm$$

i) the efficiency of the operation in the stated conditions

$$\eta = \frac{P_o}{P_{elec}} = \frac{8880W}{9720W} = 0.9136$$

2) A three-phase, two pole, 20hp, $120V_{LN}$, 60Hz, Y connected induction motor has the following parameters per phase:

- r1 = 0.15 Ohm, x1 = 0.25 Ohms
- r2 = 0.10 Ohms x2 = 0.30 Ohms

The stator core losses are 400W and the rotational losses are 200W. At no-load, the motor draws 10A with a power factor of 0.1 lagging. When the motor operates at a slip of 3%, find

First, find the currents:

```
-->V1 = 120;

-->Im = 10.0 * exp(-j*84.26*%pi/180)

    1.0001441 - 9.9498599i

-->Rm = r2 * (1-s) / s

    3.2333333

-->I2 = V1 / (r1 + r2 + Rm + j*(x1+x2) )

    33.611794 - 5.3071253i

-->I1 = Im + I2

    34.611938 - 15.256985i
```

Synchronous and actual speed:

```
-->ws = 2*%pi*60;

-->w = (1-s)*ws;
```

Power across the air gap:

```
-->Pg = 3 * (abs(I2))^2 * r2/s

    11579.182

-->Tg = Pg / w

    31.664676

-->Pm = 3 * (abs(I2))^2 * Rm

    11231.807

-->Prot = 200 * (w / 377)
 Prot  =

    193.99543

-->Po = Pm - Prot

    11037.812
```

```
-->To = Po / w

    30.184232

-->Pe = real(3*V1 * I1)
 Pe  =

    12460.298

-->eff = Po / Pe

    0.8858385
```

Checking

```
-->[To, Po, eff] = slip(0.03,0.1)
 eff  =

    0.8858385
 Po  =

    11037.812
 To  =

    30.184232
```

a) the input line current and power factor

$$V_1 = 120V_{LN}$$

$$I_1 = I_m + I_2$$

$$I_m = 10\angle -84.26^0$$

$$R_m = r_2\left(\frac{1-s}{s}\right) = (0.1\Omega)\left(\frac{1-0.03}{0.03}\right) = 3.2333\Omega$$

$$I_2 = \frac{120V}{3.233\Omega+(0.15+j0.25)+(0.1+j0.3)} = 34.03\angle -8.97^0$$

so

$$I_1 = I_m + I_2 = 37.828\angle -23.787^0$$

power factor = 0.915 lagging

b) the developed electromagnetic torque in Nm

$$n_s = 2\pi \cdot 60Hz = 377 \text{ rad/sec}$$

$$n = (1-s)n_s = 365.69 \text{ rad/sec}$$

$$P_m = 3 \cdot |I_2|^2 R_m = 7488W$$

$$T_m = \frac{P_m}{n} = \frac{7488W}{365.69 \text{ rad/sec}} = 20.478Nm$$

c) the horsepower output

$$P_o = P_m - P_{rot}$$

$$P_o = 7488W - 200W = 7288W$$

$$P_o = 9.848Hp$$

d) the efficiency

$$P_{elec} = 3 \cdot real(V_1 \cdot I_1^*)$$

$$P_{elec} = 3 \cdot 120V \cdot 37.828A \cdot \cos(-23.787^0)$$

$$P_{elec} = 12.461W$$

$$\eta = \frac{P_o}{P_{elec}} = \frac{7{,}288W}{12{,}461W} = 0.585$$

Note: Write a SciLab routine for this:

```
function [To, Po, eff] = slip(s, r2)

j = sqrt(-1);

r1 = 0.15;
r2 = 0.10;
x1 = 0.25;
x2 = 0.30;

Pc = 400;   // stator core losses
Prot = 200;  // rotational losses

V1 = 120;
Im = 10.0 * exp(-j*84.26*%pi/180);

Rm = r2 * (1-s) / s;
I2 = V1 / (r1 + r2 + Rm + j*(x1+x2) );

I1 = Im + I2;

ws = 2*%pi*60;
w = (1-s)*ws;

Pg = 3 * (abs(I2))^2 * r2/s;
Tg = Pg / w;

Pm = 3 * (abs(I2))^2 * Rm;
Prot = 200 * (w / 377);

Po = Pm - Prot;
To = Po / w;

Pe = real(3*V1 * I1);

eff = 0;
if (s > 0) eff = Po / Pe; end
if (s < 0) eff = Pe / Po; end

endfunction
```

Checking

```
-->[To, Po, eff] = slip(0.03,0.1)
 eff  =

    0.8858385
 Po  =

    11037.812
 To  =

    30.184232
```

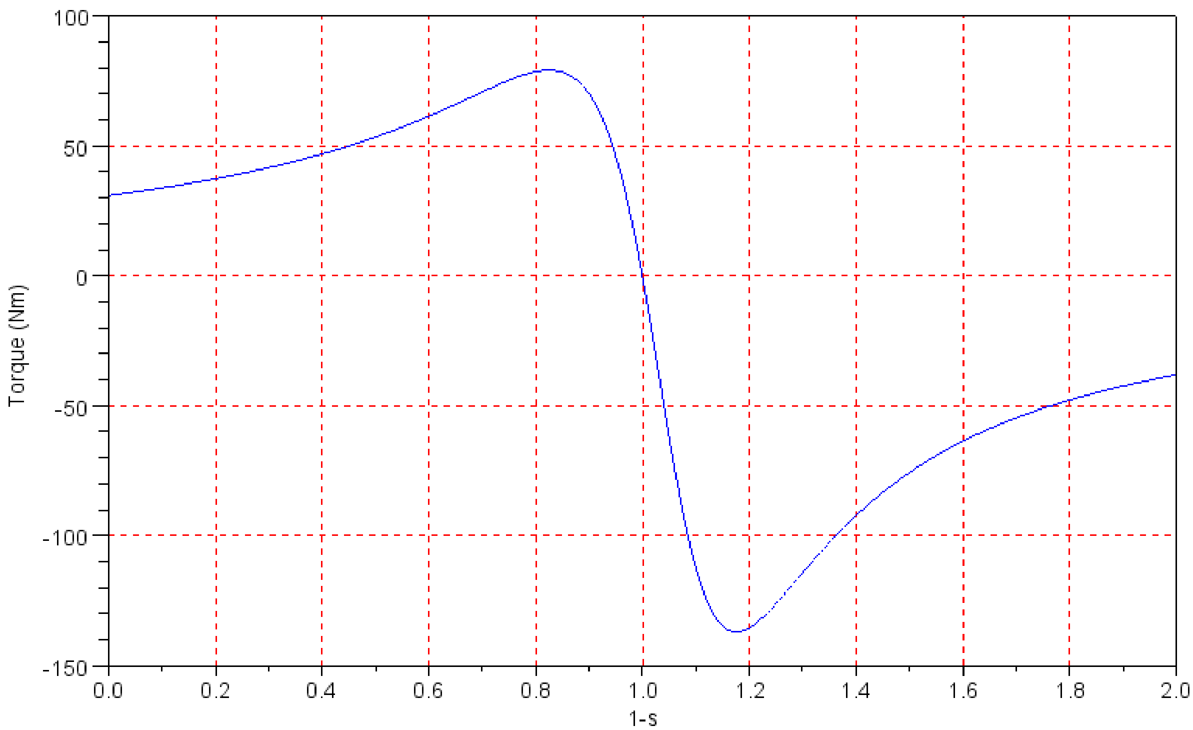
3) Plot the torque-slip speed relationship for the motor in problem 2.

```
T = 0*s;
P = 0*s;
eff = 0*s;

s = [-0.999:0.001:0.999]' + 1e-6;

for i=1:length(s)
 [T(i),P(i),eff(i)] = slip(s(i), 0.1);
 end

plot(1-s,T)
xlabel('1-s')
ylabel('Torque (Nm)')
xgrid(5)
```



4) Find r2 so that the starting torque is 70% of the peak torque.

Using trial and error

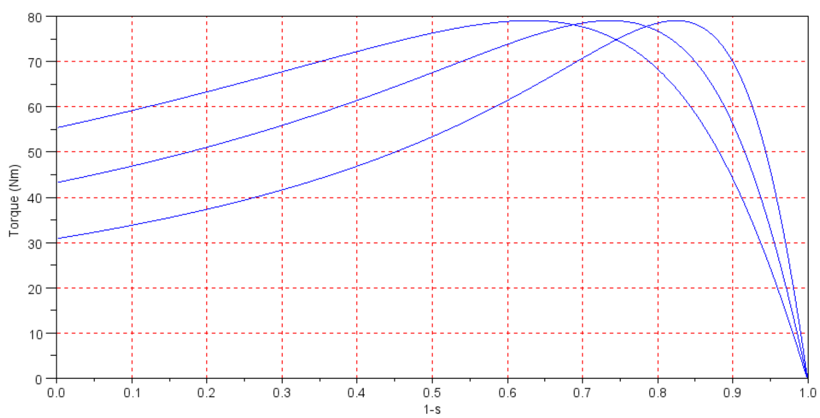
```
T = 0*s;
P = 0*s;
eff = 0*s;

s = [0.001:0.001:0.999]';

for i=1:length(s)
 [T(i),P(i),eff(i)] = slip(s(i), 0.2112);
 end

plot(1-s,T)
xlabel('1-s')
ylabel('Torque (Nm)')
xgrid(5)

T(999) / max(T)
```



r2 = 0.1 (bottom curve), 0.15, 0.2112 (top curve)

Starting torque = 70% of the peak torque for r2 = 0.2112 Ohms

r2	%
0.1	39%