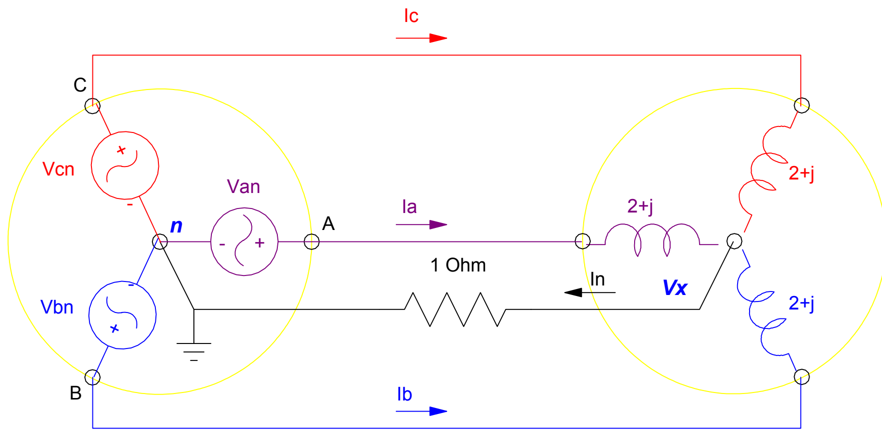
Analysis of 3-Phase Loads

Y Source, Y Load, Balanced Load



Define the voltages and impedances:

```
-->Va = 120;
-->Vb = 120 * exp(-j*120*%pi/180);
-->Vc = 120 * exp(-j*240*%pi/180);

-->Za = 2 + j;
-->Zb = 2 + j;
-->Zc = 2 + j;
```

Here you have just one unknown: Vn. Write one equation to solve for one unknown.

```
-->Vx = (Va/Za + Vb/Zb + Vc/Zc) / (1/Za + 1/Zb + 1/Zc + 1)

Vx = 0
```

Note: With a balanced load, the three phase voltages sum to zero. Solving for the line currents:

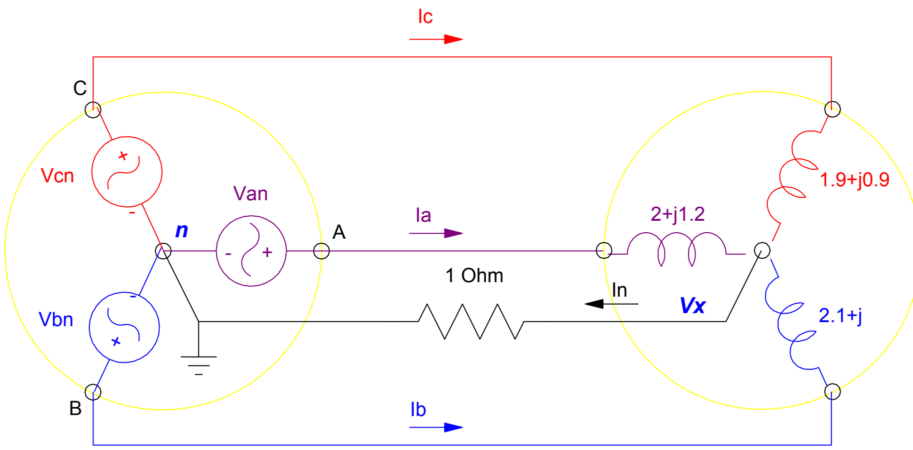
```
-->Ia = (Va - Vn) / Za;
-->Ib = (Vb - Vn) / Zb;
-->Ic = (Vc - Vn) / Zc;
-->In = Vn / 1;

-->I = [Ia; Ib; Ic; In];
-->[abs(I), atan(imag(I),real(I))*180/%pi]

     Amps           degrees
Ia    53.665631       -26.565051
Ib    53.665631      -146.56505
Ic    53.665631        93.434949
In     0.000000      -164.74488
```

Note: With a balanced load, the three phase currents sum to zero.   You can get the same result using per-phase analysis:

$$I_a = \frac{V_{an}}{Z_a} = 53.665\angle -26.565^0$$

Y Source, Y Load, Unbalanced Load

If the load is not balanced, you can't use per-phase analysis. Circuits I and II techniques always work, however. Just use voltage nodes.




```
-->Va = 120;
-->Vb = 120 * exp(-j*120*%pi/180);
-->Vc = 120 * exp(-j*240*%pi/180);

-->Za = 2 + j*1.2;
-->Zb = 2.1 + j;
-->Zc = 1.9 + j*0.9;
```

With only one unknown (Vn), write one equation for one unknown:

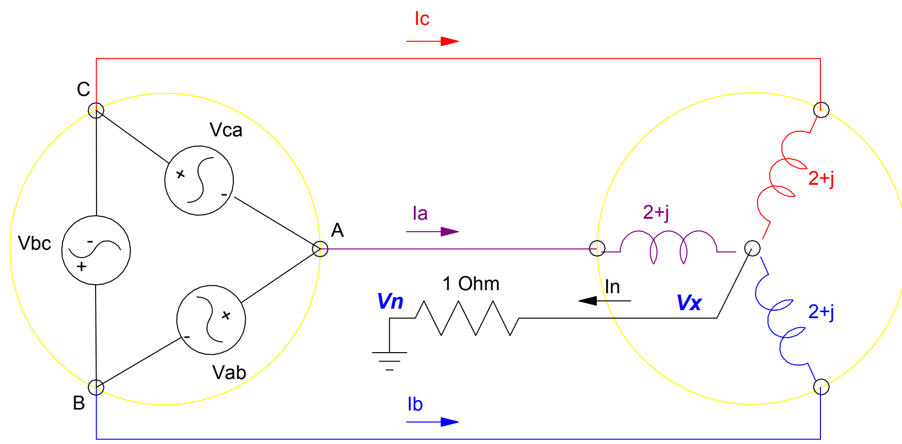
```
-->Vx = (Va/Za + Vb/Zb + Vc/Zc) / (1/Za + 1/Zb + 1/Zc + 1)
  - 1.4176461 + 0.1403446i
```

Note that the voltage at the load's ground is no longer 0V.

```
-->Ia = (Va - Vn) / Za;
-->Ib = (Vb - Vn) / Zb;
-->Ic = (Vc - Vn) / Zc;
-->In = Vn / 1;

-->I = [Ia; Ib; Ic; In];

-->[ abs(I), atan(imag(I),real(I))*180/%pi]

       Amps        degrees
Ia    52.057421   -31.029984
Ib    51.34257   -144.84063
Ic    56.685889    94.097303
In    1.4245761   174.34623
```

Note also that unbalanced loads result in currents that don't sum to zero. You have current on the ground line.

Delta Source, Y Load, Balanced Load



Define the voltages and impedances:

```
-->Vab = 120;
-->Vbc = 120 * exp(-j*120*%pi/180);
-->Vca = 120 * exp(-j*240*%pi/180);

-->Za = 2 + j;
-->Zb = 2 + j;
-->Zc = 2 + j;
```

With 4 unknown voltages, write voltage node equations to solve for 4 unknowns:

$$A\begin{bmatrix} V_a \\ V_b \\ V_c \\ V_x \end{bmatrix} = B$$

Writing the voltage node equations:

```
-->A = [1, -1, 0, 0;  0, 1, -1, 0;  -1, 0, 1, 0;  -1/Za, -1/Zb, -1/Zc,
1/Za+1/Zb+1/Zc+1]

    1.           - 1.            0              0
    0              1.          - 1.             0
  - 1.             0             1.             0
  - 0.4 + 0.2i   - 0.4 + 0.2i  - 0.4 + 0.2i     2.2 - 0.6i

-->B = [Vab; Vbc; Vca; 0]

    120.
  - 60. - 103.92305i
  - 60. + 103.92305i
    0
```

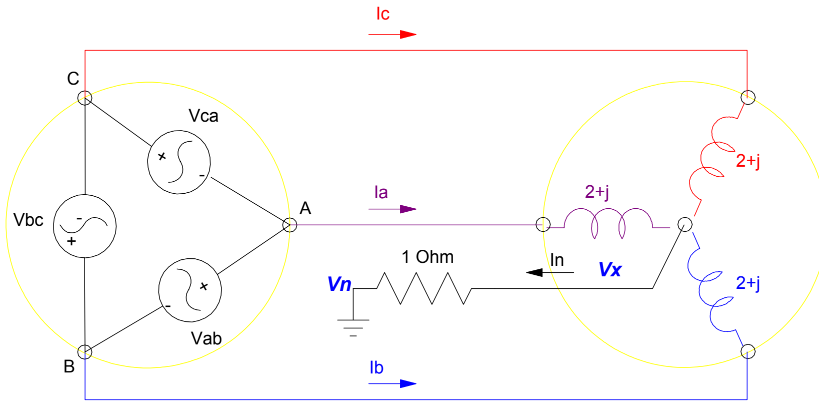
Solving

```
-->V = inv(A)*B;
             !--error 19
Problem is singular.
```

The A-matrix is singular - meaning the three transformers are redundant.

Delta Source, Y Load, Balanced Load

(take 2)



One way to fis the problem is to replace one transformer equation with

$$Va + Vb + Vc = 0$$

```
-->A = [1, 1, 1, 0;  0, 1, -1, 0;  -1, 0, 1, 0;  -1/Za, -1/Zb, -1/Zc, 
1/Za+1/Zb+1/Zc+1]

    1.              1.              1.              0
    0               1.            - 1.              0
  - 1.              0               1.              0
  - 0.4 + 0.2i    - 0.4 + 0.2i    - 0.4 + 0.2i      2.2 - 0.6i

-->B = [0; Vbc; Vca; 0]

    0
  - 60. - 103.92305i
  - 60. + 103.92305i
    0
```

Solving for the line voltages:

```
-->V = inv(A)*B;
-->[ abs(V), atan(imag(V),real(V))*180/%pi ]

        Volts     degrees
Va     69.282032   -30.
Vb     69.282032  -150.
Vc     69.282032    90.
Vx     0.            0.
```

Delta configuration is just like Y configuration, except

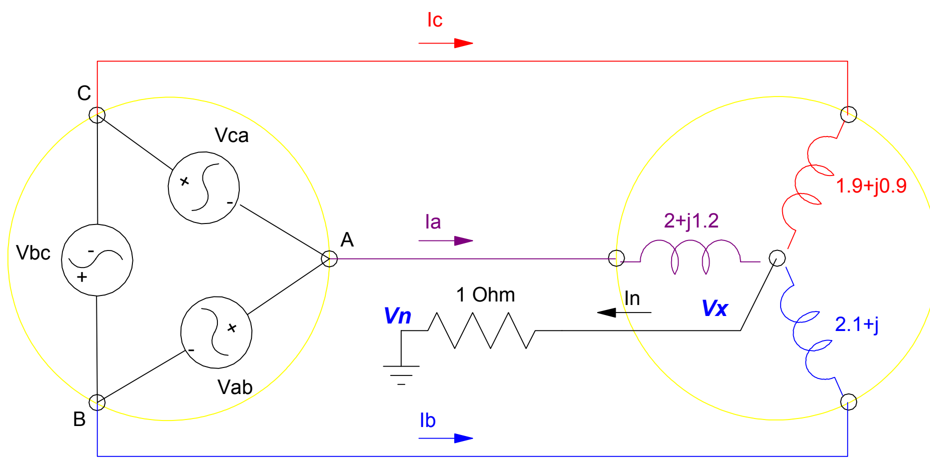
- The line-to-line voltages are $\sqrt{3}$ times larger than the line-to-neutral voltages, and
- There is a 30 degree phase shift between Van and Vab.

Solving for the line currents:

```
-->Ia = (Va - Vn) / Za;
-->Ib = (Vb - Vn) / Zb;
-->Ic = (Vc - Vn) / Zc;
-->In = Vn / 1;

-->I = [Ia; Ib; Ic; In];
-->[ abs(I), atan(imag(I),real(I))*180/%pi ]

Ia    30.983867    -56.565051
Ib    30.983867    -176.56505
Ic    30.983867     63.434949
In    0.            0.
```

With a balanced load, delta configuration still winds up with no current on the neutral line.

Delta Source, Y Load, Unbalanced Load



Define the voltages and impedances:

```
-->Vab = 120;
-->Vbc = 120 * exp(-j*120*%pi/180);
-->Vca = 120 * exp(-j*240*%pi/180);

-->Za = 2 + j;
-->Zb = 1.9 + j*1.1;
-->Zc = 2.1 + j*0.9;
```

Write 4 equations for 4 unknowns as

$$A\begin{bmatrix} V_a \\ V_b \\ V_c \\ V_x \end{bmatrix} = B$$

```
-->A = [1, 1, 1, 0;  0, 1, -1, 0;  -1, 0, 1, 0;  -1/Za, -1/Zb, -1/Zc, 
1/Za+1/Zb+1/Zc+1]

  1.             1.                1.                 0
  0              1.               -1.                 0
 -1.             0                 1.                 0
 -0.4 + 0.2i    -0.394 + 0.228i   -0.402 + 0.172i     2.196 -0.600i

  -->B = [0; Vbc; Vca; 0]

    0
  - 60. - 103.92305i
  - 60. + 103.92305i
    0
```

Solving:

```
-->V = inv(A)*B;
-->[abs(V), atan(imag(V),real(V))*180/%pi]

      Volts        Degrees
Va    69.282032     -30.
Vb    69.282032    -150.
Vc    69.282032      90.
Vx    1.4343974     156.34116
```

Note that with an unbalanced load, the voltages don't sum to zero.

Solving for the line currents:

```
-->Va = V(1);
-->Vb = V(2);
-->Vc = V(3);
-->Vx = V(4);

-->Ia = (Va - Vx) / Za;
-->Ib = (Vb - Vx) / Zb;
-->Ic = (Vc - Vx) / Zc;
-->In = Vx / 1;

-->I = [Ia; Ib; Ic; In];

-->[abs(I), atan(imag(I),real(I))*180/%pi]

       Amps           Degrees
Ia    31.621503    - 56.436675
Ib    31.174373    - 179.10129
Ic    30.077482      65.705905
In     1.4343974     156.34116
```

Note that with an unbalanced load, there is current on the neutral line