3-Phase Induction Motors

How to Make a Magnetic Field Spin

A single phase won't spin. For example, take an electromagnet along the x-axis:

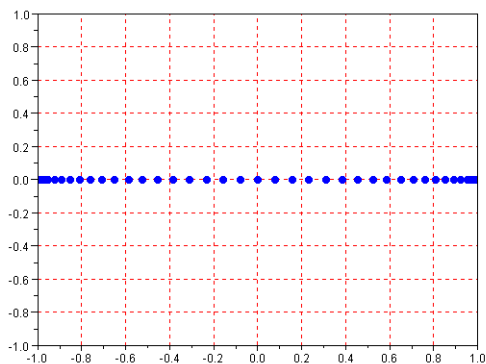$$V_a(t) = \sin(t)$$

$$P_a = (1 + j0) = 1\angle 0^0$$

The induced magnetic field will be

$$M(t) = P_a V_a$$

This is a magnetic field in the X direction, oscillating from +1 to -1.
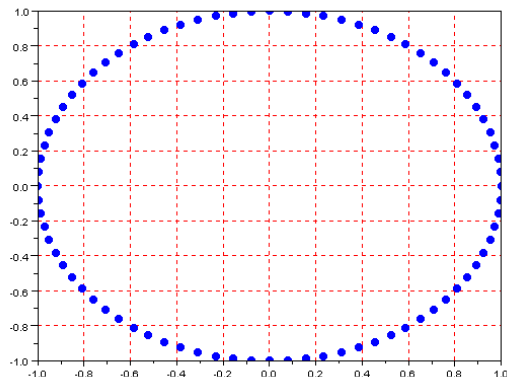
Scilab Code:

```
-->Va = sin(t);
-->Pa = 1;
-->M = Pa*Va;
-->plot(real(M),imag(M),'.')
-->xgrid(5)
```


Two phases will work. Let's use phase quadrature

```
-->Va = sin(t);
-->Vb = sin(t-%pi/2);
-->Pa = 1;
-->Pb = j;
-->M = Pa*Va + Pb*Vb;
-->plot(real(M),imag(M),'.')
-->xgrid(5)
```

Note that using two sine waves, 90 degrees out of phase, applied to two electromagnets 90 degrees out of phase results in a magnetic field that rotates. The amplitude of the net magnetic field is the maximum of either phase.
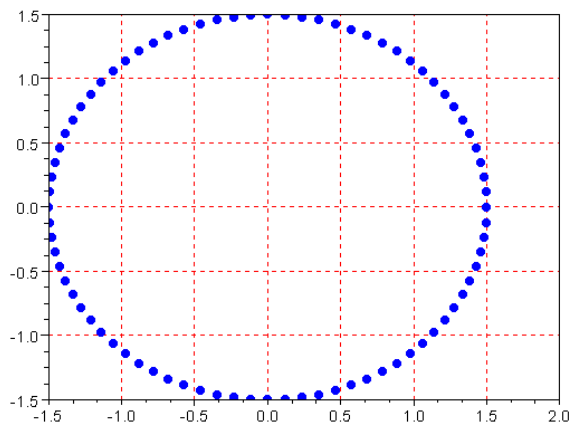
Three phases also works:

```
->Va = sin(t);
->Vb = sin(t-%pi/3);
->Vc = sin(t-2*%pi/3);

->Pa = exp(j*0*%pi/180)
->Pb = exp(j*120*%pi/180)
->Pc = exp(j*240*%pi/180)

->M = Pa*Va + Pb*Vb + Pc*Vc;
->plot(real(M),imag(M),'.')
```


Note that the induced magnetic field is rotating in a circle with a magnitude 1.5 times that of the maximum field induced by any single phase.

An AC induction motor depends upon having a rotating magnetic field. There are several types the above analysis suggests:

- A single-phase AC induction motor has no preferred direction of spinning. You need to start it some other way.
- If you pass a single-phase AC line through a capacitor, the phase is shifted 90 degrees. This lets you build a 2-phase AC induction motor using a single-phase AC line (plus a capacitor).
- If you have a 3-phase AC line, you can connect the three phases to three windings in the stator - each 120 degrees apart physically. This results in a magnetic field that spins with an amplitude 1.5x that of any one phase.

How to produce torque:

Next, assume you have a N:1 transformer with the secondary side consisting of a single shorted turn. N-amps of current on the high-side produces N amps on the low side.

These N amps will create a magnetic field which opposes (and is attracted to) the field produced by the primary (high) side. If the magnetic field moves, the seconday coil will try to follow it, producing a force of

$$F = I_2 \cdot l \cdot B$$

- This lets you provide propulsion to electromagnetic trains. Instead of the magnetic field rotating, it moves along a linear track. The train tries to keep up with the moving magnetic field.
- This can also make a rail gun. Using a capacitor to provide *lots* of current (10^6 amps), a projectile can be accelerated to 2000 m/s and more.

This also works if you create a linear (flat) stator. This is used for
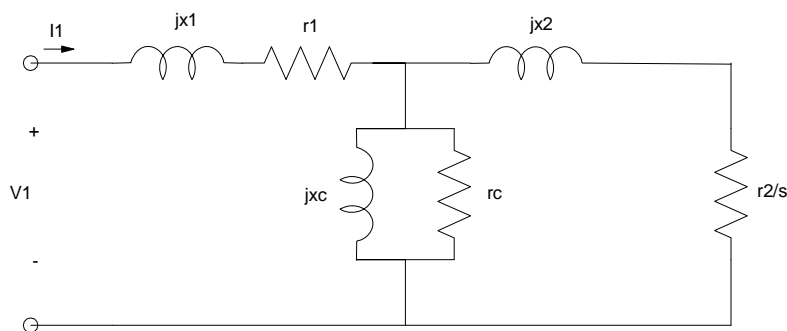- Propulsion of electromagnetic trains
- Rail guns.

Equations:

n = motor speed

Synchronous Speed $= n_s = (2\pi \cdot 60Hz)\left(\frac{2}{\#\,\text{poles}}\right)$

Slip: $s = \left(\frac{n_s - n}{n_s}\right)$
- slip = 1 means stopped (locked) rotor
- slip = 0 means the motor is running at synchronous speed

An AC induction machine is essentially a transformer and has a similar model:

- r1 + jx1 is the stator resistance and reactance
- rc + jxc is the core loss and core reactance
- r2 + jx2 is the rotor resistance and reactance

Note that at synchronous speed, slip is zero (s=0) , the core current flow is zero, which is modeled as the resistance going to infinity.

Work is proportaional to speed, so changing r2 to

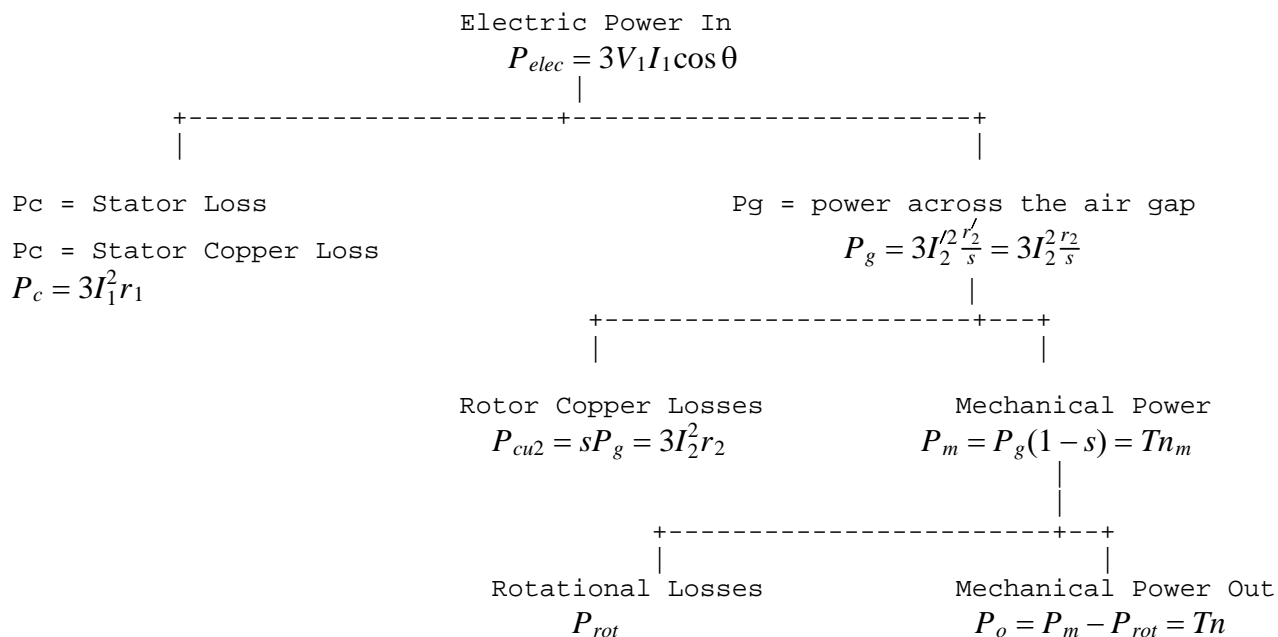$$\frac{r_2}{s} = \frac{r_2}{s}(1 + s - s) = r_2\left(\frac{1-s}{s}\right) + r_2$$

Noting that x1, r1 are small relative to the core, an approximate model is:
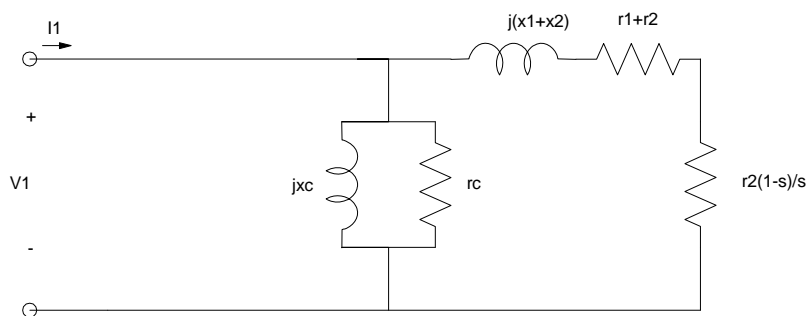
$r_2\left(\frac{1-s}{s}\right)$ is the mechanical power delivered by the motor:

A power breakdown for a 3-phase motor follows:

```
                          Electric Power In
                       Pelec = 3V₁I₁cos θ
                               |
        +----------------------+------------------------+
        |                                               |
   Pc = Stator Loss                        Pg = power across the air gap

   Pc = Stator Copper Loss
   Pc = 3I₁²r₁                             
                                           |
                              +----------------------+---+
                              |                          |
                        Rotor Copper Losses       Mechanical Power

                                          |
                              +----------------------+--+
                              |                         |
                        Rotational Losses        Mechanical Power Out
```

Electric Power In

$$P_{elec} = 3V_1 I_1 \cos\theta$$

Pc = Stator Loss

Pc = Stator Copper Loss

$$P_c = 3I_1^2 r_1$$

Pg = power across the air gap

$$P_g = 3I_2'^2 \frac{r_2'}{s} = 3I_2^2 \frac{r_2}{s}$$

Rotor Copper Losses

$$P_{cu2} = sP_g = 3I_2^2 r_2$$

Mechanical Power

$$P_m = P_g(1-s) = Tn_m$$

Rotational Losses

$$P_{rot}$$

Mechanical Power Out

$$P_o = P_m - P_{rot} = Tn$$

More equations:

P_{Cu2} = rotor copper losses

$R_m = r_2\left(\frac{1-s}{s}\right)$ = mechanical shaft load

$P = Tn$

$s = \frac{P_{Cu2}}{P_{gap}}$

Example: A 3 phase, 4 pole, 30hp, 220V line-to-line, 60Hz, Y connected induction machine draws 77A with a power factor of 0.88 lagging. The losses are: P_{cu1} = 1033W, P_{cu2} = 1299W, P_c = 485W, P_{rot} = 540W.

a) Find the power across the gap

Analyze this on a per-phase basis. The total answer will be 3 times the result per phase.

The line-to-neutral voltatage is

$$V_{LN} = \frac{V_{LL}}{\sqrt{3}} = 127V$$

The electric power input and power across the gap are

$$P_{elec} = 3 \cdot 127V \cdot 77A \cdot 0.88 = 25,820W$$

$$P_g = P_{elec} - P_{Cu1} - P_c$$

$$P_g = 25,820W - 1033W - 485W = 24,302W$$

b) The torque is

$$n_s = (2\pi \cdot 60Hz)\left(\frac{2}{4 \text{ pole}}\right) = 188.5 rad/\sec$$

$$T = \frac{P_g}{\omega_s} = \frac{24,302W}{188.5W} = 128.9Nm$$

c) The slip and slip speed are

$$P_{Cu2} = s \cdot P_g$$

$$s = \frac{P_{Cu2}}{P_g} = \frac{1299W}{24302W} = 0.053$$

$$sn_s = (0.053)(188.5 rad/\sec) = 10.07 rad/\sec$$

d) The mechanical power developed

$$P_m = P_g - P_{Cu2} = 24,302W - 1299 = 23,003W$$

e) The output power is

$$P_o = P_m - P_{rot} = 23,003W - 540W = 22,463W$$

f) The motor's speed:

$$n = (1-s)n_s = 178.5 rad/\sec$$

g) The torque

$$T = \frac{P_o}{n} = \frac{22463}{178.5} = 125.8Nm$$

h) The torque to cover meahcnical losses is

$$T_{rot} = \frac{P_{rot}}{n} = \frac{540W}{178.5 rad/\sec} = 3.02Nm$$

The efficiency is power out over power in

$$\eta = \frac{P_o}{P_{elec}} = \frac{22,463W}{25,820W} = 87\%$$

mechanical torque delivered to the load

Problem: A 3-phase, 4 pole, 50hp 480V line-to-line, 60Hz, Y connected motor has the followig parameters:

> stator: r1 = 0.1   x1 = 0.35
>
> rotor: r2' = 0.12   x2' = 0.4
>
> Pc = core loss in the stator = 1200W,
>
> Prot = rotational losses = 950W,
>
> No load current draw = 19.64A with a power factor of 0.089 lagging.

a) Find the current I1 and power factor for a slip of s = 0.025.

$$I_1 = I_m + I_2$$

The line-to-neutral is
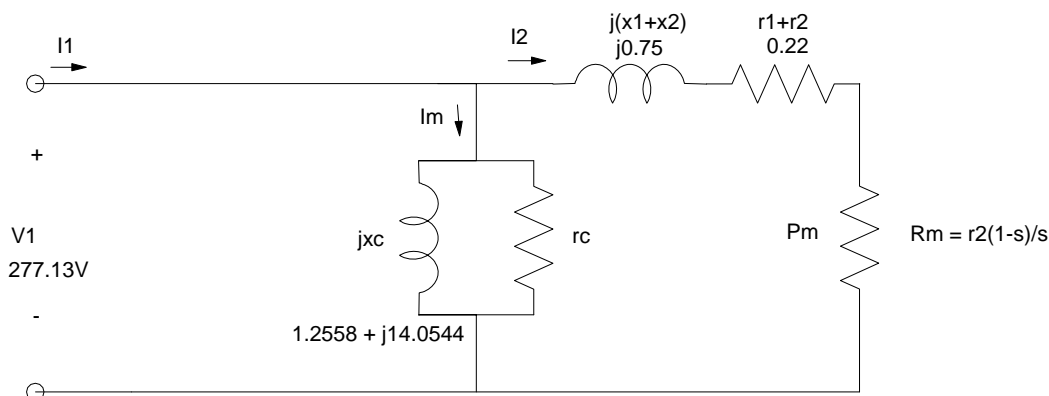
$$V_1 = V_{LN} = \frac{1}{\sqrt{3}} V_{LL} = 277.13V$$

The current through the core is the no-load current draw (approximately - this assume the slip is zero)

$$I_m = 19.94\angle - 84.89^0$$

The core impedance is then:

$$Z_c = \frac{V_1}{I_c} = 1.2588 + j14.0544\Omega$$

If you really want, you can express this as two impedances in parallel or just say the net is 1.26 + j14.05 Ohms

Next, let the slip be 0.025. The current, I2, is then

$$R_m = r_2\left(\frac{1-s}{s}\right) = 4.68\Omega$$

$$I_2 = \frac{277.13V}{(0.22+j0.75)+4.68} = 55.91\angle - 8.70^0$$

The total current is

$$I_1 = I_m + I_2 = 63.53\angle - 26.17^0$$

$$p.f. = \cos(-26.17^0) = 0.8975 \text{ lagging}$$

b) Compute the developed torque:

This essentially means: Find the resulting speed and power delivered to Rm. The resulting speed is 60Hz, divide by 2 (since there are 4 poles)

$$n_s = 2\pi \cdot 60Hz \cdot \tfrac{1}{2} = 188.49 \text{ rad/sec}$$

$$n = (1 - s)n_s = 183.78 \text{ rad/sec}$$

The mechanical power is (times three since there are 3 phases and we're analyzing a single phase)

$$P_m = 3 \cdot |I|_2^2 R_m$$

$$P_m = 3 \cdot (55.91A)^2 \cdot (4.68\Omega) = 43,888W$$

The torque is then

$$P = Tn$$

$$T = \frac{43,888W}{183.78 \text{ rad/sec}} = 238.80 \text{ Nm}$$

c) Compute the output power assuming the rotational losses are 950W.

$$P_o = 43,888W - 950W = 42,938W$$

Problem: Assume the rotational losses are proportional to speed. Compute the torque and power out as the slip varies. from 0 to 1.

Solution: Repeat the above calculations many times. MATLAB is useful for this. To do these computations one time, use the following code: (note: I'm using SciLab because it's free. The syntax is pretty close to MATLAB. http://www.scilab.org/ In case I didn't mention it before, SciLab is free.)

```
s = 0.025;
r1 = = 0.1;
r2 = 0.12;
x1 = 0.35;
x2 = 0.4;

V1 = 480 / sqrt(3);
Im = 19.64 * exp(-j*84.89*pi/180);

Rm = r2 * (1-s) / s;
I2 = V1 / (r1 + r2 + Rm + j*(x1+x2) );

I1 = Im + I2;

ns = 2*pi*60 / 2;
n = (1-s)*ns;

Pg = 3 * (abs(I2))^2 * r2/s;
Tg = Pg / n;

Pm = 3 * (ans(I2))^2 * Rm;
Prot = 950 * (n / 183.78);

Po = Pm - Prot
To = Po / n
```

To do this many times, create a subroutine in MATLAB (SciLab). Create a file with the name slip.m (slip.sci in SciLab)

```
function [To, Po] = slip(s)

j = sqrt(-1);

r1 = 0.1;
r2 = 0.12;
x1 = 0.35;
x2 = 0.4;

V1 = 480 / sqrt(3);
Im = 19.64 * exp(-j*84.89*%pi/180);

Rm = r2 * (1-s) / s;
I2 = V1 / (r1 + r2 + Rm + j*(x1+x2) );

I1 = Im + I2;

ns = 2*%pi*60 / 2;
n = (1-s)*ns;

Pg = 3 * (abs(I2))^2 * r2/s;
Tg = Pg / n;

Pm = 3 * (abs(I2))^2 * Rm;
Prot = 950 * (n / 183.78);

Po = Pm - Prot;
To = Po / n;

Pe = real(3 * V1 * I1);
eff = Po / Pe;


endfunction                        // use end in MATLAB
```

By giving this funciton the extension .m you can call it from MATLAB or SciLab as

```
-->[To,Po] = slip(0.025)
 Po  =

    42931.233
 To  =

    233.59719
```

(The answer is a little different since MATLAB / SciLab doesn't round the results like I did).

Problem: Compute the torque and power vs. slip angle from 0.1 to 0.9 slip.

```
-->s = [0.1:0.1:0.9]'
 s  =

    0.1
    0.2
    0.3
    0.4
    0.5
    0.6
    0.7
    0.8
    0.9

-->To = 0*s;
-->Po = 0*s;
-->for i=1:length(s)
-->   [To(i),Po(i)] = slip(s(i));
-->   end
-->[s,To,Po]
 ans  =

    0.1    646.00589    109592.32
    0.2    691.63456    104296.03
    0.3    596.58337    78717.322
    0.4    502.36432    56816.067
    0.5    427.44306    40285.559
    0.6    369.48517    27858.526
    0.7    324.20454    18333.335
    0.8    288.18517    10864.325
    0.9    258.99502    4881.9411
```
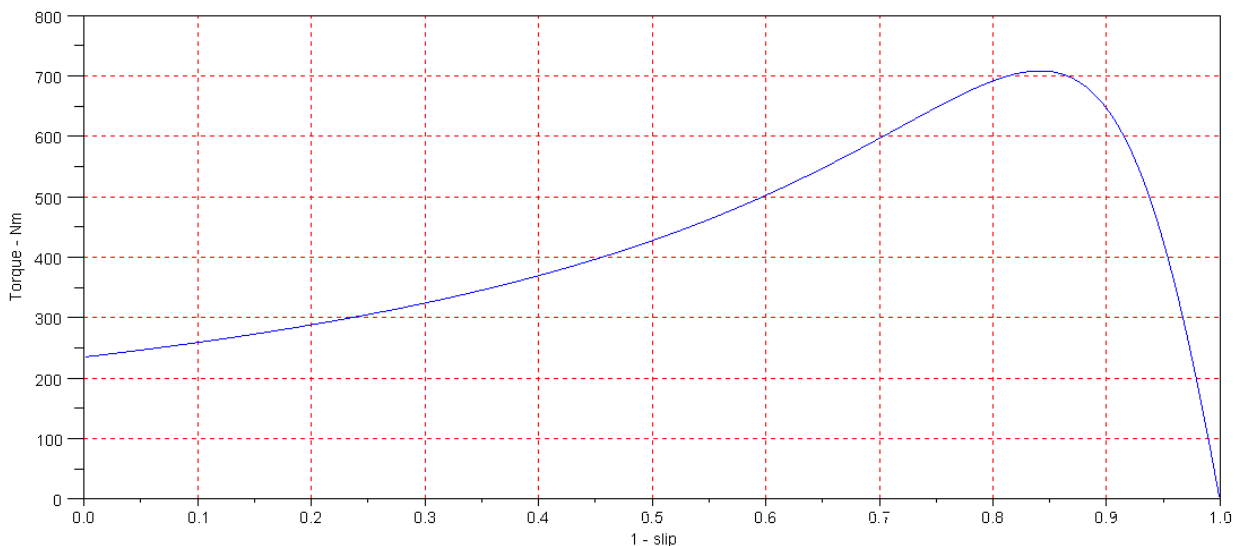
Plot the torque and slip from 0.01 to 0.99:

```
-->s = [0.01:0.01:0.99]';
-->To = 0*s;
-->Po = 0*s;
-->for i=1:length(s)
-->   [To(i),Po(i)] = slip(s(i));
-->   end

-->plot(s,To)
-->plot(1-s,To)
-->xlabel('1 - slip')
-->ylabel('Torque')
```

1-s is porportional to speed with sybchronous speed being at 1-s = 1 (s = 0). Note that as you increase torque, the motor slows down. If you apply too much torque, the motor suddenly stops and falls off the hill. This is bad - when the motor stops, the rotor current becomes very large and can damage the motor. The starting torque is 235Nm. If the load is more than this, the motor won't start.

```
-->plot(1-s,Po/1000)
-->xlabel('1 - slip')
-->ylabel('kW')
```
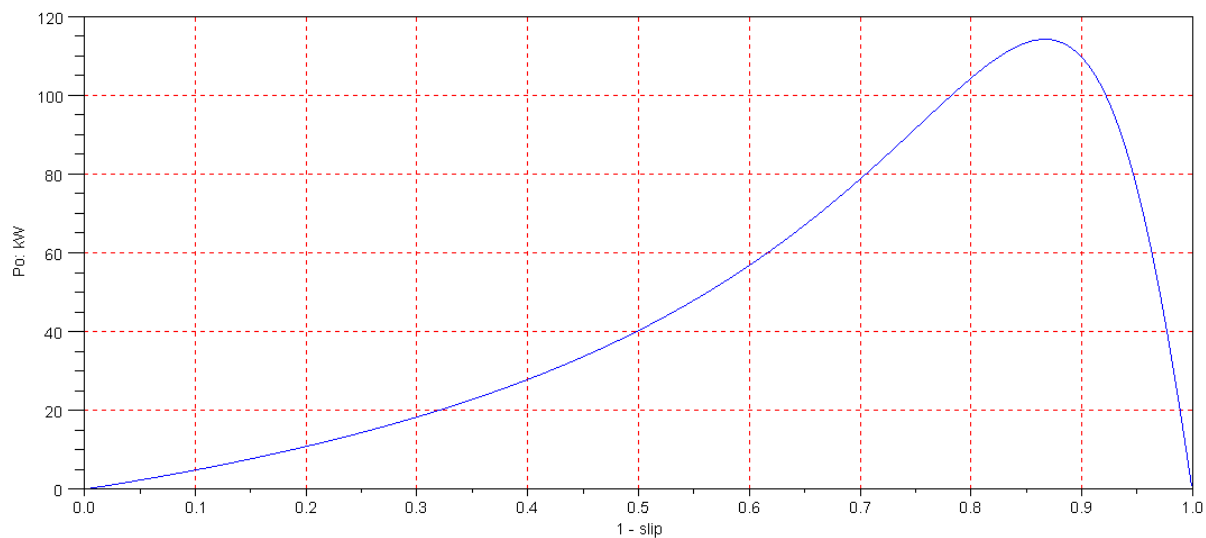

The power output vs. speed also has a similar characteristic. The maximum power out is at a slip of about 0.89. These numbers vary as you change the motor's characteristics (r1, r2, x1, x2).

The efficiency can also be computed (add another output variable in the subroutine). The resulting speed vs. efficiency curve is
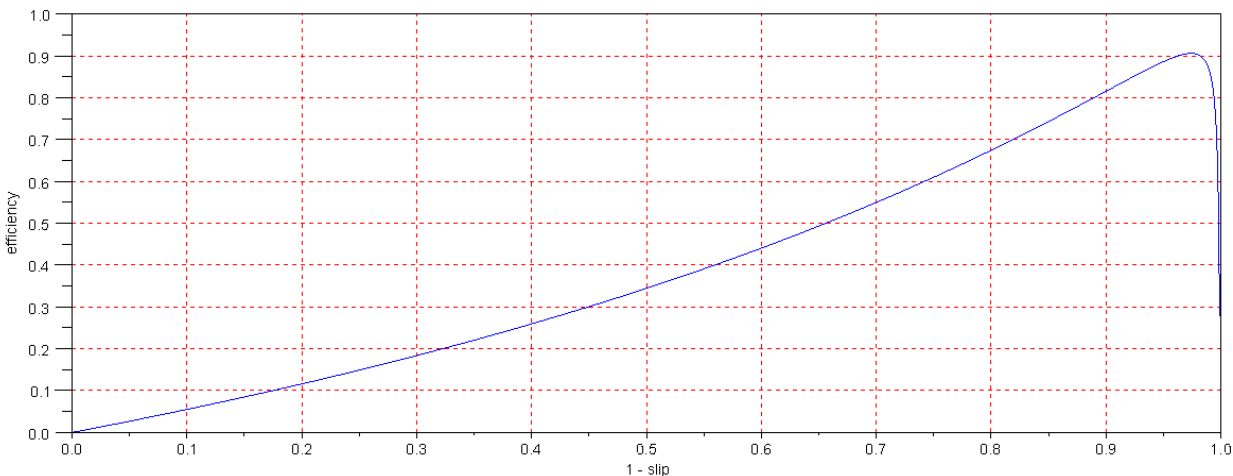
Note that the efficiency is zero at no load: you're not doing any work. At a slip of 1, you're also not doing any work: the motor isn't spinning. The efficiency peaks at 90.5% for a slip of 0.023

```
-->[s,To,Po,eff]
 ans  =
    slip        To            Po            eff
    0.001    4.9993404     941.41111     0.2791297
    0.002    15.13171      2846.5557     0.5383707
    0.003    25.225675     4740.6629     0.6586115
    0.004    35.279057     6623.3457     0.7275144
    0.005    45.289714     8494.2255     0.7718336
    0.006    55.255543     10352.932     0.8024852
    0.007    65.174478     12199.104     0.8247563
    0.008    75.044491     14032.389     0.8415191
    0.009    84.863597     15852.443     0.8544683
    0.01     94.629851     17658.934     0.8646682
    0.011    104.34135     19451.535     0.8728212
    0.012    113.99624     21229.932     0.8794095
    0.013    123.59271     22993.819     0.8847749
    0.014    133.12898     24742.902     0.8891666
    0.015    142.60334     26476.894     0.8927707
    0.016    152.01411     28195.52      0.8957288
    0.017    161.35966     29898.514     0.8981504
    0.018    170.63842     31585.622     0.9001219
    0.019    179.84885     33256.597     0.9017121
    0.02     188.98948     34911.205     0.9029765
    0.021    198.05888     36549.221     0.9039606
    0.022    207.05565     38170.431     0.9047016
    0.023    215.97847     39774.63      0.9052307
```
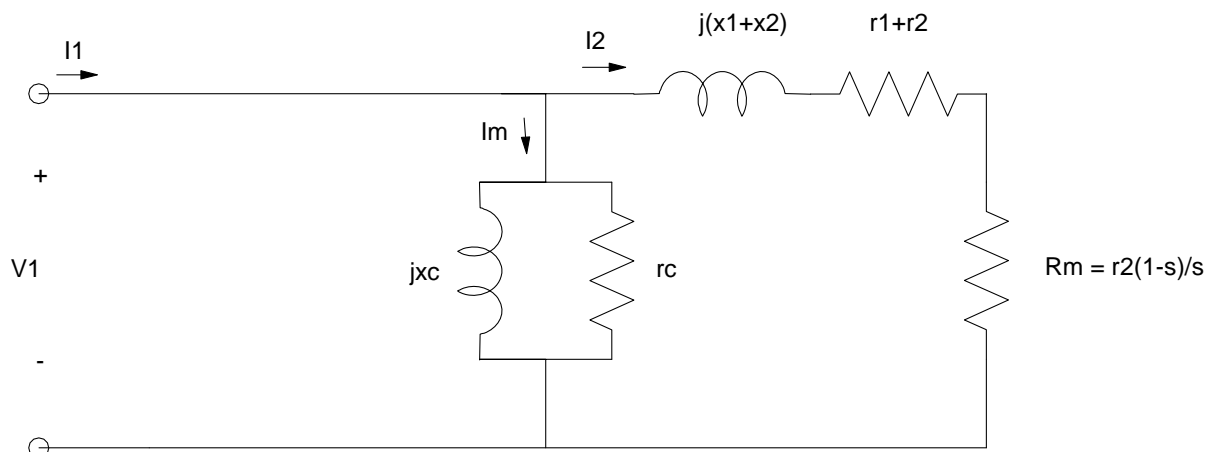
Transformer Testing:

(added February 26, 2010)

Problem: How do you measure the induction motor parameters?

Solution:

1) No-Load test. (Transformer equivalent: open circuit test): Remove any load from the induction motor. This causes the slip to become close to 1 and Rm becomes large (infinity ideally). This sets I2 = 0. Measuring V1, I1, and input power (or power factor) lets you determine the core resistance and reactance (rc and xc).

2) Locked Rotor Test: (Transformer equivalent: short circuit test). Lock the rotor or apply enough voltage to get rated current but not enough to make the motor spin. This results in the slip being 1, and Rm = 0. Measure the input's V1, I1, power factor (or power). Assuming that the core losses are small, this gives you r1+r2 and x1+x2.

3) DC test. With the motor turned off, measure the resistance of one phase at DC. This tells you r1.

AC Induction Motors as Generators

Motors and generators are one and the same thing. The only difference is motors run with a slip between 0 and 1 (speed less than synchronous speed), generators run with a slip less than zero (speeds greater then synchronous speeds). This is seen in the model with Rm being negative (it produces energy rather than absorbs it).
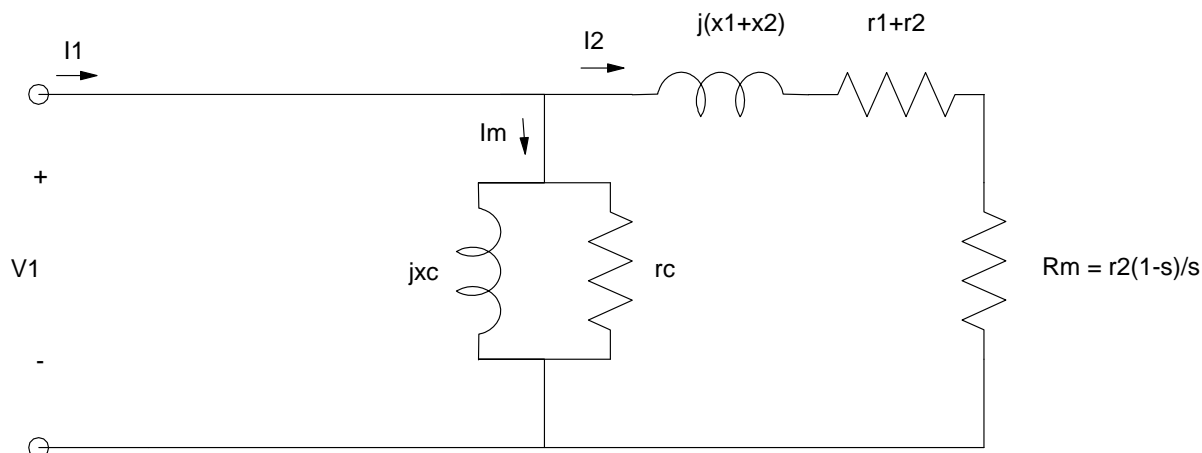
For example, repeat analysis for slips between -1 and 0:

Problem: A 3-phase, 4 pole, 50hp 480V line-to-line, 60Hz, Y connected motor has the followig parameters:

- stator: r1 = 0.1   x1 = 0.35
- rotor: r2' = 0.12   x2' = 0.4
- Pc = core loss in the stator = 1200W,
- Prot = rotational losses = 950W,
- No load current draw = 19.64A with a power factor of 0.089 lagging.

Solution: Use the same code as before but send a slip between 0 and -1. You need to modify the efficiency, slightly, for negative s. When s is negative, the power in is Po and the power out is Pe:

```
eff = 0;
if (s > 0) eff = Po / Pe; end
if (s < 0) eff = Pe / Po; end
```
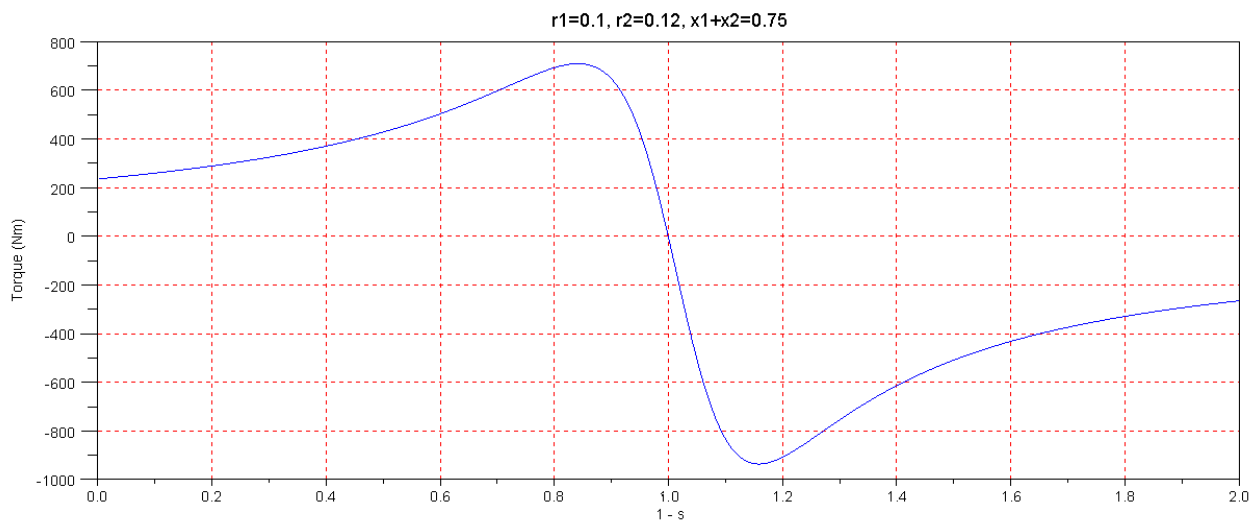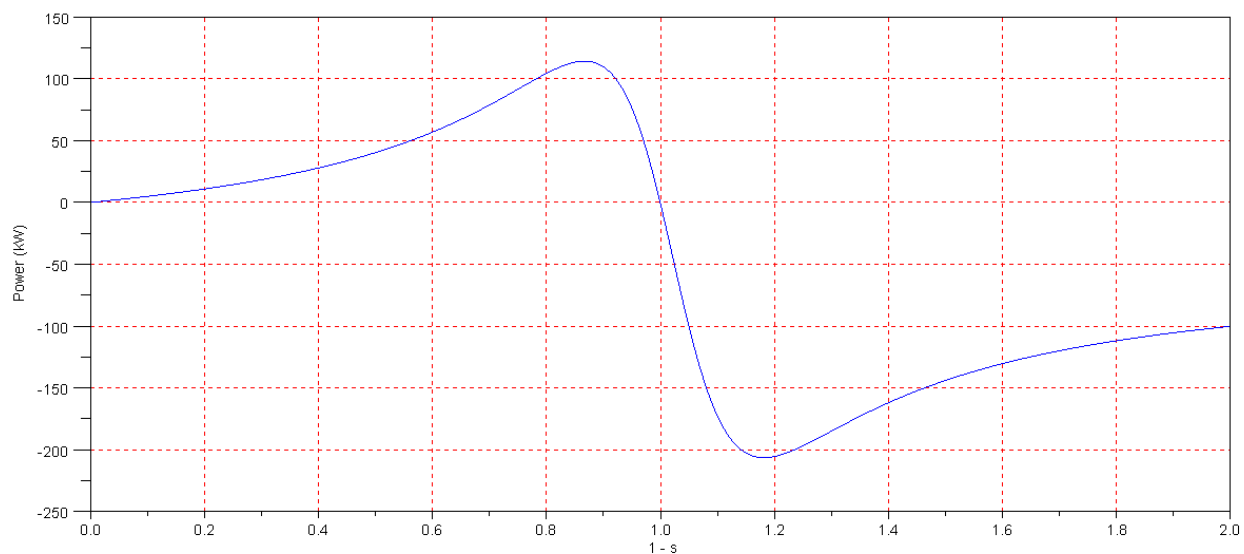
Developed torque vs slip. 1-s greater than 1 is the motor acting as a generator.

Note:

- When used as a motor, if you apply too much torque, you'll stall the motor. This is bad - it produces too much current in the rotor and burns out the motor.
- When used as a generator, if you apply too much torque (T < -935), the speed (1-s) goes to infinity. That is also bad - it send pieces flying.

Developed power vs slip. 1-s greater than 1 is the motor acting as a generator.
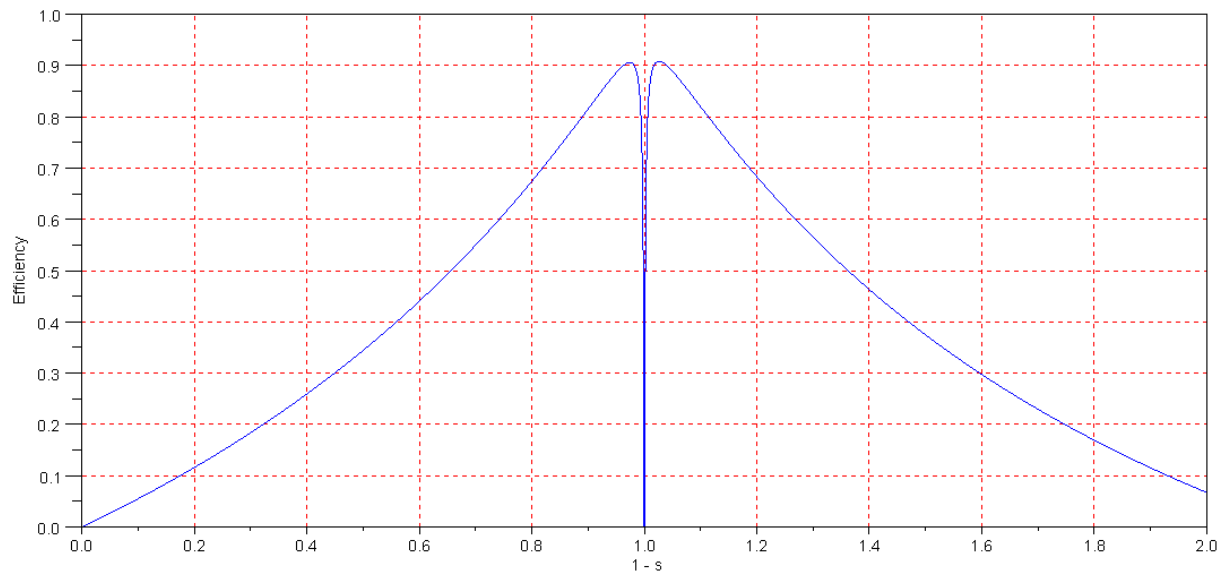
Efficiency vs. Speed: 1-s greater than 1 is the motor acting as a generator.

Note: If you spin an AC induction motor without energizing its stator, it just spins. No power is generated.

The generated frequency is fixed at 60Hz and is set by the frequency you're using to energize the stator. The stator is also in phase with the power grid automatically. This allows you to conect a spinning AC induction motor to a power grid without having to worry about phase mis-alignment.

- To get maximum efficiency, you want the speed to be close to 1-s = 1.05.
- To get the maximum power generated, you want the slip to be close to and maximum power, you really want the speed of the motor to be around 1-s = 1.15

You need to somehow control the speed to get maximum efficiency or maximum power.

This implies that if you're building a wind turbine with an AC induction motor / generator, it helps a great deal if you can vary the pitch of the blades to control the speed of the wind turbine to control the speed of the AC generator. Alternatively, you can use a variable gear box so that the motor spins at a constant speed while the wind turbine spins at different speeds.