Single-Phase Induction Motors

How to make a single-phase motor's magnetic field spin.

Option 1: Don't.

If a 3-phase motor is spinning and you remove one of it's phases (as you did in lab last week), the motor keeps spinning. The single-phase exciation induces current onto a spinning rotor and likewise produces torque. The only catch is

- The motor is less efficient
- It's noisier (you should have heard a loud humm when you removed a lead). The magnetic field induced from a 3-phase balanced source is constant magnitude, resulting in a quiet motor. The magnetic field induced with a single-phase line bounces back and forth, causing this hum.
- The motor can't start.

At rest, there is no reason for the motor to spin clockwise or counterclockwise. By symmetry, the motor has no torque.

Option 2: Add a small motor to get it started.

Option 3: Create a second phase that's 90 degres out of phase, creating a 2-phase motor.

Two ways to do this are:

Resistance Start Split Phase:

- Make the main winding primarily reactive.
- Make the second winding primarily resistive.

Capacitive Start Split Phase:

- Make both windings primarily reactive
- Add a capacitor in series with the secondary winding.

Either method creates a phase shift, which creates a spinning magnetic field. Both methods don't have a lot of starting torque, however.

- Adding a resistor in series with a resistance start split phase motor limits the current in the second winding. The spinning field is more of an elipse than a circle as a result.
- Adding a capacitor in series results in a high impedance in the second winding. Again, the spinning field is more of an ellipse than a circle.

Example: Assume the stator (phase B) has an impedance of

$$r1 + jx1 = 0.12 + j0.35$$

Compute the phase differnnce of a second winding (phase B) which is coupled with

a) A 2.65 Ohm resistor in series, or

b) A 1000uF capacitor

Solution: Phase A has an impedance of

$$Z_A = 0.12 + j0.35 = 0.37\angle 71.1^0$$

a) Adding a 2.65 Ohm resistor in series with phase B results in

$$Z_B = 2.77 + j0.35 = 2.79\angle 7.2^0$$

which is 63.8 degrees apart. This isn't 90 degrees, so the phases are not in quadrature. The amplitude of the current in phase B will also be smaller, resulting in the rotating field being an oval, not a circle.

b) Add a 7000uF capacitor in series with phase B. The impedance of 7uF at 60Hz is

$$Z = \frac{1}{j\omega C} = -j0.38$$

which results in

$$Z_B = (0.12 + j0.35) - j0.38 = 0.12\angle -14.0^0$$

which has a phase difference of 85 degrees. The rotating field is closer to a uniform circle, but still not perfect.

MATLAB (SciLab) Code:

Plot 100 points in one cycle (0 to 2π)

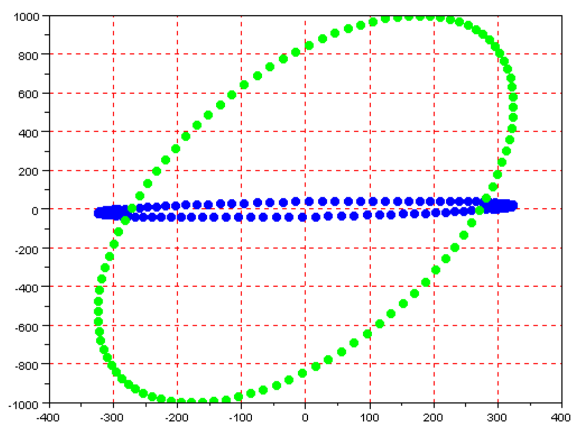
```
-->t = [0:0.01:1]' * 2*%pi;
-->j = sqrt(-1);
```

Resistive Start Split Phase

```
-->Ia = (120/0.37)*sin(t - 71.1*%pi/180);
-->Ib = (120/2.79)*sin(t - 7.2*%pi/180);
-->plot(Ia,Ib,'.')
-->xgrid(5)
```

Capacitive Start Split Phase

```
-->Ib2 = (120/0.12)*sin(t - 14*%pi/180);
-->plot(Ia,Ib2,'g.')
```



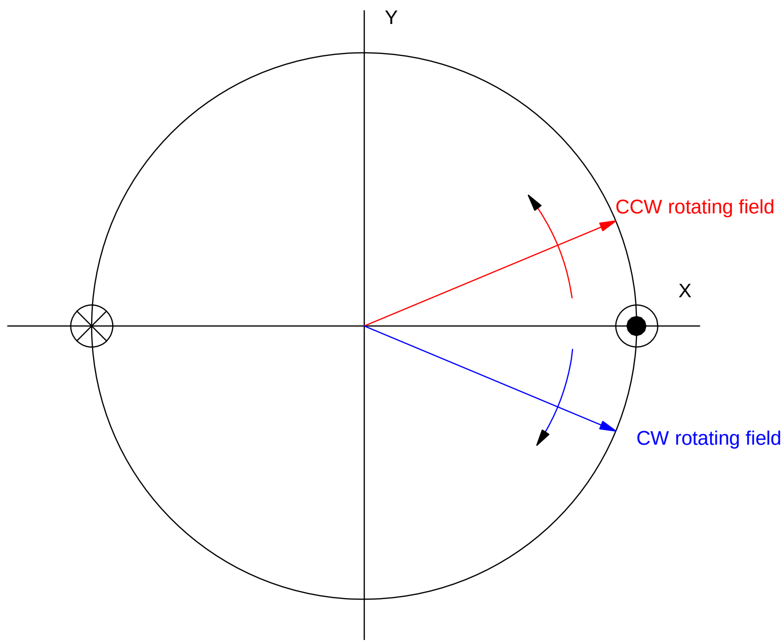
Rotating Magnetic Field for Resistive (blue) and Capacitive (green) Start Split Phase

Electrical Models for a Single-Phase Induction Motor

One way to model a single-phase induction motor is to use the model for a 3-phase induction motor twice:

- One for the field rotating clockwise
- One for the field rotating counterclockwise

The counter rotating fields result in the fields cancelling in the Y direction, resulting in a net field that bounces left and right in the figure below.

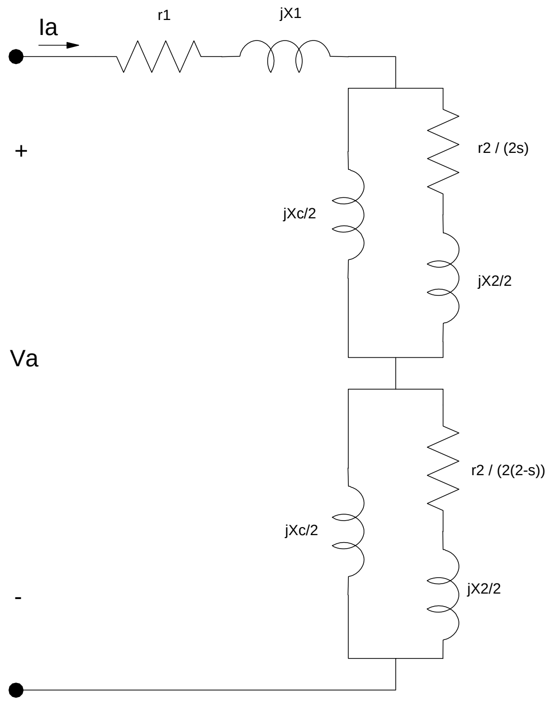


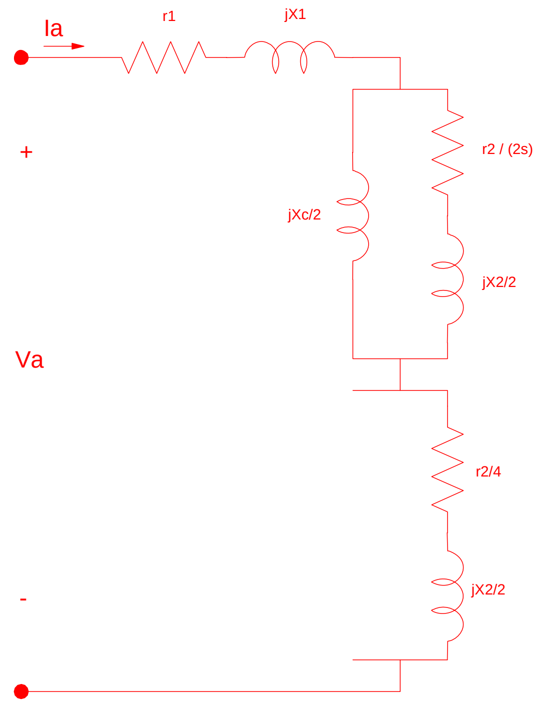
If the motor is spinning at synchronous speed in the CCW direction,

- The slip relative to the CCW field is zero
- The slip relative to the CW field is 2

So, you have two different slip's resulting in two different impedances for the circuit equivalent. This gives the rotor portion of the circuit model two sections as shown below to the left:

- The stator (r1 + jx1) are the same as a 3-phase motor
- The core reactance is split into two portions in series by symmetry: one for the CW and one for the CCW rotating field
- The rotor's reactance is also split into two portions in series by symmetry.
- The rotor's resistance is split into two portions (r2/2):
- The slip for the two portions are (s) and (2-s).





Appoximate Model for s < 0.15

Normally, the slip will be small. In this case, you can approximate

$$2(2-s) \approx 4$$

$$j\tfrac{X_c}{2} \,||\, \left(\tfrac{r_2}{4} + j\tfrac{X_2}{2}\right) \approx \left(\tfrac{r_2}{4} + j\tfrac{X_2}{2}\right)$$

and use the circuit to the right. The torque comes from the power dissipated in Rm1 minus the torque dissipated in Rm2, which is in this case only comes from the forward rotating field:

$$\frac{r_2}{2s} = \frac{r_2}{2s}(1+s-s) = \frac{r_2}{2}\left(\frac{1-s}{s}\right) + \frac{r_2}{2} = R_{m1} + \frac{r_2}{2}$$

$$R_{m1} = \frac{r_2}{2}\left(\frac{1-s}{s}\right)$$

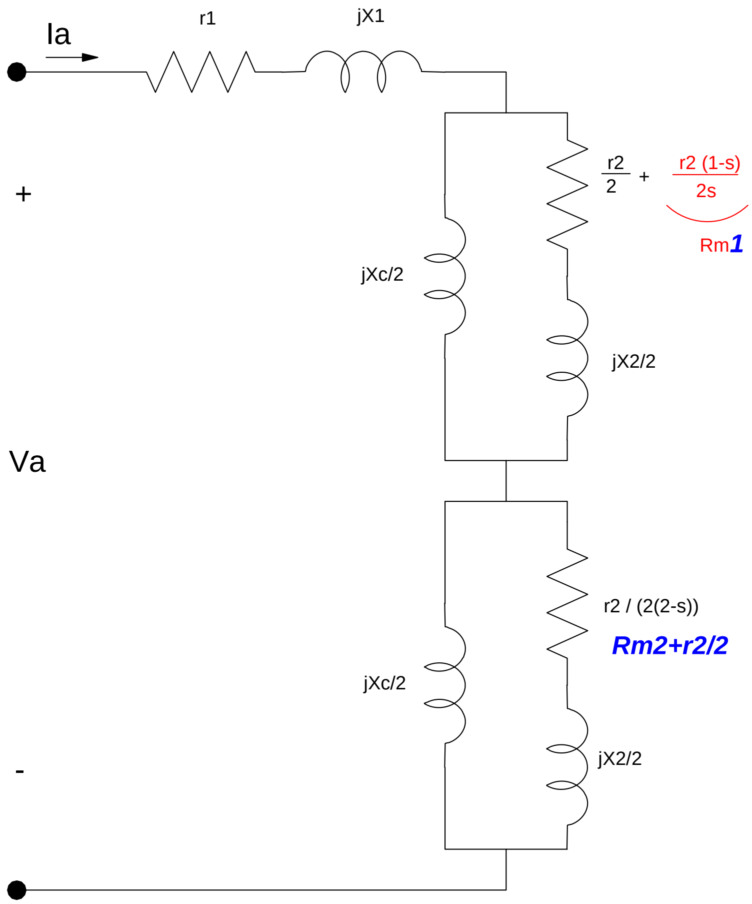
Opposing this is the power delivered from the field rotating the opposite direction

$$\frac{r_2}{2(2-s)} = R_{m2} + \frac{r_2}{2}$$

$$R_{m2} = \frac{r_2}{2(2-s)} - \frac{r_2}{2} = \left(\frac{r_2}{2}\right)\left(\frac{1}{2-s} - 1\right)$$

$$R_{m2} = \left(\frac{r_2}{2}\right)\left(\frac{1-s}{2-s}\right)$$

Note that Rm2 is negative. It's power is negative and opposes the power (and torque) from Rm1. At a slip of 1 (standstill), Rm1 = - Rm2 and the net toque is zero.



The power out is the power delivered to Rm1 plus the power delivered to Rm2 (which is negative and will oppose the torque from Rm1)

Torque vs. Speed Curves

Compute the power dissipated in Rm1 and Rm2 vs slip

Note that

- Torque = 0 at s = 0 as before
- Torque = 0 at s = 1.

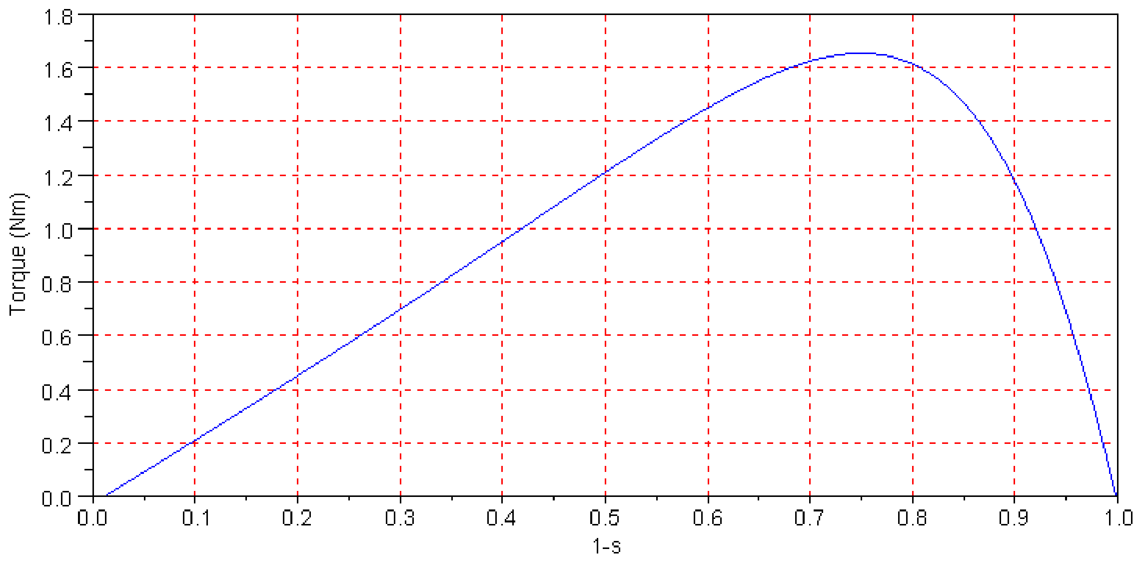
There is no starting torque for a single-phase induction motor.

SciLab Code:

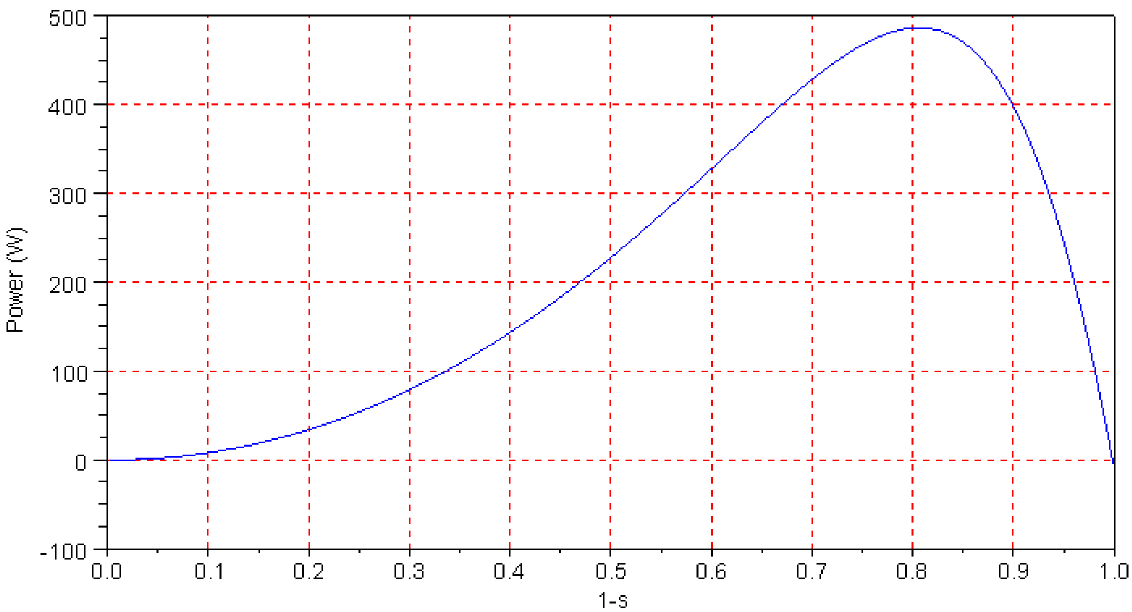
```
function [To,Po] = slip1(s)

   r1 = 1.3;
   r2 = 3.0;
   x1 = 2.5;
   x2 = 2.0;
   xc = 50;

   Va = 110;

   Rm1 = r2*(1-s)/(2*s);
   Rm2 = r2/2/(2-s) - r2/2;


   j = sqrt(-1);
   Z1 = r1 + j*x1;
   Zc = j*xc/2;
   Zf = r2/2 + Rm1 + j*x2/2;
   Z21 = 1/(1/Zc + 1/Zf);

   Zr = r2 / (2*(2-s)) + j*x2/2;
   Z22 = 1/(1/Zc + 1/Zr);

   Za = Z1 + Z21 + Z22;

   Ia = Va / Za;

   I21 = (Zc / (Zc + Zf))*Ia;
   Pm1 = (abs(I21))^2 * Rm1;

   I22 = (Zc / (Zc + Zr))*Ia;
   Pm2 = (abs(I22))^2 * Rm2;

   Pm = Pm1 + Pm2;

   ns = 2*%pi*60;
   n = (1-s)*ns;

   Prot = 10*(n/ns);

   Po = Pm - Prot;

   To = Po / n;


endfunction
```

Sample Output:

```
-->for i=1:length(s)
-->   [To(i),Po(i)]=slip1(s(i));
-->   end

-->[s(N),To(N),Po(N)]
 ans  =

    0.1    1.1729252    397.96415
    0.2    1.6132279    486.53806
    0.3    1.6243379    428.65267
    0.4    1.4497729    327.93091
    0.5    1.2091029    227.91053
    0.6    0.9523943    143.61768
    0.7    0.6980582    78.948521
    0.8    0.4509929    34.00406
    0.9    0.2104691    7.9344965
```



Note that the torque is zero at synchronous speed (1-s = 1) and at standstill (1-s = 0). There is no start-up torque.



Performance Analysis

Electric Power In

$P_i = V_a I_a \cos\theta$

Stator Copper Loss

$Pc = I_a^2 r_1$

Gap Power

reverse

$P_{g2} = (I_{22})^2 \left(\frac{r_2}{2(1-s)}\right)$

forward

$P_{gf} = (I_{21})^2 \left(\frac{r_2}{2s}\right)$

Rotor Copper Loss

$(I_{21})^2 \frac{r_2}{2}$

Mechanical Power

$P_{m1} = P_{g1}(1-s)$

Rotational Losses

P_{rot}

Output Power

$P_o = P_{m1} - P_{rot}$

Example: A 1/4hp, 110V, 60Hz, 2-pole, single phase induction motor has a rotational loss of 10W at normal speeds. The equivalent circuit parameters are

r1 = 1.3 Ohm r2 = 3.0 Ohm

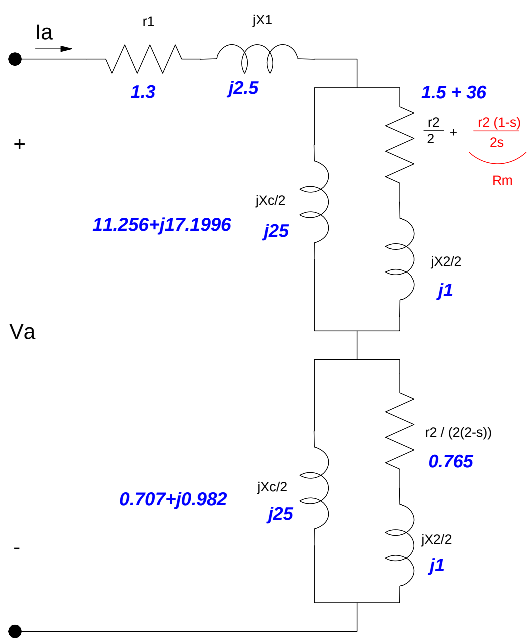
x1 = 2.5 Ohm x2 = 2.0 Ohm

xc = 50 Ohms

Determine the line current, line power factor, power out, and efficiency at a slip of 4%

Solution: (Note: this isn't how the book does it. Being more comfortable with my circuit analysis, I prefer drawing the circuit and computing the power to Rm using circuits techniques.)

Draw the circuit:



The goal is to find the current and power dissipated in Rm

$$R_m = \frac{r_2}{2}\left(\frac{1-s}{s}\right) = 36\Omega$$

The net impedance (adding stuff in series and parallel) is

$$Z = 13.263 + j20.678$$

The current is then

$$I_a = \frac{110V}{13.263+j20.678} = 4.478\angle -57.325^0$$

The line current is 4.478 Amps

The line power factor is 0.54 lagging

Power Out: Compute the power to Rm1:

By current divicion

$$I_{Rm1} = \left(\frac{j25}{j25+(37.5+j1)}\right) 4.478A = 2.453\angle 55.26^0 A$$

$$P_{m1} = |I_{Rm1}|^2 R_{m1} = (2.453)^2 \cdot 36\Omega = 216.68W$$

The power to Rm2 opposes the rotation (at a slip of 1, it is equal and opposite, resulting in no start-up torque.)

$$I_{Rm2} = \left(\frac{j25}{j25+(0.765+j1)}\right) 4.478A = 4.304\angle 1.65^0 A$$

$$R_{m2} = \frac{r_2}{2(2-s)} - \frac{r_2}{2} = \frac{r_2}{2}\left(\frac{s-1}{2-s}\right) = -0.735\Omega$$

$$P_{m2} = |I_{Rm2}|^2 R_{m2} = (4.304)^2 \cdot (-0.735)\Omega = -13.6W$$

So the net mechanical power developed is

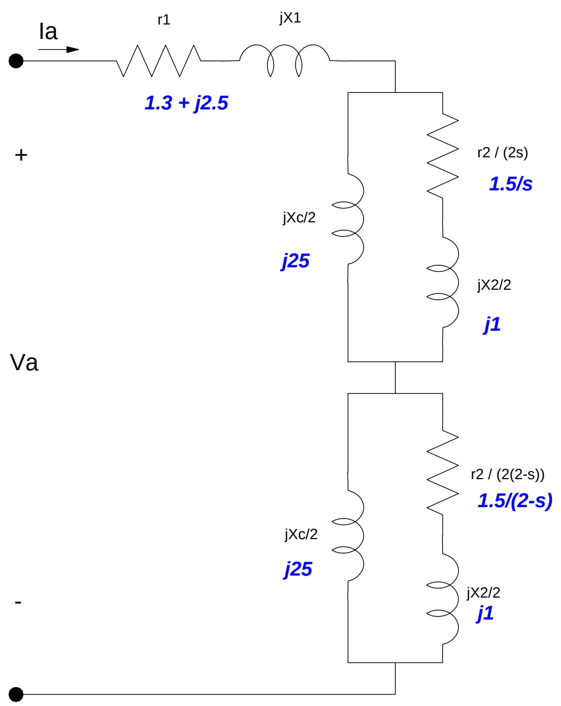
$$P_m = 216.68W - 13.6W = 203.06W$$

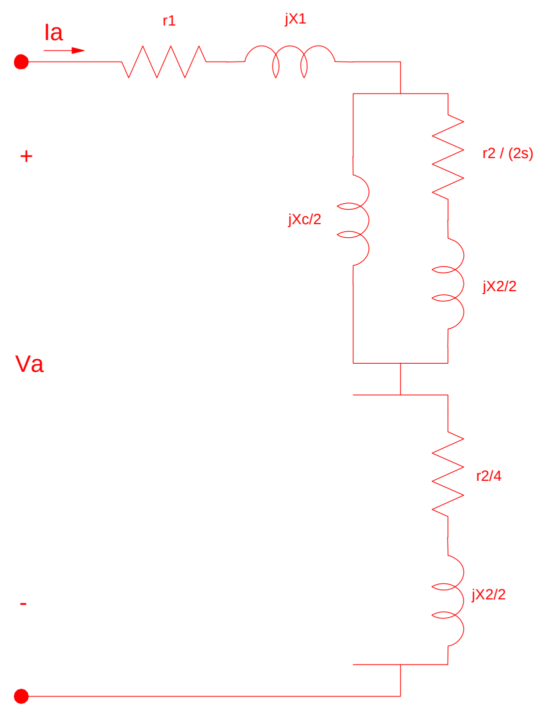
The power out is the mechanical power minus the rotational losses

$$P_o = P_m - P_{rot} = 193.06W$$

Determining the Motor's Parameters

To measure the parameters for a single-phase AC induction motor, consider the model with typical parameters shown to the left:





Appoximate Model for s < 0.15

1) DC Test: Measure the DC resistance of the stator using an Ohm meter (or like device). This gives you r1.

2) Blocked Rotor Test: Apply voltage until you get rated current while the motor is locked (not spinning). This sets the slip equal to 1 and the impedance is

$$Z_a = (r_1 + jx_1) + (jx_c || r_2 + jx_2)$$

but due to the large impedance of xc relative to r2 and x2

$$Z_a \approx (r_1 + jx_1) + (r_2 + jx_2)$$

3) No-Load Test: Remove the load from the rotor. Let the motor spin freely, resulting in the slip being approximately zero. The impedance measured is then

$$Z_a = (r_1 + jx_1) + \left(\frac{jx_c}{2} || \left(\frac{r_2}{2s} + \frac{jx_2}{2}\right)\right) + \left(\frac{jx_c}{2} || \left(\frac{r_2}{2(2-s)} + \frac{jx_2}{2}\right)\right)$$

$$Z_a \approx (r_1 + jx_1) + \left(\frac{jx_c}{2}\right) + \left(\frac{r_2}{4} + \frac{jx_2}{2}\right)$$

Example: Determine an approximate model for a single-phase AC induction motor with the following test results:

- Blocked Rotor Test: Vsc = 110V, Isc = 17.67A, Psc = 1342W
- No-Load Test: Vn = 110V, In = 3.84 A, Pn = 53.9W
- Rotational Losses 17W
- DC Resistance 1.3 Ohms

r1 is the DC resistance:

r1 = 1.3 Ohms

The blocked rotor test gives

$$pf = \frac{1342W}{110V \cdot 17.67A} = 0.691 = 46.3^0$$

$$(r_1 + r_2) + j(x_1 + x_2) = \frac{110V}{17.67A} \angle 46.3^0 = (4.3 + j4.5)\Omega$$

so

r2 = 3.0 Ohms

The no-load test gives

Stator and core losses = 53.9W - 17W = 36.9W

$$pf = \frac{36.9W}{110V \cdot 3.84A} = 0.087 = 84.99^0$$

$$(r_1 + jx_1) + \left(\frac{jx_c}{2}\right) + \left(\frac{r_2}{4} + \frac{jx_2}{2}\right) = \frac{110V}{3.84A} \angle 84.99^0 = (2.05 + j28.5)\Omega$$

This gives two equations for two unknowns for the real part:

$$r_1 + r_2 = 4.3\Omega$$

$$r_1 + \frac{r_2}{4} = 2.05\Omega$$

$$r_1 = 1.3\Omega$$

$$r_2 = 3.0\Omega$$

and for the complex part.

$$x_1 + x_2 = 4.5\Omega$$

$$x_1 + \frac{x_2}{2} + \frac{x_c}{2} = 28.5\Omega$$

If x1 = x2

$$x_1 \approx x_2 \approx 2.25\Omega$$

$$x_c \approx 25.12\Omega$$