Voltage, Current, Resistance, and Power

Current:

Current is the flow of electrons with 1 Amp being 1 Coulomb of electrons flowing past a point every second. (1 Coulomb = $6.02 \cdot 10^{23}$). Current is what actually carries the energy in a circuit and what causes heating (and damage).

Voltage:

Voltage is the force which tries to force current flow. 1 Amp flowing across 1 Volt delivers 1 Watt of power.

Resistance:

When you apply a voltage across a device, the current flow is not infinite. The material limits the current flow, with a mathematical model being

$$V = I R$$

The resistance is a function of the material being used as well as the dimensions of the device as

$$R = \frac{\rho L}{A}$$

where ρ is the conductivity of the material, L is the length, and A is the cross-sectional area.

Material	Resistivity (Ohm-meters)	
Silver	1.59 x 10-8	Best conductor
Copper	1.68 x 10-8	
Silicon	6.40 x 102	
Aluminum	2.82 x 10-8	
Iron	1.0 x 10-7	
Sea Water	2 x 101	
Teflon	1 x 1022	Insulator

from www.Wikipedia.com

For small circuits, the resistnace of copper can be assumed to be zero. For example, determine the resistance of a copper trace on a circuit board with dimensions:

Length = 10cm

Cross-sectional area = 0.1mm x 1mm

Solution:

$$R = \frac{\rho L}{A} = \frac{(1.68 \cdot 10^{-8} \Omega m)(0.01 m)}{(0.0001 m) x (0.001 m)} = 0.0017 \Omega$$

In this class, the resistance of copper can become significant.

Problem 2: An transformer has 100 windings of 12 gage copper wire around the core. Assume the core is 5cm x 5cm in cross section. Find the resistance of the copper wire.

Solution: From Wikipedia, 12 gage copper wire has a diameter of 2mm

$$R = \frac{\rho L}{A} = \frac{(1.68 \cdot 10^{-8} \Omega m)(100 \cdot 0.2m)}{\pi \cdot (0.001m)^2} = 0.107\Omega$$

If you're dealing with 100 Amps, even 0.107 Ohms dissipates 1kW of heat.

Problem 3: Determine the resistance of a copper transmission line. Assume the line has a length of 1000km (Montana to Minneapolis) and has a cross-sectional area of 1cm2

Solution

$$R = \frac{\rho L}{A} = \frac{(1.68 \cdot 10^{-8} \Omega m)(10^6 m)}{(0.01m)(0.01m)} = 168\Omega$$

168 Ohms might seem small, but if you consider that a large power plant can produce 1,000 MW, the transmission line losses (I^2R) can be significant. The losses depend upon the voltage:

Voltage	Current for 1000MW	Transmission Line Losses (I2R)
120V	8.33MA	1.1 x 1016 W
10kV	100 kA	1.68 x 1012 W
1MV	1000 A	168 MW

Even at 1MV, you're losing 16% of the energy produced in transmission line losses. This is part of the reason for this course:

- You need to increase the voltage to reduce the transmission line losses (i.e. you need transformers)
- You need more area than 1cm x 1cm (i.e. you need more than one transmssion line), and
- You need to move the generators closer to the demand.

Power & RMS:

Power is the rate at which energy is delivered (Joules / second).

For DC voltages:

$$P = VI = \frac{V^2}{R} = I^2R$$

For AC sourses with a peak voltage of Vp,

$$P = \frac{1}{2}V_pI_p = \frac{1}{2}\frac{V_p^2}{R} = \frac{1}{2}I_p^2R$$

So that you don't need to use different equations for DC and AC, rms (root-mean-squared) voltage is used. Essentially, the 1/2 is distributed amoung the voltag and current so that

$$V_{rms} = \frac{1}{\sqrt{2}}V_p$$

$$I_{rms} = \frac{1}{\sqrt{2}} I_p$$

If you use rms voltage and current, then AC formulas are the same as DC formulas:

$$P = V_{rms} I_{rms} = \frac{V_{rms}^2}{R} = \frac{1}{2} I_{rms}^2 R$$

You likewise need to watch and note whether you are talking about peak of rms voltages when dealing with AC circuits.

Resistors in Parallel and Series

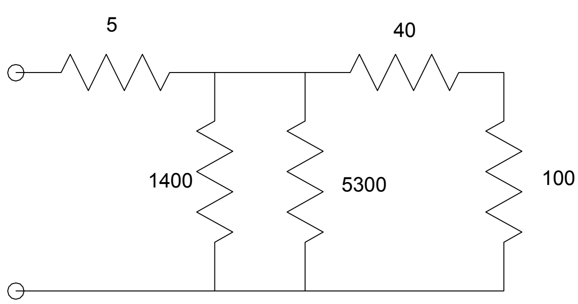
Resistors in series add:

$$R_{total} = R_1 + R_2 + R_3$$

Resistors in perallel add as the sum of the invreses inverted:

$$R_{total} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$$

Example: Determine the resistance of the the following circuit:



Solution: Combine the resistors in series and parallel:

100 + 40 = 140	(series)
140 || 5300 || 1400 = 124.29	(parallel)
124.29 + 5 = 129.29	(series)

This looks like a 129.29 Ohm resistor.

Kirchoff's Laws

Kirchoff's laws simply restate the conservation of voltage and current:

- If you sum the voltages around any closed path, the sum must be zero.
- If you sum the current flowing away from a point, the sum must be zero.

This allows you to solve for the voltages or currents in a circuit by writing N equations to solve for N unknowns.

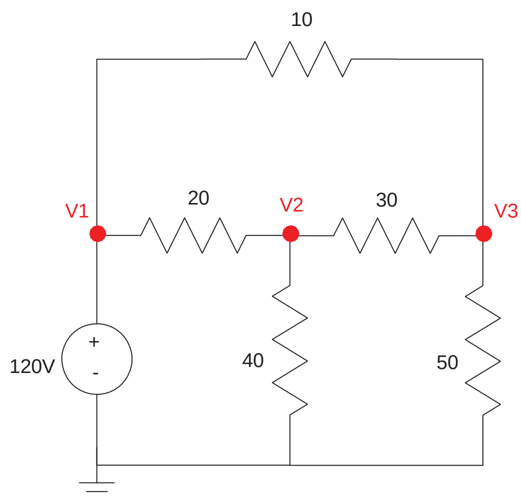
Kirchoff's Voltage Nodes (KVN):

Objective: Write N voltage node equations to solve for N unknown voltages.

Procedure:

- Step 1: Define ground for a circuit.
- Step 2: Define N voltage nodes.
- Step 3: Write N equations to solve for N unknowns
  - For each voltage source, write an equation Vp - Vm = Voltage Source
  - For the remaining N equations, sum the current to zero at each node
- Step 4: Solve.

Example: Find the voltages in the following circuit:



Step 1 & 2: Done already.

Step 3: Write N equations to solve for N unknowns

Nove V1: Voltage Source:

$$V_1 - 0 = 120$$

Node V2: Sum the current flowing from node V2 to zero.

$$\left(\frac{V_2 - V_1}{20}\right) + \left(\frac{V_2 - 0}{40}\right) + \left(\frac{V_2 - V_3}{30}\right) = 0$$

Node V3: Sum the current flowing from node V3 to zero.

$$\left(\frac{V_3-V_1}{10}\right)+\left(\frac{V_3-V_2}{30}\right)+\left(\frac{V_3-0}{50}\right)=0$$

Step 4: Solve.

This is the sort of problem MATLAB does extremely well. First group terms:

$$V_1 = 120$$

$$\left(\frac{-1}{20}\right)V_1+\left(\frac{1}{20}+\frac{1}{40}+\frac{1}{30}\right)V_2+\left(\frac{-1}{30}\right)V_3=0$$

$$\left(\frac{-1}{10}\right)V_1+\left(\frac{-1}{30}\right)V_2+\left(\frac{1}{10}+\frac{1}{30}+\frac{1}{50}\right)V_3=0$$

Put in matrix form

$$\begin{bmatrix} 1 & 0 & 0 \\ \left(\frac{-1}{20}\right) & \left(\frac{1}{20}+\frac{1}{40}+\frac{1}{30}\right) & \left(\frac{-1}{30}\right) \\ \left(\frac{-1}{10}\right) & \left(\frac{-1}{30}\right) & \left(\frac{1}{10}+\frac{1}{30}+\frac{1}{50}\right) \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}=\begin{bmatrix} 120 \\ 0 \\ 0 \end{bmatrix}$$

Solve in MATLAB

```
->A=[1,0,0;-1/20,1/20+1/40+1/30,-1/30;-1/10,-1/30,1/10+1/30+1/50]
 A  =

    1.        0.           0.
  - 0.05      0.1083333  - 0.0333333
  - 0.1    - 0.0333333    0.1533333

-->B = [120;0;0]
 B  =

    120.
    0.
    0.

-->V = inv(A)*B
 V  =

    120.
    85.16129
    96.774194
```

Meaning

- V1 = 120V
- V2 = 85.162V
- V3 = 96.774V

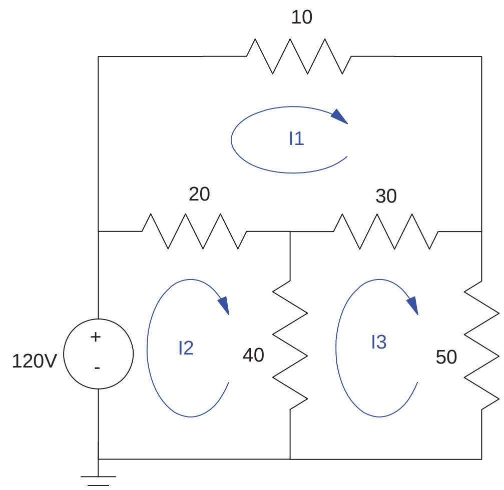
Kirchoff's Current Loops (KCL):

Objective: Write N current loop equations to solve for N unknown currents.

Procedure:

- Step 1: Define N current loops
- Step 2: Write N equations to solve for N unknowns
  - For each current source, write an equation for the current
  - For the remaining N equations, sum the voltages to zero around a closed-loop
- Step 3: Solve.

Example: Find the currents for the following circuit:



Solution:

Step 1: Done already.

Step 2: Sum the voltages around each loop to zero

Loop I1:

$$10I_1 + 30(I_1 - I_3) + 20(I_1 - I_2) = 0$$

Loop I2:

$$-120 + 20(I_2 - I_1) + 40(I_2 - I_3) = 0$$

Loop I3:

$$40(I_3 - I_2) + 30(I_3 - I_1) + 50I_3 = 0$$

Step 3: Solve.

Again, MATLAB is useful. Group terms:

$$60I_1 - 20I_2 - 30I_3 = 0$$

$$-20I_1 + 60I_2 - 40I_3 = 120$$

$$-30I_1 - 40I_2 + 120I_3 = 0$$

Place in matrix form:

$$\begin{bmatrix} 60 & -20 & -30 \\ -20 & 60 & -40 \\ -30 & -40 & 120 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 120 \\ 0 \end{bmatrix}$$

Solve using MATLAB

```
-->A = [60,-20,-30;-20,60,-40;-30,-40,120]
 A  =

    60.  - 20.  - 30.
  - 20.    60.  - 40.
  - 30.  - 40.    120.

-->B = [0;120;0]
 B  =

    0.
    120.
    0.

-->I = inv(A)*B
 I  =

    2.3225806
    4.0645161
    1.9354839
```

Meaning

- I1 = 2.32 A
- I2 = 4.06 A
- I3 = 1.93 A