Transformer Design

note: All units rms

Problem: Deliver power to a customer who lives 400km away and has a load of 10kVA with a power factor of 1.0.

Solution: Use two transformers

- 138kV carries the power for 300km to a substation
- 7200V carries power to a local transformer
- 240V is delivered to the customer.

Assume a 20:1 fan-out for each transformer.

- 20 customers are served by the 7200V-240V transformer. Size it to 150kVA.
- 20 substations are served by the 138kV-7200V transformer. Size it to 2,000kVA

Problem: Design a 7200V-240V 150kVA transformer for 60Hz. Design for a flux density of 1.5T. Assume Silicon Steel.

Solution: Note that

$$B = \frac{\phi}{A} = \frac{1}{A}\left(\frac{NI}{R}\right)$$

$$I \approx \frac{V}{\omega L}$$

$$L = \frac{N^2}{R}$$

$$B \approx \left(\frac{N}{AR}\right)\left(\frac{V}{\omega}\right)\left(\frac{R}{N^2}\right) = \left(\frac{V}{377AN}\right)$$

On the high side,

Vh = 7200V

Ih = 20.83A

So, assuming 600 turns on the high-side:

$$B = 1.5T = \left(\frac{V}{377AN}\right) = \left(\frac{7200V}{(377)(A)(600)}\right)$$

If l = 1 meter

$$A = 0.021m^2$$

The core could have dimensions of 14cm x 14cm with a circumference of 1m .- which is reasonable.

The reluctance of the core is

$$R = \frac{l}{\mu A} = \frac{1m}{(7000\mu_o)(0.021m^3)} = 5413$$

$$L = \frac{N^2}{R} = \frac{600^2}{5413} = 66.5H$$

$$jX_c = j\omega L = j(377)(66.5H) = j25.1k\Omega$$

The hysteresis losses are

$$P_h = k_h f v B_m^2$$

$$P_h = \left(350 \frac{J}{m^3}\right)(60Hz)(0.014m^3)(1.5T)^2 = 661W$$

The eddy current losses don't work out. Assume they're half the hysteresis losses (you can adjust the thickness of the plates to reduce the Eddy current losses.)

$$P_e \approx 330W$$

The core resistance accounts for this 991W loss in the core. A resistor which has such a loss at 7200V is

$$991W = \frac{V^2}{R} = \frac{7200^2}{52.3k\Omega}$$

$$R_c = 52.3k\Omega$$

Next, design the windings. Assume the high-side and low-side have i^2R losses equal to 1% of rated load.

$$i_h = \frac{150kVA}{7200V} = 20.8A$$

$$i_h^2 R_h = 1.5kW$$

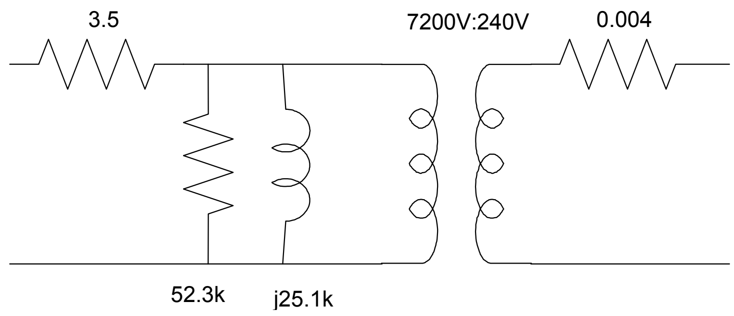
$$R_h = 3.5\Omega$$

$$i_l = \frac{150kVA}{240V} = 625A$$

$$R_l = 0.004\Omega$$

This determines the diameter of the copper wire used in the windings.

The net model for this transformer is:



Problem: Design a 138kV-7200V, 2000kVA transformer. Design for Silicon Steel with a maximum flux of 1.5T.

Solution: Assume 600 turns. Assume the length of the transformer is 1m:

$$B = 1.5T = \left(\frac{V}{377AN}\right) = \left(\frac{138kV}{(377)(A)(600)}\right)$$

$$A = 0.407m^2$$

A core that has a cross section of 64cm x 64cm would work. The length of the core would need to be about 7 meters.

$$R = \frac{l}{\mu A} = \frac{7m}{7000\mu_0 \cdot 0.407m^2} = 1955$$

$$L = \frac{N^2}{R} = 184H$$

$$jX = j377L = j69.4k\Omega$$

The core losses are

$$P_h = \left(350\frac{J}{m^3}\right)(60Hz)(2.85m^3)(1.5T)^2 = 134kW$$

Assuming the eddy currents losses are half the hysteresis losses (?)

$$P_e \approx 67kW$$

$$P_c = 202kW$$

The core acts as if it has a resistance of

$$\frac{V^2}{R} = \frac{(138kV)^2}{R} = 202kW$$

$$R = 94k\Omega$$

Sizing the windings so that the i2R losses are 1% of rated load:

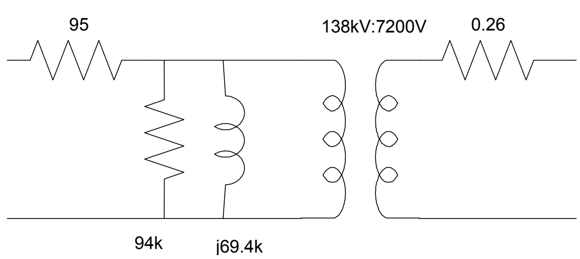
$$i_h = \frac{2000kVA}{138kV} = 14.5A$$

$$i_h^2 R_h = 20kVA$$

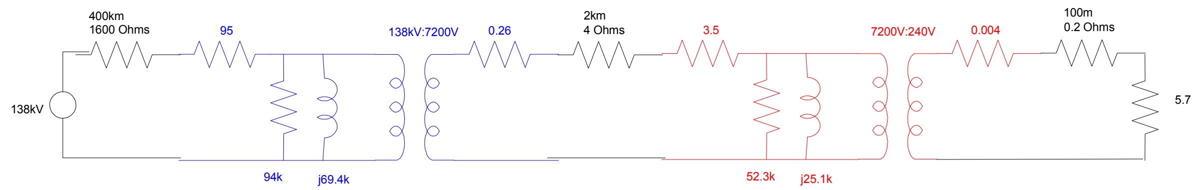
$$R_h = 95\Omega$$

$$i_l = \frac{2000kVA}{7200V} = 278A$$

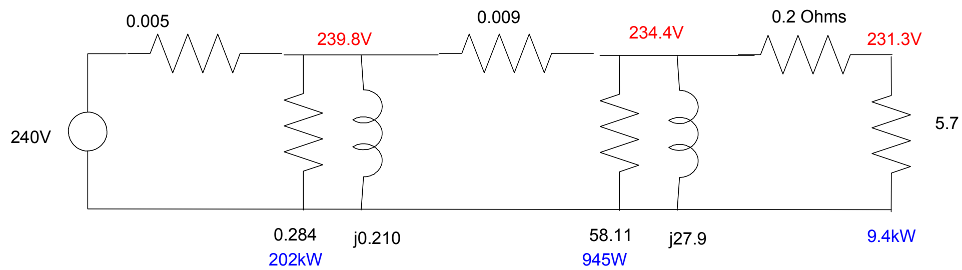
$$R_l = 0.259\Omega$$



Problem: Connect these to a single customer drawing 10kVA at pf=1.0. Assume copper wire connects a 138kV power plant to a customer 400km away.



To analyze, bring all impedances to one node. Let's use the customer:



Note a couple of things:

- The line resisance is so small relative to the core loss resistances you can almost ignore the core losses when computing voltages (the red numbers).
- If you ignore the core losses (blue numbers), 96% of the resistance is in the load. 96% of the power gets to the customer.
- If you include the core losses, 4.4% of the power gets to the customer.

It doesn't make economic sense for a utility to build transmission lines for a single customer 400km away: only 4.4% of the power gets to the customer. In remote locations, a local generator is more economical.

If you assume a 20:1 fanout, the core losses are shared amoung 8000 customers. The core losses then become only 25W per customer. It makes sense to build long transmission lines if you have a sizable load.

The cost of the transmission lines, the power to energize the cores, and the cost of keeping the power available are not separated by utilities. These are lumped into the cost-per-killowatt the customer is charged.