Delta and Y Connections

Transformer Polarity

- Terminal H1 has the same polarity as L1
- Current entering at terminal H1 leaves at terminal L1. Ditto for H2 and L2.
- A dot indicates the same number (current into the dot terminal leaves the other dot terminal)

Nameplates

- Dash (240-120): Two voltages from two separate windings
- Slant (240/120): Two voltages from the same winding (there's a center tap)
- Cross (240x120): Two ways to connect the windings. One way (series) results in 240V. The other way (parallel) results in 120V.
- Wye (Y): Three phase Y connection
- Delta (Δ): Three phase delta connection

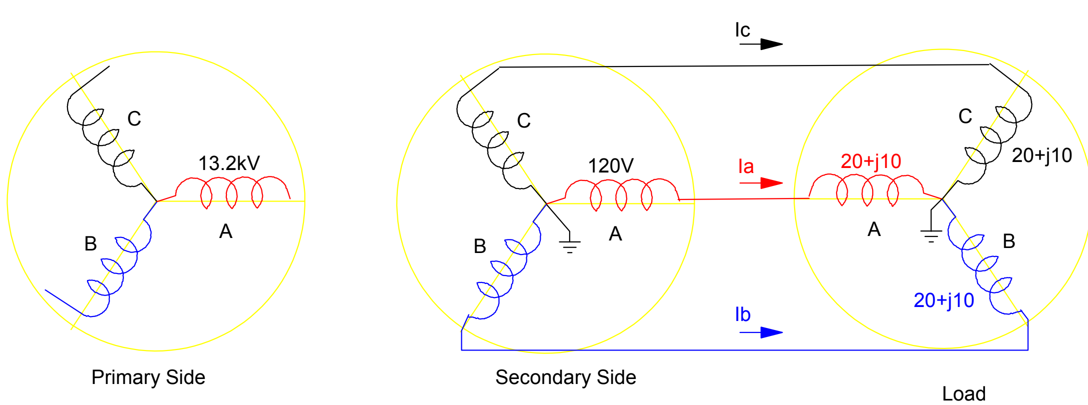
Problem: Use three separate transformers to convert 3-phase 13.2kV rms line-neutral) to 120V rms line to neutral. Determine the line currents if the load is

- a) 20+j10 Ohms line to neutral
- b) 20+j10 Ohms line to line.

Solution: There are several ways to do this.

Y-Y Configuration

a) Y - Y: Gives you a common ground - which in theory carries no current. You can detect a fault in a phase by monitoring the current on the ground line: if it isn't zero, there's an imbalance load.



Y - Y Configuration

The rms line to neutral voltages at the load are

- $V_{AN} = 120\angle 0^0$
- $V_{BN} = 120\angle -120^0$
- $V_{CN} = 120\angle -240^0$

The line currents are then

$$I_A = \frac{V_{AN}}{20+j10} = 5.37\angle -26.5^0$$

$$I_B = \frac{V_{BN}}{20+j10} = 5.37\angle -146.5^0$$

$$I_C = \frac{V_{CN}}{20+j10} = 5.37\angle -266.5^0$$

The power to the load is

$$P = |I|^2 Z = (5.37)^2 \cdot (20 + j10) = 576 + j288 W$$

Each load dissipates 576 Watts of heat. It also 'absorbs' 288VA reactive.

The ground current is

$$I_N = I_A + I_B + I_C = 0.$$

With a balanced load, there is no current on the ground line. You'll cover unbalanced loads in ECE 433.

What happens if you lose phase C?

- The voltage on line C is zero. These customers lose power.
- The neutral current is

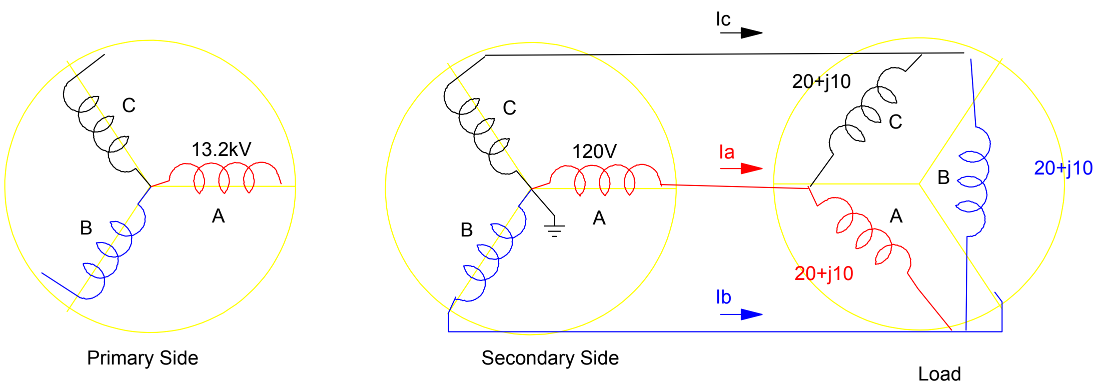
$$I_N = I_A + I_B$$

$$I_N = (5.37\angle -26.5^0) + (5.37\angle -146.5^0)$$

$$I_N = 5.37\angle -86.5^0$$

There is now current on the ground line. You can use this to detect faults.

What happens if you connect the load line-to-line?



Y - Y Configuration

The current is then

$$I_{AC} = \frac{V_{AN}-V_{CN}}{20+j10} = 9.295\angle -56.6^0$$

$$P = |I_{AC}|^2 \cdot (20+j10) = 1728 + j864\,\mathrm{W}$$

Each load now consumes 1728 Watts (vs 576 Watts). The line-to-line voltage is $\sqrt{3}$ larger than the line-to-neutral voltage. This increases the current by $\sqrt{3}$ and the power by 3 times.

If you connect your load line-to-neutral, it will absorb 3x as much power.

The line current are:

$$I_A = I_{AB} + I_{AC}$$

$$I_A = 9.295\angle 3.43^0 + 9.295\angle -56.56^0$$

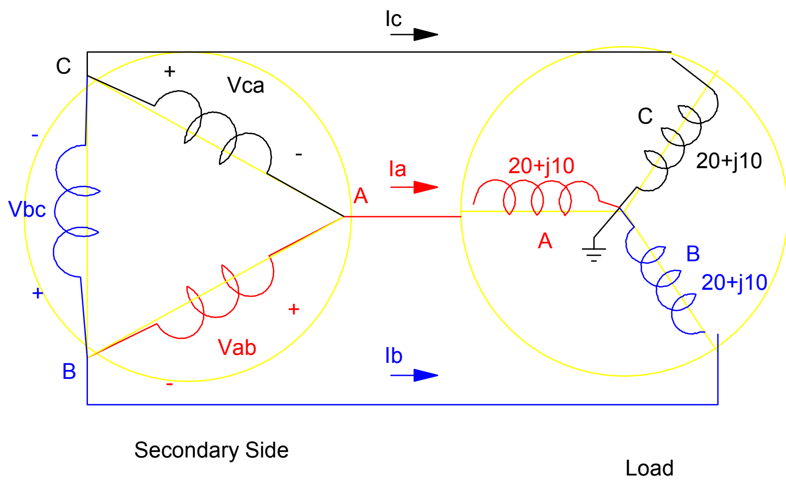
$$I_A = 16.1\angle -26.5^0$$

Connecting the load in delta configuration increases the line currents from 5.71A to 16.1A

Delta-Delta Configuration

Delta - Delta: Able to operate with one transformer removed (turning it into a V-V configuration). This allows for inspection, maintenance, testing, replacement, etc.

V - V: Sometimes used when you plan on having a larger load in the future (such as a new housing development). Add a third transformer and you have a Delta-Delta transformer.



If you want the line-to-neutral voltage to be 120V, the line-to-line voltage on the transformer needs to be $\sqrt{3}$ times larger (208V).

If the load is connected in a Y configuration, the voltages at A, B, C are the same as before, the line currents are the same as before, and the power is the same as before. By symmetry, each part of the transformer delivers 1/3rd of the total power:

$$P_A = 576 + j288W$$

$$P_{total} = 3P_A = 1728 + j864W$$

$$P_{ab} = \tfrac{1}{3}P_{total} = 576 + j288W$$

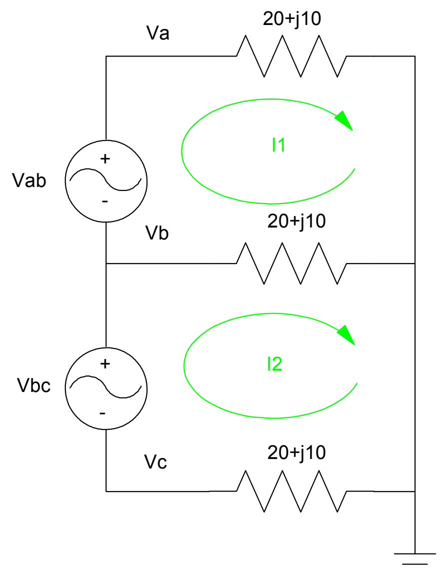
Since the voltage is $\sqrt{3}$ larger, the current in each leg will be $\sqrt{3}$ smaller:

$$|I_{AB}| = \frac{|576+j288W|}{208V} = 3.10A = \frac{5.37A}{\sqrt{3}}$$

Likewise, there will be less I^2R heating in the transformer with a delta configuration (good).

A second advantage of a delta-delta configuration is you can remove one transformer without affecting the service to the customer.

What happens if transformer C (black) is removed from service?



note: There normally is a third transformer (which looks like a voltage source) from C to A. This has been removed.

The current loop equatios are:

$$V_{AB} = 208\angle 30^0$$

$$V_{BC} = 208\angle -90^0$$

i1: $(20+j10)(I_1 - I_2) - V_{AB} + (20+j10)I_1 = 0$

i2: $(20+j10)I_2 - V_{BC} + (20+j10)(I_2 - I_1) = 0$

Solving

$$\begin{bmatrix} 40+j20 & -(20+j10) \\ -(20+j10) & 40+j20 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_{AB} \\ V_{BC} \end{bmatrix}$$

```
-->Va = 120;
-->j = sqrt(-1);
-->Vb = 120*exp(j*2*%pi/3);
-->Vc = 120*exp(j*2*%pi*2/3);

-->A = [40+j*20,-(20+j*10);-(20+j*10),40+j*20]
```

```
        40. + 20.i   - 20. - 10.i
      - 20. - 10.i     40. + 20.i

   -->I = inv(A)*[Va-Vb;Vb-Vc]
        4.8 - 2.4i
        4.478461 + 2.9569219i
```

$$I_A = I_1 = 4.8 - j2.4 = 5.37\angle -26.5^0$$

$$I_B = I_2 - I_1 = -0.32 + j5.36 = 5.37\angle 93.4^0$$

$$I_C = -I_2 = -4.48 - j2.95 = 5.37\angle -146.6^0$$

Note that the currents to the load are the same as what you had with the Y-Y configuration (5.37 Amps). If the currents are the same, the voltages are the same (120V line to neutral).

A delta-delta configuration allows you to lose one transformer without affecting the service to the customers.

This allows you to

- Put the transformer under repairs without affecting the customers
- Only install 2/3rds of the transformer so save money. When the load increases with future developments, the third transformer can be added.

Note that with only two transformers, each one no longer delivers 576 + j288W:

$$P_{AB} = (V_{AB})(I_{AB})^* = (V_{AB})(I_2)^*$$

$$P_{AB} = (208\angle 30^0)(4.478 - j2.956)$$

$$P_{AB} = 1113.4 - j66.8W$$

$$P_{BC} = (V_{BC})(I_{BC})^* = (V_{BC})(I_1)^*$$

$$P_{BC} = (208\angle -90^0)(4.8 + j2.4)$$

$$P_{BC} = 498.8 - j997.7$$

The current likewise increases from the 3.10A from before by $\sqrt{3}$:

$$|I_{AB}| = \left|\frac{P_{AB}}{V_{AB}}\right| = \left|\frac{1115VA}{208V}\right| = 5.37A$$

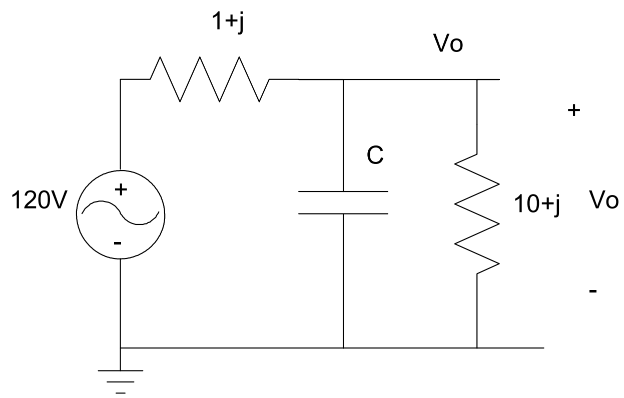
$$|I_{BC}| = \left|\frac{P_{BC}}{V_{BC}}\right| = \left|\frac{1115VA}{208V}\right| = 5.37A$$

Likewise, you don't want to remove one part of a delta transformer for repair when the transformer is fully loaded. That will draw too much current from the transformer. Repairs should take place under low-load conditions, such as spring and fall when it's mild weather.

Capacitors Add Voltage

Another common saying in utilities is that capacitors add voltage. This is somewhat true - it depents upon having an inductive load and inductance in the transmission lines.

For example, find the voltage at the load for the following circuit:



The transformers, transmission lines, etc. are modeled as a 1+j line impedance to the load (the Thevenin equivalent as seen by the customer). If C=0, the output voltage is by voltage division:

$$V_o = \left(\frac{10+j}{(10+j)+(1+j)}\right)120V = 107.8\angle -4.6^0$$

If you add a capacitor, it's reactance is -jX:

$$Z_c = \frac{1}{j\omega C} = \frac{1}{j377C} = -jX$$

This negative reactance can cancel the positive reactance of the transmission lines, increasing current flow and voltage at the load. To see this, compute the voltage at the load for some value of C (-jX):

$$Z_L = -jX || (10+j)\Omega$$

$$Z_L = \left(\frac{Z_1 Z_2}{Z_1+Z_2}\right) = \left(\frac{-jX\cdot(10+j)}{10+j-jX}\right)$$

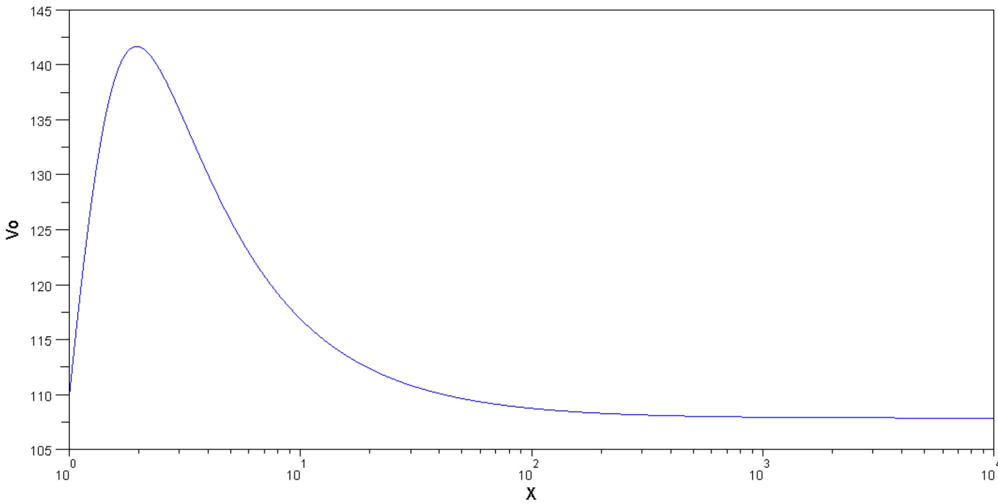
$$V_o = \left(\frac{Z_L}{Z_L+(1+j)}\right)208V$$

in SciLab:

```
-->X = -j*logspace(0,4,1000)';
-->ZL = 1 ./ (1 ./X + 1/(10+j));

-->Vo = ZL ./ (ZL + 1+j) * 120;

-->plot(abs(X),abs(Vo))
-->xlabel('X');
-->ylabel('Vo');
```



For C small (X large), you're at 107.8V. As you increase C, X decreases and the voltage rises. Hence, capacitors add voltage.

This only works due to the inducatance in the system. The impedance of a capacitor is -jX $\left(\frac{1}{j\omega C}\right)$. This negative reactance cancels with the positive reactance of the inductors, reducing the impedance of the transmission lines, increasing the voltage at the load. This only works up to a point: once you've cancelled out the reactance of the transmission lines, the capacitors just make things works. This happens for X < 2 in this case.