Magnetic Circuits & Inductors

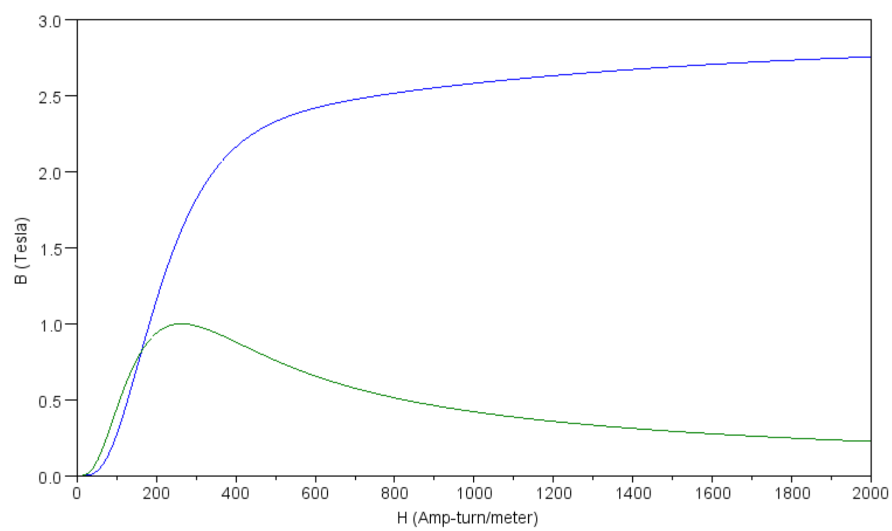
Volts = Magnetomotive Force

Ohms = Reluctance

Amps = Flux

Material	Permiability @ 20 Gauss (2mT)	Maximum Permiability	Saturation Flux Density B (Tesla)	Hysteresis Loss ergs / cm3	Coercive Force Oersteds
Cold Rolled Steel	180	2,000	2.1		1.8
Iron	200	5,000	2.15	5,000	1
Purified Iron	5,000	180,000	2.15	300	0.05
4% Silicon Iron	500	7,000	1.97	3,500	0.5
78 Permalloy	8,000	100,000	1.07		0.05
Superalloy	100,000	800,000	0.8	12,000	0.002

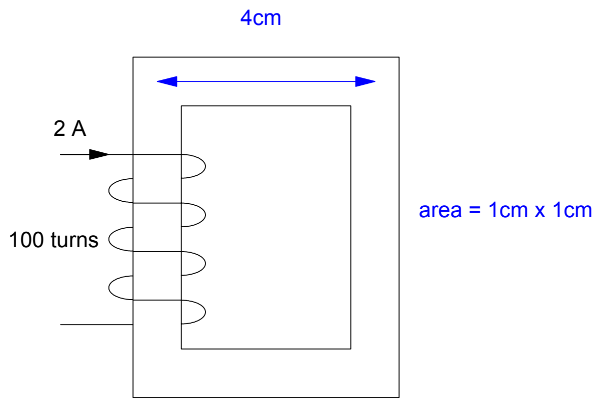
(from CRC Handbook of Chemistry and Physics - 58th Edition)



Magnetization Curve for Iron (blue) and it's relative permiability / 5000 (green)

Magnetic Circuits

Example 1: An iron core has a cross sectional area of 1cm^2. 0.2A is applied with 100 turns. Determine the flux density, B. Assume $\mu_r = 5000$.



Solution: Find the reluctance:

$$R = \frac{l}{\mu A} = \frac{0.16m}{(5000)(4\pi\cdot 10^{-7})(0.01m)^2} = 254,647$$

The magnetic flux is

$$\Phi = \frac{NI}{R} = \left(\frac{(100\text{ Turns})(0.2\text{A})}{254{,}647}\right) = 78.54\ \mu Wb$$

The magnetic flux density is

$$B = \frac{\Phi}{A} = \frac{78.54\mu Wb}{(0.01m)^2} = 0.785\frac{Wb}{m^2} = 0.785T$$

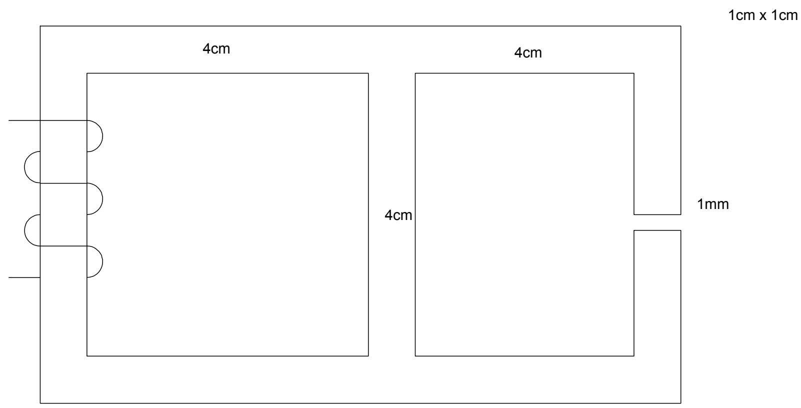
Properties of Iron & Saturation

The relative permiability of purified iron is only 5000 for a limited range. The iron saturates at approximately 2.15 Teslas. Likewise, for this circuit, the flux will clip at approximately

$$\Phi_{\max} \approx \left(2.15\frac{Wb}{m^2}\right)(0.01m)^2 = 215\mu Wb$$

To prevent saturation, you can increase the current to 547mA.

Problem 1: An iron core has a cross sectional area of 1cm2. 500mA is applied with 100 turns. Draw the circuit equivalent and find the flux in each section.



Solution:

The reluctance of each branch is

$$R = \frac{l}{\mu A} = \frac{0.04m}{(5000)(4\pi \cdot 10^{-7})(0.01m)^2} = 63,661$$

The metal with the air gap is

$$R = \frac{l}{\mu A} = \frac{0.039m}{(5000)(4\pi \cdot 10^{-7})(0.01m)^2} = 62,070$$

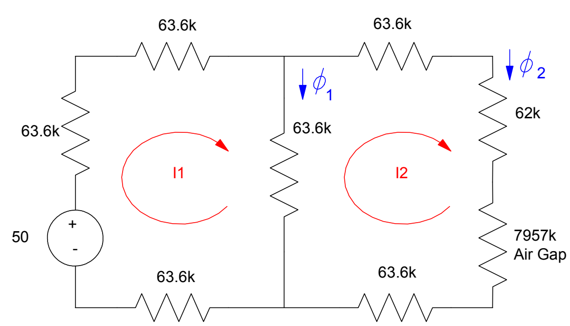
The reluctance of the air gap is

$$R = \frac{l}{\mu A} = \frac{0.001m}{(1)(4\pi \cdot 10^{-7})(0.01m)^2} = 7,957,747$$

The magnetomotive force (mmf) is

F = (0.5A)(100 Turns) = 50 Amp turns

This gives us the following circuit:



The flux (I1 and I2) are then from solving two loop equations

$$(63.6k + 63.6k + 63.6k + 63.6k)I_1 - (63.6k)I_2 = 50$$

$$(63.6k + 62k + 63.6k + 7.9M + 63.6k)I_2 - (63.6k)I_1 = 0$$

Solving in MATLAB:

```
-->A = [63.6*4,-63.6;-63.6, 63.6*4+7900] * 1000
    254400.   - 63600.
  - 63600.     8154400.

-->B = [50;0]
    50.
    0.

-->I = inv(A)*B
    0.0001969
    0.0000015

-->Phi = [I(1) - I(2), I2]
    0.0001954      Webers (ϕ1)
    0.0000015      Webers (ϕ2)
```

The flux density is

```
--> B = [Phi1; Phi2]  / (0.01^2)     (Teslas)

    1.9538895     Teslas (the iron bar)
    0.0153591     Teslas  (the air gap)
```

Note that the magnetic fields stick to iron. The path with a small air gap has almost no flux. We'll use this later to assume all flux lines go through the iron.

Problem 2: Increase the current 10x to 5A. Computer the flux ϕ_1 and ϕ_2.

Solution: If this was a linear circuit, increasing the current 10x will increase the flux 10x. The problem is that by doing so, the flux density becomes 19.5 Teslas - which is more than the saturation flux.

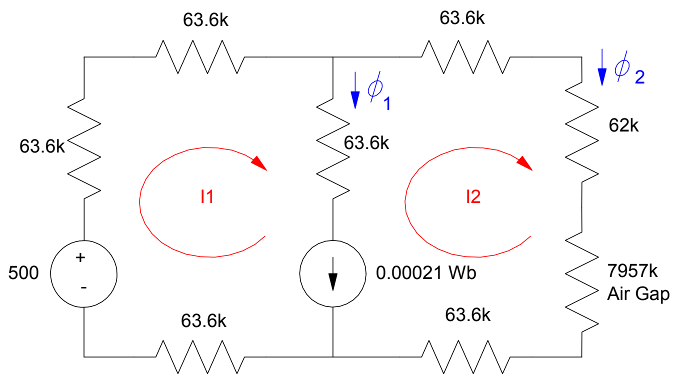
So, assume the flux density and flus in the iron bar are:

$$B_1 = 2.1 \text{ Teslas.}$$

$$\phi_1 = (2.1 Tesla) \cdot (0.01m)^2$$

$$\phi_1 = 0.00021 \text{ Webers}$$

Change the circuit to fix ϕ_1 at its saturation current:



Write the loop equations

$$I_1 - I_2 = 0.00021$$

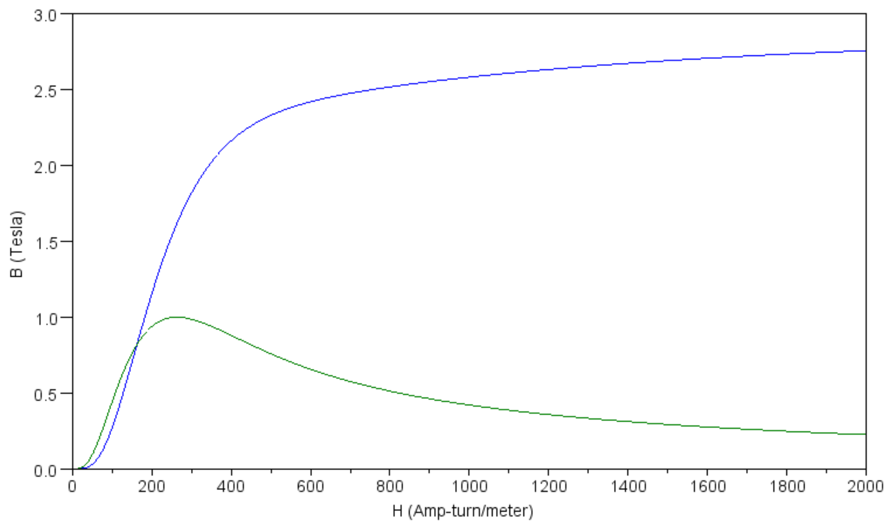
$$-500 + (63.6k + 63.6k + 63.6k)I_1 + (63.6k + 63.6k + 62k + 7957k)I_2 = 0$$

Solve in MATLAB:

```
-->A = [1,-1;63600*3,63600*2+6200+7900000]
    1.         - 1.
    190800.     8033400.


-->B = [0.00021;500]
    0.00021
    500.

-->I = inv(A)*B
    0.0002659      I1
    0.0000559      I2

-->Phi = [I(1)-I(2);I(2)]
    0.00021         Webers   (φ1)
    0.0000559       Webers   (φ2)

-->B = Phi / (0.01 ^ 2)
    2.1             Teslas   (the iron bar)
    0.5592422       Teslas   (the air gap)
```

Note that by increasing the current 10x, the magnetic flux didn't increase very much. The iron is saturated.

Problem 3: Assume the magnitization curve has the following characteristic. Determine the flux ϕ_1 and ϕ_2.



Magnetization Curve for Iron (blue) and it's relative permiability / 5000 (green)

Solution: You're almost forced to use a numerical solution:

- Guess ϕ_1 and ϕ_2
- Given the cross sectional area, compute B for each leg
- From the above curve (blue line), find the voltage drop across each element
- The voltages around each loop won't add up to zero.
- Adjust ϕ_1 and ϕ_2 and repeat until the voltages sum to zero around each loop.

This will give the most accurate estimate for the magnetic flux in the circuit, but requires numerical solutions.