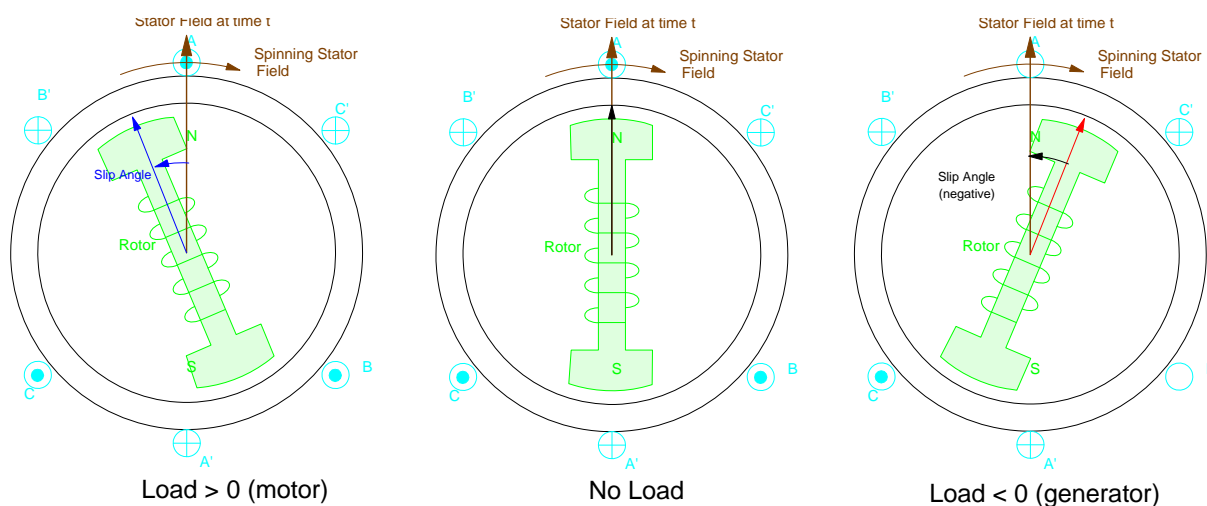


AC Synchronous Generators

If you have an AC synchronous motor

- If the motor has a load, the rotor will lag the rotating stator field by the slip angle, δ .
- If the load torque is reduced to zero, the slip angle, δ , will be zero degrees.
- If the load torque is negative (meaning you're applying torque *to* the motor), the rotor will lead the rotating stator field by the slip angle, δ .



If you don't mind using negative numbers, a motor looks just like a generator, only with

- negative load (meaning you're feeding energy to the motor rather than taking it from the generator), and
- negative slip (meaning the rotor leads the stator field.)

If you hate negative numbers, you can come up with a new set of equations for an AC synchronous generator.

In any event, the synchronous speed for a generator is the same as a motor:

Synchronous Speed: (P = # of poles in the rotor)

$$n_s = \frac{2\pi \cdot 60 \text{ Hz}}{P/2}$$

AC synchronous generators only work at synchronous speed. Likewise, this causes problems at start-up. To get up to synchronous speed:

- Use an external motor to get close to synchronous speed then power up the rotor
- Use an external power supply (such as a steam turbine) to get up to synchronous speed
- Imbed a squirrel cage in side the motor so it runs like an asynchronous motor on startup. Remove the load until you get close to synchronous speed (slip is close to zero). Power up the rotor to lock on at synchronous speed.

Circuit Equivalent: Let's follow the text and pretend we hate negative numbers, This results in a whole new set of equations....

On second thought, let's pretend that we don't mind negative numbers and stick with one set of equations.

As before with an AC synchronous motor, E_f is the input from the generator, V_t is the output, and the armature current goes from the generator to the load (V_t):

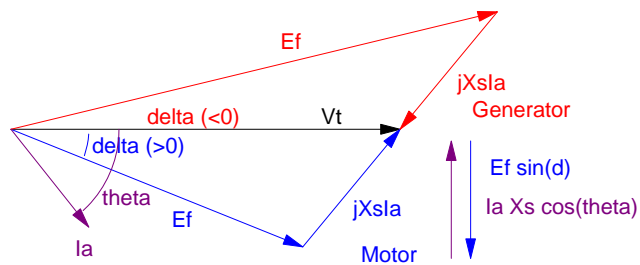
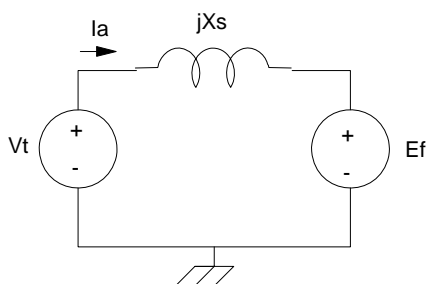
$$V_T = (R_a + jX_s)I_a + E_f$$

- R_a = armature resistance
- X_s = Synchronous Reactance = armature leakage reactance (X_l) + armature reaction reactance (X_{ar}). The motor acts like a big inductor - X_s is the net inductance (reactance) of the motor.
- E_f = Excitation voltage per phase

Assume $R_a = 0$:

$$V_T = jX_s \cdot I_a + E_f$$

Note that as a motor, I_a is negative, meaning current is flowing to the load (V_t). This results in $E_f > V_t$ as you'd expect.



The excitation voltage can be any amplitude and is set by the voltage you apply at the rotor's electromagnets.

The rotor will be running at synchronous speed, but will lag slightly behind the rotating field by an angle δ . When the power delivered is zero, this lag will be zero as well:

$$-E_f \cdot \sin(\delta) = I_a X_s \cos(\theta) \quad (8-11)$$

Multiply by V_t and divide by X_s to get power per phase (times 3 since it's three phase)

$$P = 3 \cdot V_t I_a \cdot \cos(\theta)$$

$$P = 3 \cdot \left(\frac{-V_t E_f}{X_s} \right) \cdot \sin(\delta)$$

Example 8-1: A 74.6kW (100hp), generator, 3 phase, 4-pole, 60Hz, 265V_{LN}, X_s = 2.72 Ohms is delivering 100hp with a power factor of 0.8 leading.

a) Find the developed torque:

$$\text{Speed: } n_s = (2\pi \cdot 60\text{Hz})\left(\frac{1}{2}\right) = 188.5 \text{ rad/sec}$$

$$\text{Power: } P = -100\text{hp} \cdot \left(\frac{746\text{W}}{\text{hp}}\right) = -74.6\text{kW}$$

$$\text{Torque: } T = \left(\frac{-74.6\text{kW}}{188.5\text{rad/sec}}\right) = -395.7\text{Nm}$$

(negative since we're using motor equations. Negative power absorbed is power delivered. Ditto for torque.)

b) Find the input current:

$$3 \cdot V_t I_a \cos(\theta) = -74.6\text{kW}$$

$$3(265\text{V})(I_a)(0.8) = -74.6\text{kW}$$

$$I_a = -117.29\text{A}$$

or

$$I_a = -117.29 \angle 36.87^\circ$$

(negative again since current actually flows to V_t rather than E_f).

c) Find the slip (delta): First, find the excitation voltage:

$$V_T = jX_s \cdot I_a + E_f$$

$$265\text{V} = (j2.72)(-117.29 \angle 36.87^\circ) + E_f$$

$$E_f = 265.14 \angle 73.78^\circ \text{ Volts}$$

The slip is

$$P = 3 \cdot \left(\frac{-V_t E_f}{X_s}\right) \cdot \sin(\delta)$$

$$-74.6\text{kW} = 3 \cdot \left(\frac{-(265\text{V})(265.14\text{V})}{2.72\Omega}\right) \sin(\delta)$$

$$\delta = 80.3^\circ$$

The rotor is leading the stator's rotating magnetic field by 80.3 degrees. The maximum power you can generate is when δ is -90 degrees: (The negative extream since we're treating generators as motors with negative power)

$$P = 3 \cdot \left(\frac{-V_t E_f}{X_s}\right) \cdot \sin(\delta)$$

$$P_{\max} = 3 \cdot \left(\frac{-265\text{V} \cdot 265.14\text{V}}{2.72\Omega}\right) = -77.49\text{kW} \quad (\text{generate } 77.49\text{kW})$$

$$T_{\max} = \frac{P_{\max}}{188.5} = -411\text{Nm}$$

Effect of Changing the Shaft Load:

Assume a 74.6kW (100hp) generator, 3 phase, 4-pole, 60Hz, 265V_{LN}, X_s = 2.72 Ohms, E_f = 522V. (as per example 8-1)

Find the slip angle, δ , for a load of 100hp:

$$P = 3 \cdot \left(\frac{-V_t E_f}{X_s} \right) \cdot \sin(\delta)$$

$$-74.6kW = 3 \cdot \left(\frac{-265V \cdot 522V}{2.72\Omega} \right) \cdot \sin(\delta)$$

$$\delta = +29.2^\circ$$

The rotor is leading the stator field (as it should for a generator) by 29 degrees. Find the input current:

$$V_T = jX_s \cdot I_a + E_f$$

$$265V = (j2.72)(I_a) + (522\angle +29.2^\circ)V$$

$$I_a = 116.95\angle 143.17^\circ = -116.95\angle -36.82^\circ$$

We're delivering 116.95Amps (hence the -I to the right) with a power factor of 0.8 lagging.

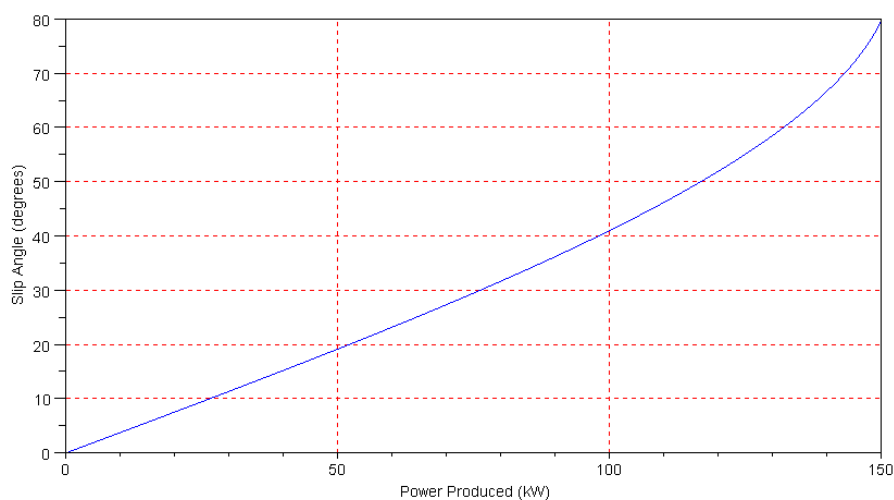
In SciLab, compute the current when generating 0 to 75kW

Power goes from 0 to -75kW consumed (i.e. 75kW produced)

```
-->P = [0:0.1:1]' * (-75000)
0.
- 7500.
- 15000.
- 22500.
- 30000.
- 37500.
- 45000.
- 52500.
- 60000.
- 67500.
- 75000.
```

The slip angle is then:

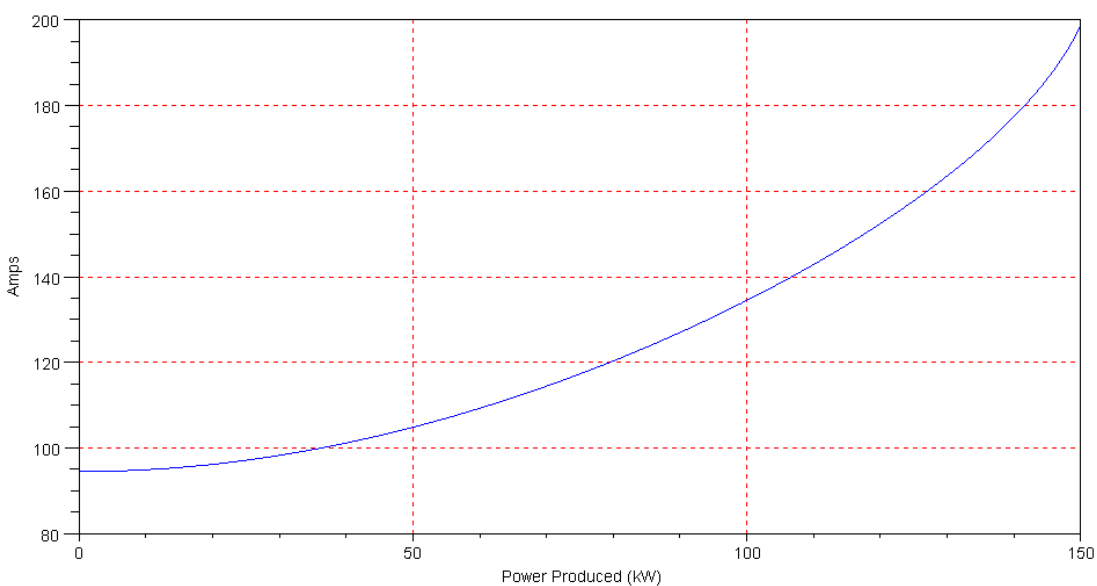
```
-->Vt = 265;
-->Ef = 522;
-->Xs = 2.72;
-->delta = asin(P*Xs / (-3*Vt*Ef));
-->j = sqrt(-1);
-->Ia = (Vt - Ef) / (j*Xs);
-->Ef = 522*exp(j*delta);
-->Ia = (Vt - Ef) / (j*Xs);
```



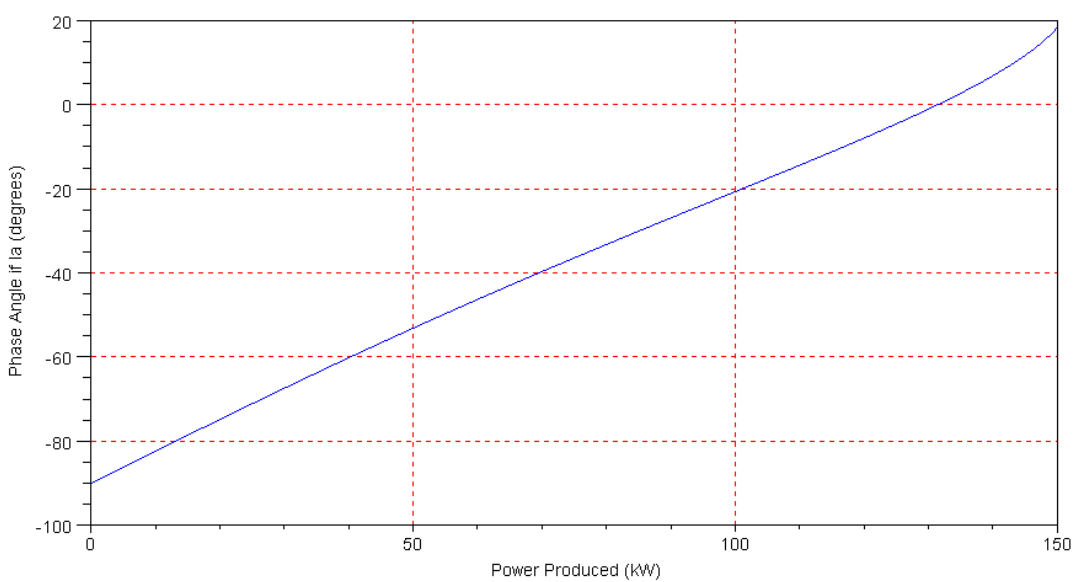
Slip Angle vs. Power Produced: Note that the slip angle approaches 90 degrees at _____ (the maximum power you can produce with an excitation field of 522V).

```
-->[P,abs(-Ia), atan(imag(-Ia),real(-Ia))*180/%pi]
```

kW	amps	phase
0.	94.485294	- 90.
- 7500.	94.724231	- 84.28421
- 15000.	95.439179	- 78.597752
- 22500.	96.62475	- 72.968037
- 30000.	98.27261	- 67.418853
- 37500.	100.37236	- 61.969066
- 45000.	102.91267	- 56.631785
- 52500.	105.88252	- 51.413993
- 60000.	109.27256	- 46.316527
- 67500.	113.07651	- 41.334282
- 75000.	117.29278	- 36.456442



Current Delivered for Loads from 0 to 150kW.



Phase of I_a vs. Power Produced. Note that the power factor is 1.00 (zero degrees phase) at 132kW.

Effect of Changing the Excitation Voltage (E_f):

With E_f large (522V) and you were producing 75kW, the current had a power factor of 36 degrees lagging. What happens if you change E_f ?

Lets repeat the previous example, assume a 75kW load, and vary E_f :

Load = -75kW, 3 phase, 4-pole, 60Hz, 265V_{LN}, $X_s = 2.72$ Ohms,

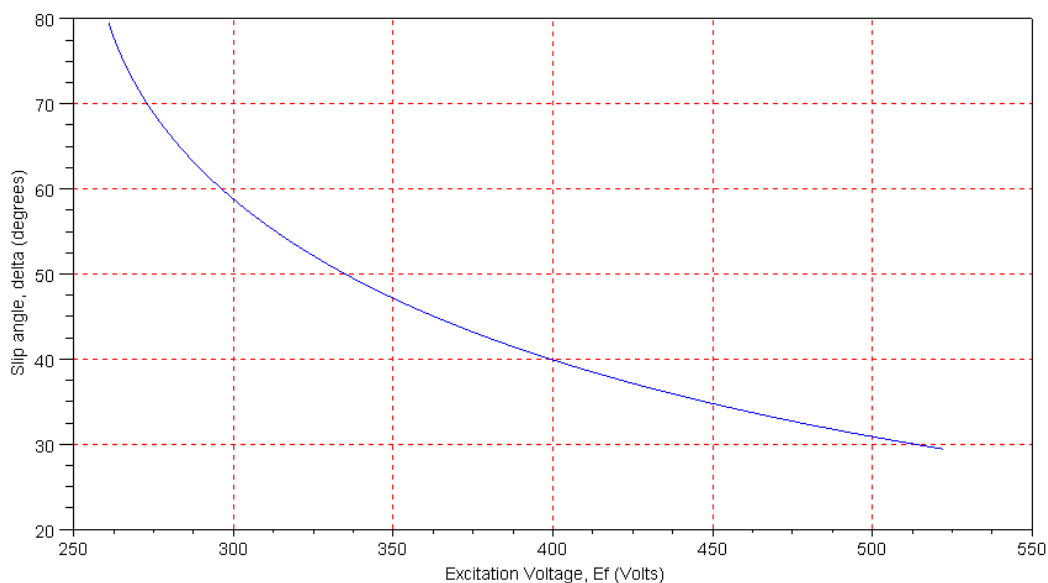
First, find the slip angle:

```
-->Ef = [0.01:0.01:1]' * 522;
-->P = -75000;
-->Xs = 2.72;
-->Vt = 265;
-->delta = (P*Xs) ./ (-3*Vt*Ef);
-->[Ef, delta]
```

E_f	$\sin(\delta)$	
5.22	49.157811	no solution
57.42	4.4688919	(E_f isn't large enough to deliver 75kW)
109.62	2.3408481	
161.82	1.5857358	
214.02	1.198971	
266.22	0.9638786	
318.42	0.8058658	
370.62	0.6923635	
422.82	0.6068866	
475.02	0.5401957	

note: you actually need to take the arcsin() of the last column. Numbers bigger than 1.00 are telling you that it won't work. You can't generate 75kW if the excitation voltage is too small. So, let's take E_f from 261V to 522V

```
-->Ef = [261:522]';
-->delta = asin(-(P*Xs) ./ (3*Vt*Ef));
-->plot(Ef,delta*180/%pi)
```

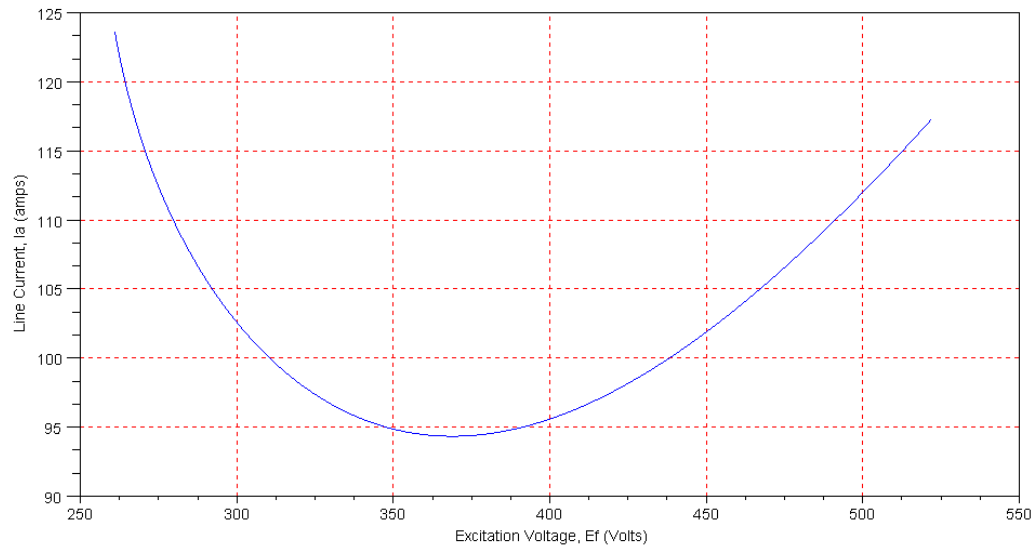


As you would expect, when you increase E_f , the rotor's magnetic field becomes stronger and the slip angle decreases.

The slip increases as you decrease E_f . Once the slip goes past 90 degrees, the motor no longer is able to generate the torque needed to provide 100hp. (note: This also means if you increase E_f , the slip decreases. The magnets get stronger.)

Plot the current vs. Excitation Voltage:

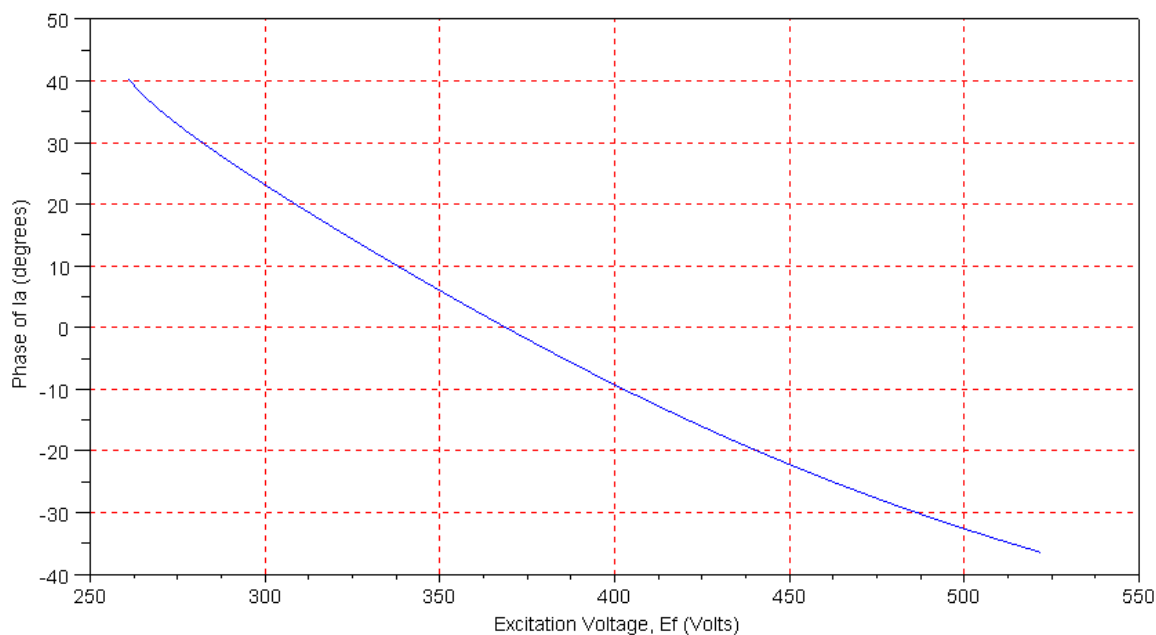
```
-->Ia = (265 - Ef.*exp(j*delta))/(j*2.72);  
-->plot(Ef,abs(Ia))  
-->plot(Ef,abs(Ia))  
-->xlabel('Excitation Voltage, Ef (Volts)')  
-->ylabel('Line Current, Ia (amps)')
```



Note there is an 'optimal' excitation voltage that minimizes the stator current.

Plot the angle if Ia (power factor) vs. Excitation Voltage:

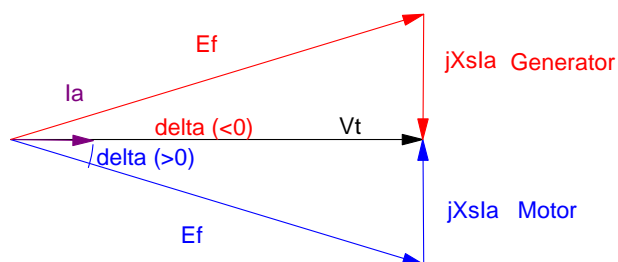
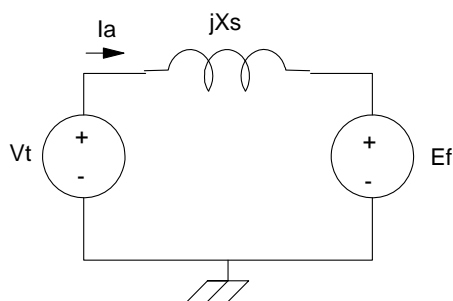
```
-->Q = atan(imag(-Ia), real(-Ia))*180/%pi;  
-->plot(Ef,Q)  
-->xgrid(5);  
-->xlabel('Excitation Voltage, Ef (Volts)')  
-->ylabel('Phase of Ia (degrees)')
```



Note:

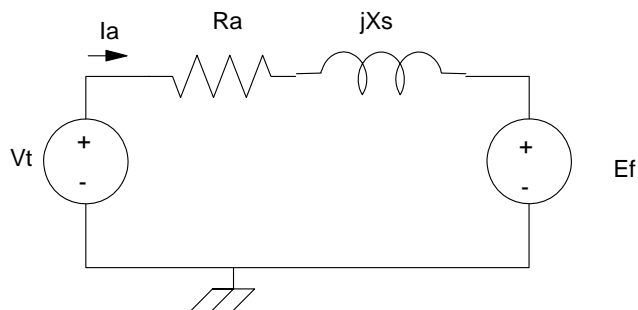
- For large excitation voltages, the generator produces current with a lagging power factor (inductive source).
- For small excitation voltages, the generator produced current with a leading power factor (capacitive source).
- You can generate current with a power factor 1.00. This happens at $E_f = 368\text{V}$ (found numerically)
- As the load changes, you need to keep adjusting the excitation voltage, E_f , to keep the power factor equal to 1.00.

Note that what you're doing is finding the excitation voltage (shown below) which is slightly larger than the load voltage and above it in a phasor diagram. This results in the voltage $jX_s \cdot I_a$ having a phase of -90 degrees, resulting in I_a being real (meaning a power factor of 1.00) and negative (meaning you're providing power to the load, V_t , and acting like a generator).



Determination of Synchronous Machine Parameters

You really only have two unknowns for an AC synchronous generator / motor: R_a and X_s .



You can pretty much take any two measurements that give you two equations for two unknowns. In the lab, the procedure is:

DC Test: (Lab: Step 1 & 2)

- Disconnect power
- Apply a small DC voltage across one phase and measure the current.
- The resistance is approximately 1.2 times V/I to account for skin effects.
- This tells you R_a (the real part of Z_s)

Short Circuit Test: (Lab step 3)

- Short the three stator leads
- Externally drive the motor at rated speed (or a little lower. In the lab you spin the motor at 1200rpm, creating a 40Hz sine wave at the output.)
- This measures the magnitude of Z_s .

Voltage Regulation:

Recall

$$V_t = E_f + jX_s \cdot I_a$$

$$P = 3V_t I_a \cos(\theta) = 3 \frac{V_t E_f}{X_s} \sin(\delta)$$

Assume you have a generator with a load of Z_L . Then:

$$V_t = -I_a Z_L$$

(The minus sign comes from using the same equations for a synchronous generator.) Substituting for I_a :

$$V_t = E_f + jX_s \cdot \left(\frac{-V_t}{Z_L} \right)$$

$$E_f = \left(1 + \frac{jX_s}{Z_L} \right) V_t$$

By adjusting the excitation voltage, you can control the voltage at the load. This is what you're doing in lab when you 'adjust I3 (E_f) such that $E1 (V_t)$ is 208V'.

Note that if you do not change E_f , the load voltage, V_t , changes by voltage division:

$$V_t = \left(\frac{Z_L}{Z_L + jX_s} \right) E_f$$

The term 'percent regulation' refers to the drop in voltage under load

$$\% \text{ Regulation} = \left(\frac{V_{\text{no load}} - V_{\text{load}}}{V_{\text{load}}} \right) \cdot 100\%$$