Inductors

A coil creates an inductor. For an ideal inductor:

$$v = L\frac{di}{dt}$$

$$W = \frac{1}{2}Li^2$$

$$L = \frac{N\Phi}{i} = \frac{N^2\mu A}{l} = \frac{N^2}{R}$$

If you're designing an inductor, you can increase the inductance by

- Increasing the number of turns (helps as the square of the number of turns)
- Increase the area
- Decrease the length (radius)

Hysteresis Losses: $P_h = vfK_hB_m^n$ Watts

- v = volume of the ferromagnetic material
- f = the frequency
- Kh depends upon the material. Cast steel = 0.025, Silicon Steel = 0.001, permalloy = 0.0001
- n depends upon the material ($1.5 < n < 2.5$)
- B = the maximum flux density

Eddy current losses: $P_e = K_e f^2 B_m^2 \tau^2 v$

- Ke is a material constant
- f = frequency
- Bm = the maximum flux density
- tau = lamination thickness
- v = the total volume of the material

Hysteresis losses and Eddy current losses together are called 'core losses'.

Example: Find the inductance of the previous design and the losses at 60Hz and 500mA. Assume Kh = 50, Ke = 50, tau = 1mm.

$$L = \frac{N^2}{R} = \frac{(100)^2}{254{,}400} = 39mH$$

A 1 Henry inductor is not very common. This one is saturating its core and is only 39mH.

$$P_h = (0.01m \cdot 0.01m \cdot 0.16m)(60Hz)(50)(1.96T)^2 = 0.184W$$

$$P_e = (50)(60Hz)^2(1.96T)^2(0.001m)(0.000016m^3) = 0.011W$$

The losses aren't that bad at 60Hz. Note that the losses increase with frequency, however:

- Hysteresis losses increase proportional to frequency
- Eddy current losses increase proportional to frequency squared (!)

Problem: Design a 1H inductor at 120V rms, 60Hz

Solution: To prevent saturation, B must be kept less than 2.15T. At 60Hz, the current will be

$$I = \frac{V}{Z} = \frac{120V}{j377\Omega} = 318mA$$

Using the previous design, you can adjust terms to make it 1H:

$$L = \frac{N^2}{R} = \frac{N^2 \mu A}{l} = 1H$$

Design for a peak flux density of 2T (1.4T rms)

$$B = \frac{\phi}{A} = \frac{NI\mu}{l} = \frac{2}{\sqrt{2}}T$$

Solving

N = 113 turns

$A = 1.98 \cdot 10^{-3} m^2 = 1cm \cdot 19cm$

To calculate the losses, first find the wire size. From http://www.powerstream.com/Wire_Size.htm, you need 34 gage wire to carry 318mA, which has a diameter of 0.16mm. The length of the wire is

l = 2*(1cm x 19cm) * 113 turns = 0.429m

The resistance is then

$$R = \frac{\rho L}{A} = \frac{(1.68 \cdot 10^{-8}\Omega m)(0.429m)}{\pi(0.00008m)^2} = 0.359\Omega$$

$$P_{cu} = I^2 R = (0.318A)^2(0.359\Omega) = 36mW$$

The Eddy current losses are:

$$P_h = v f K_h B_m^n$$

$$P_h = (0.01m \cdot 0.19m \cdot 0.16m)(60Hz)(50)(2T)^2 = 3.648W$$

The hysteresis losses (assuming 1mm thickness lamination) are:

$$P_h = v f K_h B_m^n$$

$$P_e = (50)(60Hz)^2(2T)^2(0.001m)(0.01m \cdot 0.19m \cdot 0.16m) = 0.219W$$

A model for this industor then is

- j377 Ohms (the inductance), in parallel with
- 3720 Ohms (to dissipate 3.8W, modeling the losses).