RL Circuits

In general, the impedance of an inductor is

$$Z = j\omega L$$

This tells you that the impedance changes as frequency changes. In this class, however, frequency is almost always 60Hz (377 rad/sec). Since the frequency is fixed, the impedance of an inductor is often written simply as

$$Z = jX$$

Since the frequency is fixed, the impedance of a load (be it an inductor, tranformer, motor) really only has two degrees of freedom: the amplitude and phase shift (in polar form) or real and complex impedance (in rectangular form).

$$Z\angle\theta = R + jX$$

As long as the complex part is positive (meaning the angle is between 0 and 90 degrees), you can model such a load with an RL circuit.

Example 1: The voltage, current, and power to an inductor at 60Hz was measured as

$$V = 120V_{rms}$$

$$I = 120mA_{rms}$$

$$P = 2.4W$$

Determine a model dor the inductor.

Solution: The impedance is

$$|Z| = \left|\frac{V}{I}\right| = \frac{120V_{rms}}{0.12A_{rms}} = 1000\Omega$$

The angle is

$$P = VI \cdot \cos\theta$$

$$2.4W = (120V)(0.12A) \cdot \cos\theta$$

$$pf = \cos\theta = 0.1667$$

$$\theta = 80.40^0$$

A model for the inductor is any circuit with an impedance of

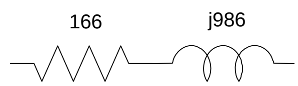
$$Z = 1000\angle 80.4^0$$

Series Model:

$$Z = 166.66 + j986.01$$

R = 166.66

jX = j986.01

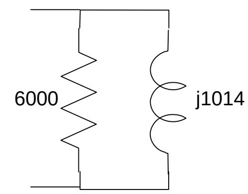


Parallel Model:

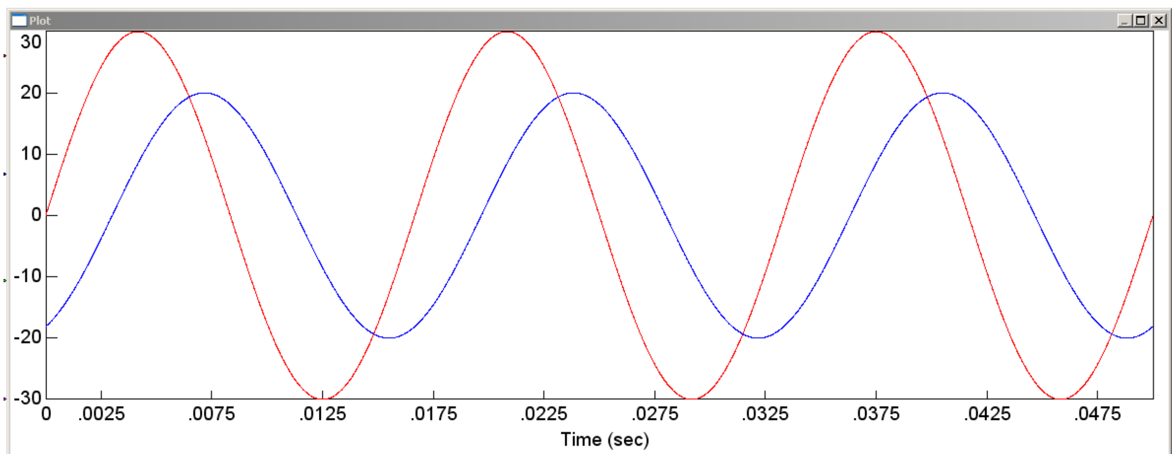
$$\frac{1}{Z} = 0.0001667 - j0.000986 = \frac{1}{R} + \frac{1}{jX}$$

R = 6000

jX = j1014



Example: The voltage and current to an inductor was measured as follows:



Voltage (V, red) & Current (mA, blue)

Determine a model for this inductor.

Solution: The impedance is

$$Z = \frac{V}{I} = \frac{30V_p}{20mA_p} = 1500\Omega$$

Note that you can use peak or rms measurements. The units cancel as long as both current and voltage are measured the same way.

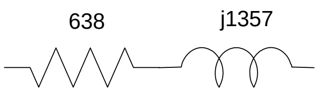
The angle is from the delay. Current is delayed by 0.003 seconds. Converting to degrees

$$\theta = (-0.003\,\text{sec}) \cdot \frac{360^0}{0.0167\,\text{sec}} = -64.8^0$$

The impedance is then

$$Z = \frac{30\angle 0^0}{20mA\angle -64.8^0} = 1500\angle 64.8^0\Omega$$

Series Model:

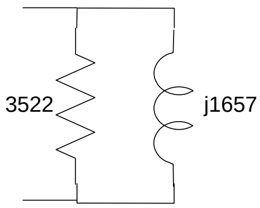


$$Z = 638.7 + j1357$$

R = 638.7

jX = j1357

Parallel model:

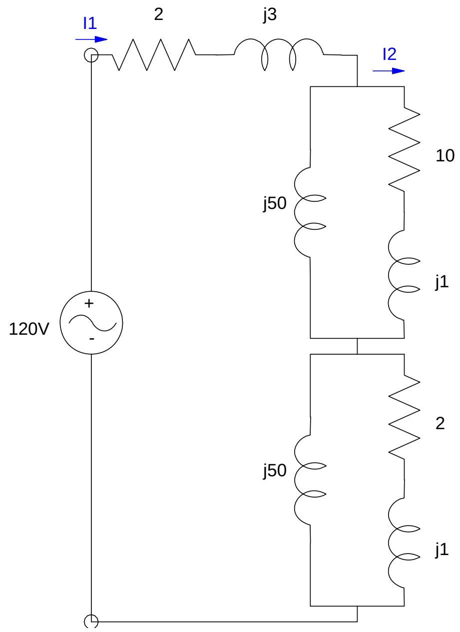


$$\frac{1}{Z} = 0.0002839 - j0.0006032$$

$$R = \frac{1}{0.0002839} = 3522$$

$$jX = \frac{1}{-j0.0006032} = j1657$$

Example 3: Determing the total impedance and currents I1 and I2:



The total impedance is

(j50) || (10+j1) = 9.2558 + j2.7953

(j50) || (2+j1) = 1.9194 + j1.0557

Z = (2 + j3) + (9.2558 + j2.7953) + (1.9194 + j1.0557)

Z = 13.1752 + j6.8509

The current I1 is then

$$I_1 = \frac{120V}{(13.1752+j6.8509)\Omega} = 7.1695 - j3.7280$$

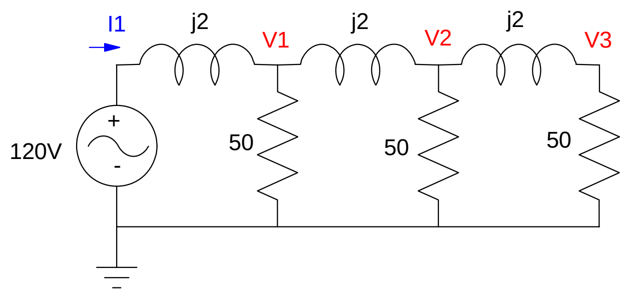
$$I_1 = 8.0808\angle -27.47^0$$

By current division, the current I2 is then

$$I_2 = \left(\frac{(j50)}{(j50)+(10+j1)}\right) I_1$$

$$I_2 = 7.7743\angle -16.38^0$$

Example 4: Determine the current, I1, and voltages V1, V2, and V3



To find the voltages, write the voltage node equations:

$$\left(\frac{V_1-120}{j2}\right) + \left(\frac{V_1}{50}\right) + \left(\frac{V_1-V_2}{j2}\right) = 0$$

$$\left(\frac{V_2-V_1}{j2}\right) + \left(\frac{V_2}{50}\right) + \left(\frac{V_2-V_3}{j2}\right) = 0$$

$$\left(\frac{V_3-V_2}{j2}\right) + \left(\frac{V_3}{50}\right) = 0$$

Group terms

$$\left(\frac{1}{j2}+\frac{1}{50}+\frac{1}{j2}\right)V_1-\left(\frac{1}{j2}\right)V_2=\frac{120}{j2}$$

$$\left(\frac{-1}{j2}\right)V_1+\left(\frac{1}{j2}+\frac{1}{50}+\frac{1}{j2}\right)V_2+\left(\frac{-1}{j2}\right)V_3=0$$

$$\left(\frac{-1}{j2}\right)V_2+\left(\frac{1}{j2}+\frac{1}{50}\right)V_3=0$$

Place in matrix form

$$\begin{bmatrix} \left(\frac{1}{j2}+\frac{1}{50}+\frac{1}{j2}\right) & \left(\frac{-1}{j2}\right) & \\ \left(\frac{-1}{j2}\right) & \left(\frac{1}{j2}+\frac{1}{50}+\frac{1}{j2}\right) & \left(\frac{-1}{j2}\right) \\ & \left(\frac{-1}{j2}\right) & \left(\frac{1}{j2}+\frac{1}{50}\right) \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}=\begin{bmatrix} \left(\frac{120}{j2}\right) \\ 0 \\ 0 \end{bmatrix}$$

Solve in MATLAB

```
-->A =
[1/j+1/50,-1/(j*2),0;-1/(j*2),1/j+1/50,-1/(j*2);0,-1/(j*2),1/(j*2)+1/50]

    0.02 - i      0.5i          0
    0.5i          0.02 - i      0.5i
    0             0.5i          0.02 - 0.5i

-->B = [120/j/2;0;0]

  - 60.i
    0
    0


-->inv(A)*B

V1:    117.41619 - 13.883439i
V2:    115.38772 - 23.070231i
V3:    114.28206 - 27.641513i
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or in polar form

V_1 =118.23414∠-6.74^0

V_2 =117.67142∠-11.30^0

V_3 =117.57739∠-13.59^0