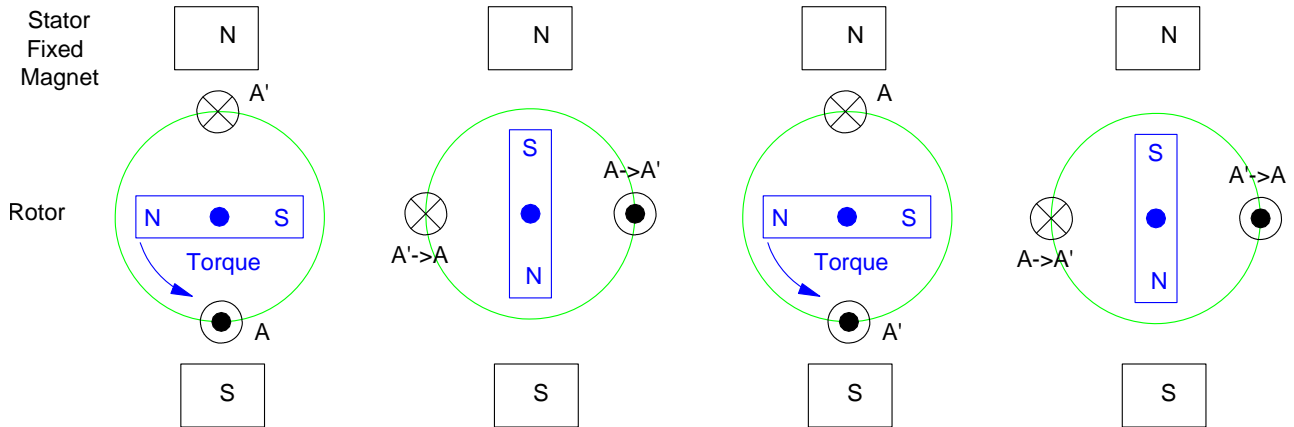


DC Shunt Excited Motor

A DC motor has

- A constant (DC) magnetic field for the stator, and
- A constant (DC) magnetic field in the rotor,
- That switches as the motor rotates.

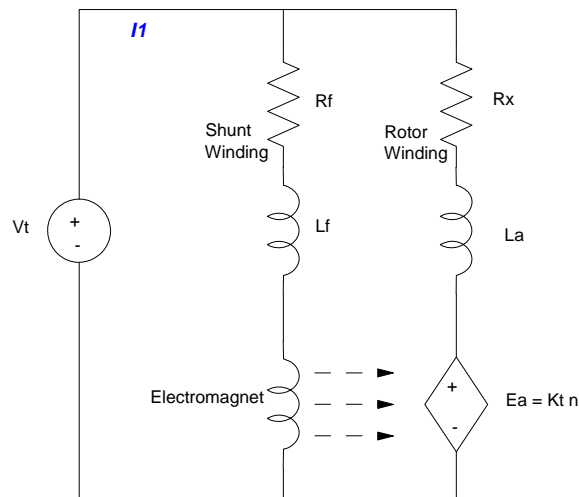
This switching results in a constant torque in one direction.



For small motors, the stator is made from permanent magnets. This is useful since it takes no energy to maintain a magnetic field in a permanent magnet. For larger motors, you can get stronger magnets using electromagnets, resulting in higher torques.

DC Shunt Excited Motor:

- Replace the permanent magnet in the stator with an electromagnet.
- Energize the stator's electromagnet with the power supply, powered in parallel with the rotor.



If you're worried about the dynamic behaviour of the DC motor (as you are in ECE 461 Controls Systems), include the inductance. If you're worried about steady-state behavior, ignore the inductors since we're using DC signals.

This results in

$$V_t = I_a R_x + E_a$$

and the back EMF:

$$E_a = K_t \omega$$

where ω is the speed of the motor in rad/sec and K_t is the torque constant. A motor also produces torque:

$$T = K_t I_a$$

It isn't obvious that the torque constants are the same (in mks units that is)

$$\frac{\text{Volt}\cdot\text{sec}}{\text{rad}} = \frac{\text{Nm}\cdot\text{s}}{\text{q}} = \frac{\text{Nm}}{\text{q/s}} = \frac{\text{Nm}}{\text{Amp}}$$

hence:

$$K_t = \frac{\text{Volt}}{\text{rad/sec}} = \frac{\text{Nm}}{\text{Amp}}$$

The torque constant can be found using the Faraday-Lenz relationship (grrrr! The text keeps alternating between n and ω for speed - I'll use ω here meaning the units are rad/sec for speed.)

$$E_a = \frac{2\omega N_a \Phi_P}{\pi} = \left(\frac{2N_a \Phi_P}{\pi} \right) \omega = K_t \omega \quad (10-5)$$

$$K_t = \left(\frac{2N_a \Phi_P}{\pi} \right)$$

where N_a is the number of windings and Φ_P is the pole flux in Wb.

Note that the pole flux is proportional to the field current (the current going through the shunt windings to the rotor's electromagnet.)

$$\Phi_P = \frac{N_f I_f}{\text{Rel}}$$

(Rel = reluctance of the shunt magnet). If you keep the field current constant, Φ_P is constant as is K_t .

Example: Assume a DC motor with $V_t = 120\text{VDC}$, $R_f = 120\text{ Ohms}$, $R_x = 1\text{ Ohm}$, $N_f = 20$, $N_a = 20$, and a reluctance of 800. Plot the speed vs. load torque relationship.

Solution: First, find the torque constant

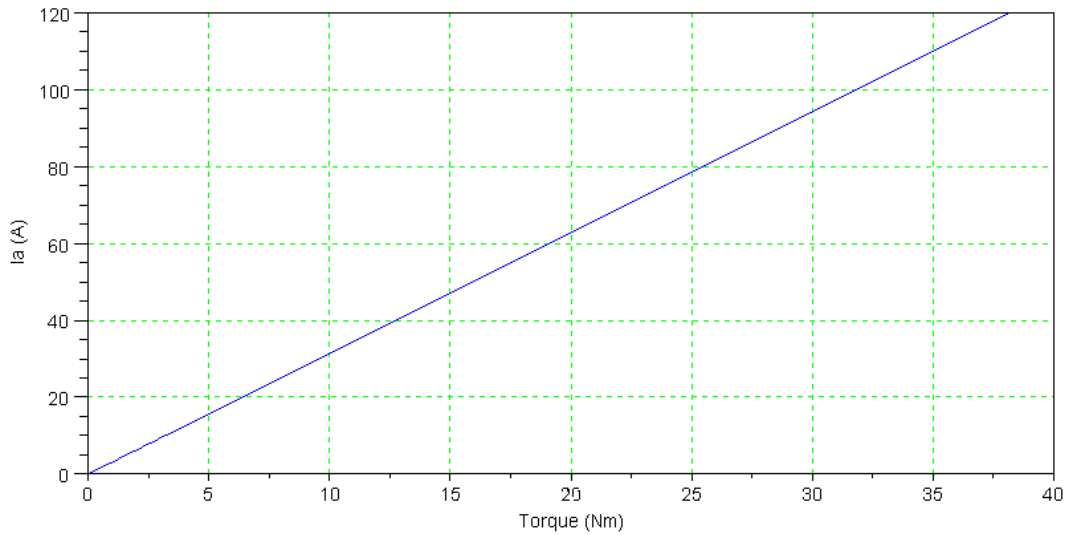
$$\Phi_P = \frac{N_f I_f}{\text{Rel}} = \frac{(20)(1\text{A})}{800} = 0.025\text{Wb}$$

$$K_t = \left(\frac{2N_a \Phi_P}{\pi} \right) = \left(\frac{2(20)(0.025\text{Wb})}{\pi} \right) = 0.3183 \frac{\text{Volt}}{\text{rad/sec}}$$

The torque vs. current relationship is simply:

$$T = K_t I_a$$

Double the torque requires you to double the current.



The speed vs. voltage relationship is

$$V_t = I_a R_x + E_a$$

The torque (and power) must also balance:

$$T = K_t I_a$$

Substituting for I_a :

$$V_t = \left(\frac{T}{K_t}\right) R_x + K_t \omega$$

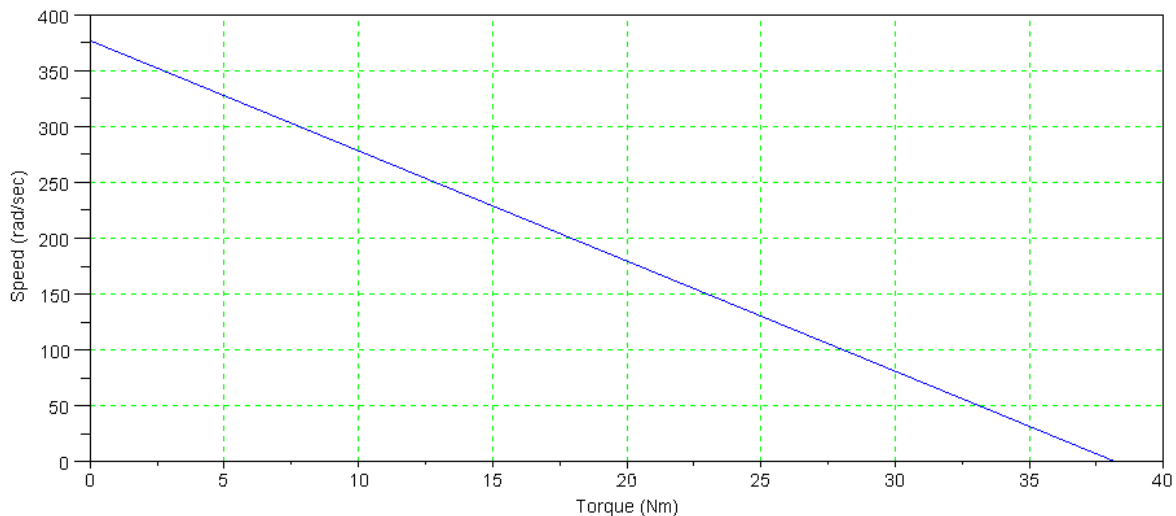
$$\omega = \left(\frac{V_t}{K_t}\right) - \left(\frac{R_x}{K_t^2}\right) T$$

Plugging in numbers:

$$\omega = 377 - 9.87T \quad \text{rad/sec}$$

or in rpm

$$\omega = 3600 - 94.2T \quad \text{rpm}$$



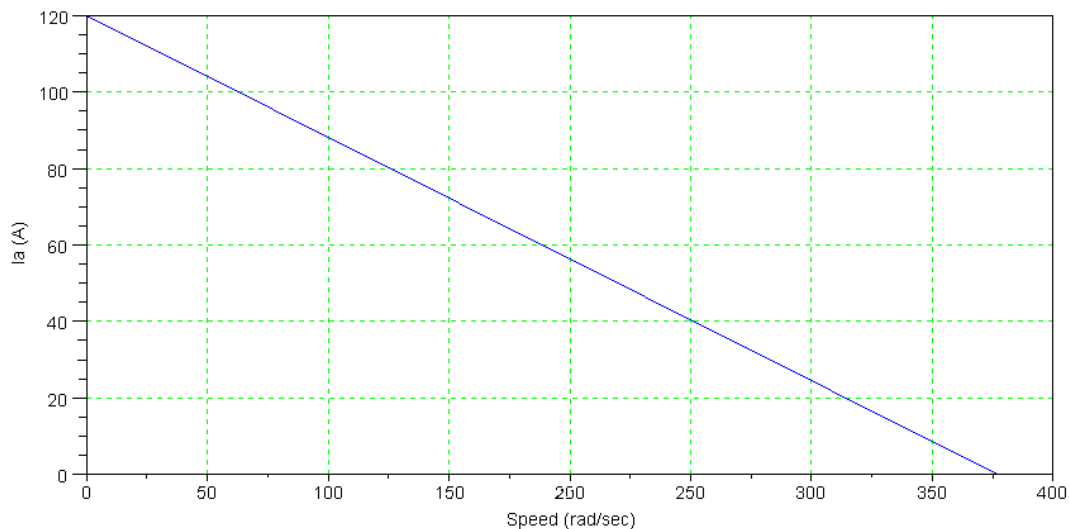
Note that the current also varies with speed:

$$V_t = I_a R_x + K_t \omega$$

$$I_a = \frac{1}{R_x} (V_t - K_t \omega)$$

Plugging in numbers:

$$I_a = 120 - 0.3183\omega \text{ Amps}$$



Note that when starting the motor, the current will be very large (termed in-rush current.) Then the motor speeds up, the current drops. At equilibrium, power in equals power out.

$$P = T\omega = E_a I_a$$

This large in-rush current means you sometimes need a starting procedure for large motors.

- Add a resistor in series to increase R_x and limit the in-rush current, I_a .
- Let the motor get up to speed
- Start removing the series resistors, decreasing R_x
- Let the motor get up to speed
- Eventually remove all added resistors, decreasing R_x to it's nominal value

Dynamic Modeling of Motor Acceleration:

Using MATLAB (or VisSim, SciLab), you can compute the speed of a motor vs. time as it accelerates. The trick is to compute the net torque on the motor.

$$J \frac{d\omega}{dt} = T_{net}$$

From Torque = Inertia * Acceleration, you can compute the speed by integrating:

$$\omega = \int_0^t \left(\frac{d\omega}{dt} \right) dt = \int_0^t \left(\frac{T_{net}}{J} \right) dt$$

For example, assume you have a DC motor with

- $K_t = 0.3183 \frac{Nm}{A}$
- $J = 2.0 \text{ kg m}^2$
- $R_x = 1 \text{ Ohm}$.

Plot the speed vs. time for this motor when connected to 120VDC.

First, compute the torque on the motor:

$$I_a = \left(\frac{V_t - E_a}{R_x} \right) = \left(\frac{120 - K_t \omega}{1} \right)$$

The torque is then

$$T = K_t I_a$$

$$T = 0.3183(120 - 0.3183\omega)$$

The acceleration is

$$\frac{d\omega}{dt} = \frac{1}{J} \cdot T = 31.83(120 - 0.3183\omega)$$

In MATLAB, solving every 0.001 second gives:

```
Vt = 120;
Rx = 1;
Kt = 0.3183;
J = 0.02;

t = 0;
w = 0;
```

```
-->Ia = (Vt - Kt*w) / Rx
      120.

-->T = Kt*Ia
      38.196

-->dw = T / J
      1909.8

-->w = w + dw*dt
      1.9098
```

Time is not at 0.001 second. Repeating:

```
-->Ia = (Vt - Kt*w) / Rx
      119.39211

-->T = Kt*Ia
      38.002509

-->dw = T / J
      1900.1254

-->w = w + dw*dt
      3.8099254
```

Repeat again for 2 seconds. This suggests you do this in a loop.

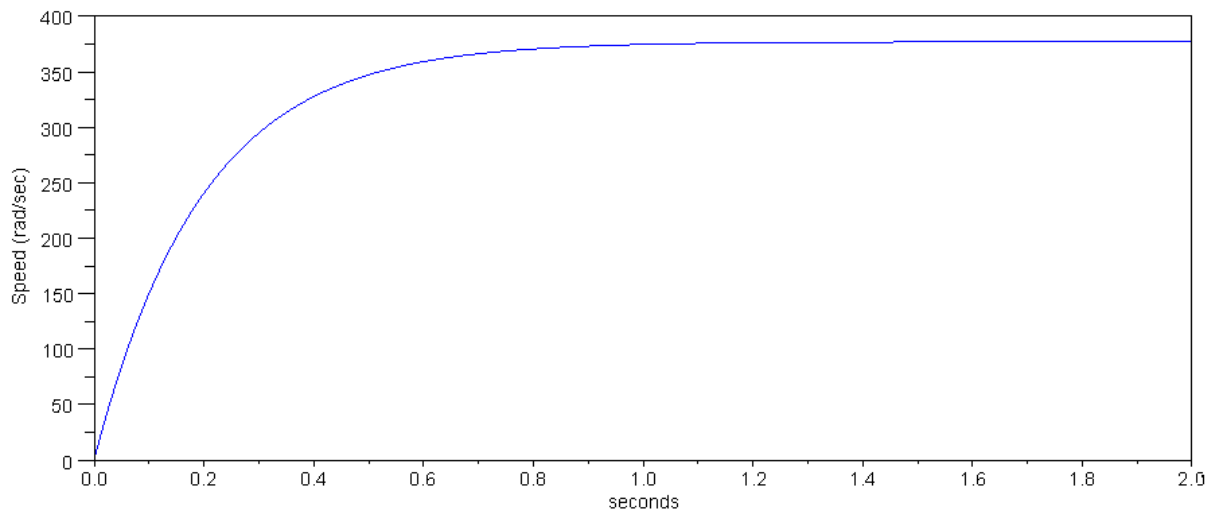
Set up arrays for t, Ia, T, w so you can plot them later:

```
Kt = 0.3183;
J = 0.02;
Rx = 1;

dt = 0.001;
t = [0:dt:2]';
Ia = 0*t;
T = 0*t;
w = 0*t;

for i=2:length(t)
    t(i) = t(i-1) + dt;
    Ia(i) = (Vt - Kt*w(i-1)) / Rx;
    T(i) = Kt*Ia(i);
    dw = T(i) / J;
    w(i) = w(i-1) + dw*dt;
end

plot(t,w)
```



Problem: Assume the rotational losses produce a torque of 4kW at 400 rad/sec. Compute the speed vs. time.

Solution: The net torque now needs to subtract the rotational losses. These are proportional to speed. At 400 rad/sec,

$$T = \frac{4000W}{400\text{rad/sec}} = 10Nm$$

$$T_{rot} \approx 0.025 \frac{Nm}{\text{rad/sec}}$$

are 2W at 350 rad/sec.

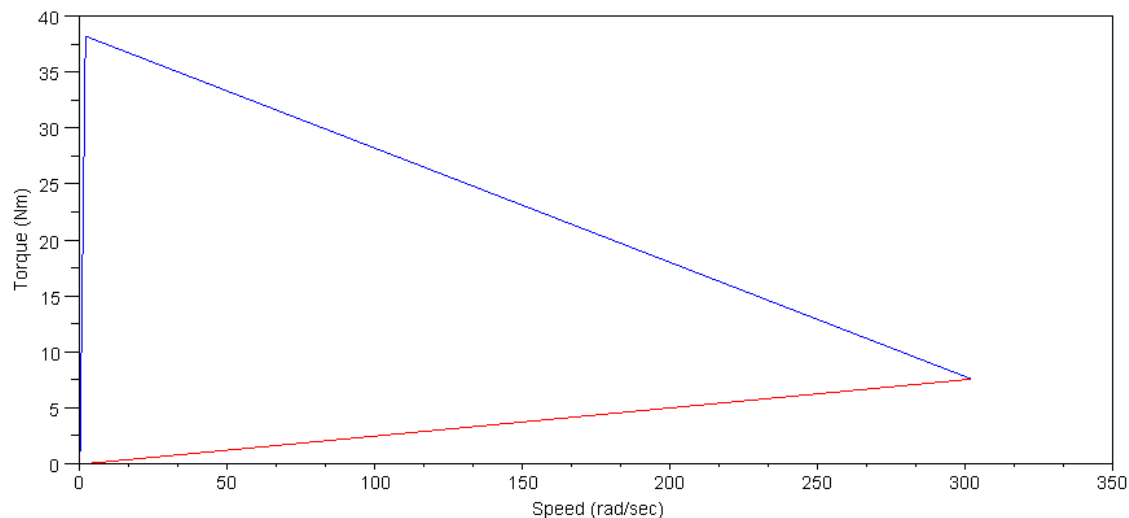
In MATLAB, compute the rotational losses at each time step and subtract them out from the net torque:

```
Kt = 0.3183;
J = 0.02;
Rx = 1;

dt = 0.001;
t = [0:dt:2]';
Ia = 0*t;
T = 0*t;
Trot = 0*t;
w = 0*t;

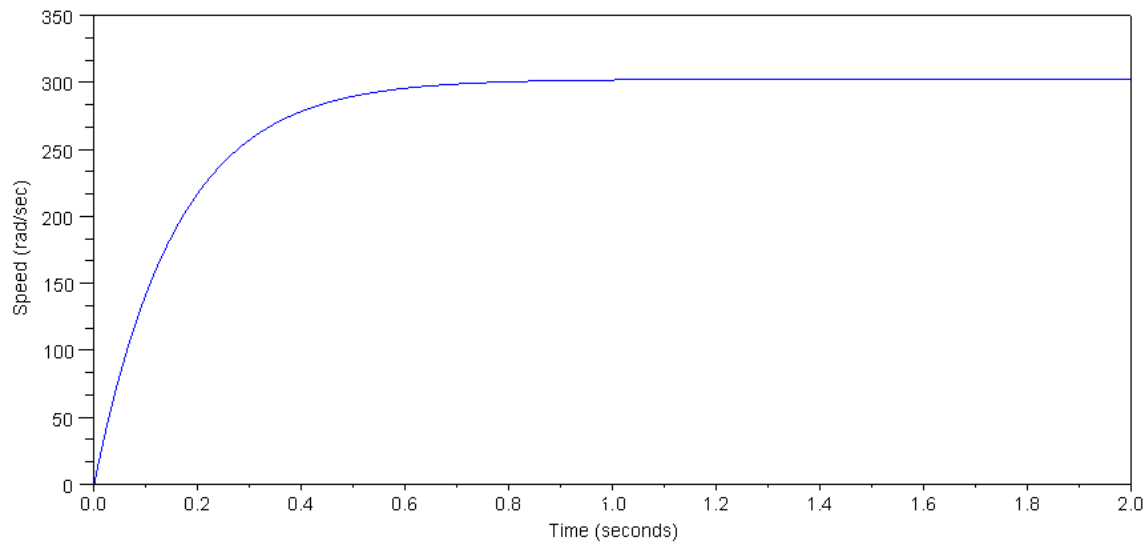
for i=2:length(t)
    t(i) = t(i-1) + dt;
    Ia(i) = (Vt - Kt*w(i-1)) / Rx;
    Trot(i) = 0.025*w(i-1);
    T(i) = Kt*Ia(i) - Trot(i);
    dw = T(i) / J;
    w(i) = w(i-1) + dw*dt;
end

plot(t,w)
```



The net torque is the difference between $K_t I_a$ (blue curve) and the rotational losses (red curve). When these meet, the net torque is zero and the motor is at steady-state

The resulting speed vs. time is then show below:



Note as a sidelight: with numerical integration you don't really care if the torque vs. speed is linear. All you care about is that you can compute the net torque at any given time point.

DC Motor Efficiency:

$$P_{out} = T\omega = E_a I_a$$

$$P_{in} = V_t I_1$$

In MATLAB:

```
-->Vt = 120;
-->Rf = 120;
-->Rx = 1;
-->Kt = 0.3183;

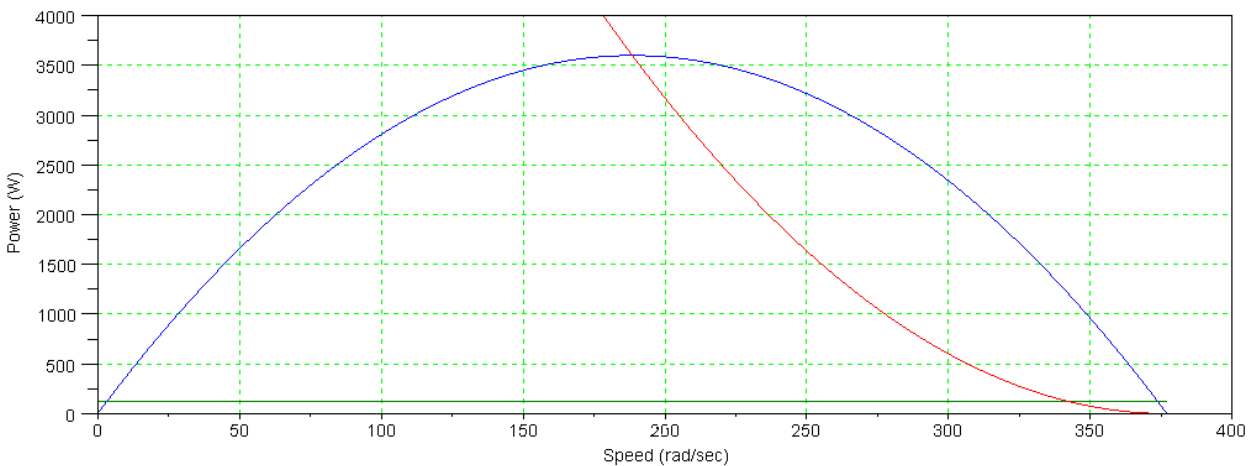
-->w = [0:0.01:1]' * Vt/Kt;
-->Ef = Kt*w;

-->Ia = (Vt - Ef)/Rx;
-->If = Vt / Rf;

-->T = Kt*Ia;
```

The output power and input power can be computed by $P = V \cdot I$

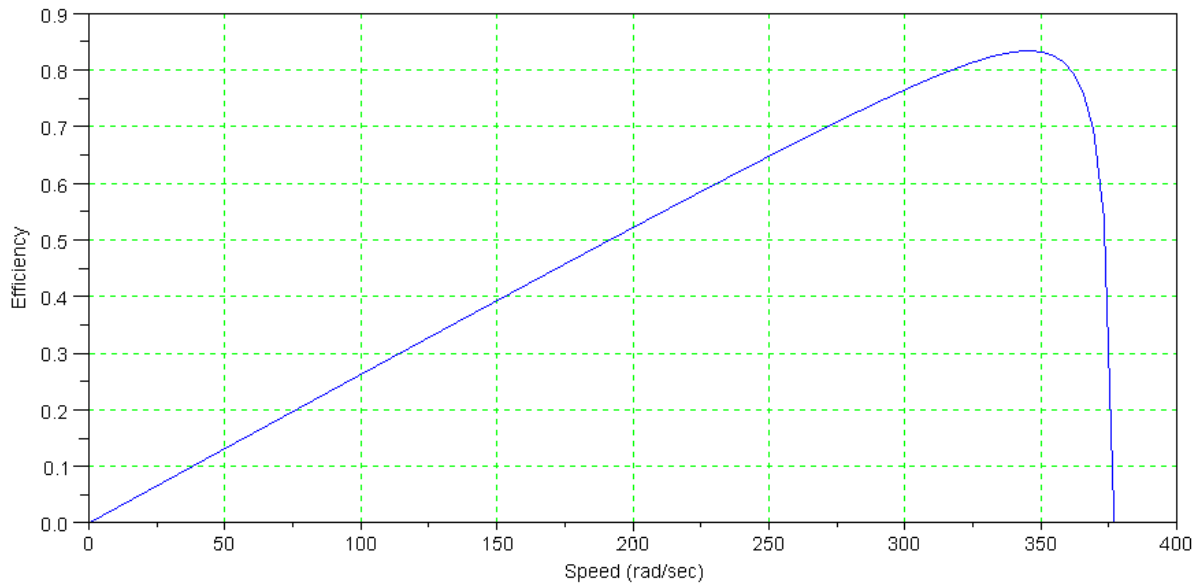
```
-->Pout = Ia .* Ef;
-->Pin = Vt * (Ia + If);
-->plot(w,Pout,w,(If.^2)*Rf,w,(Ia.^2)*Rx)
```



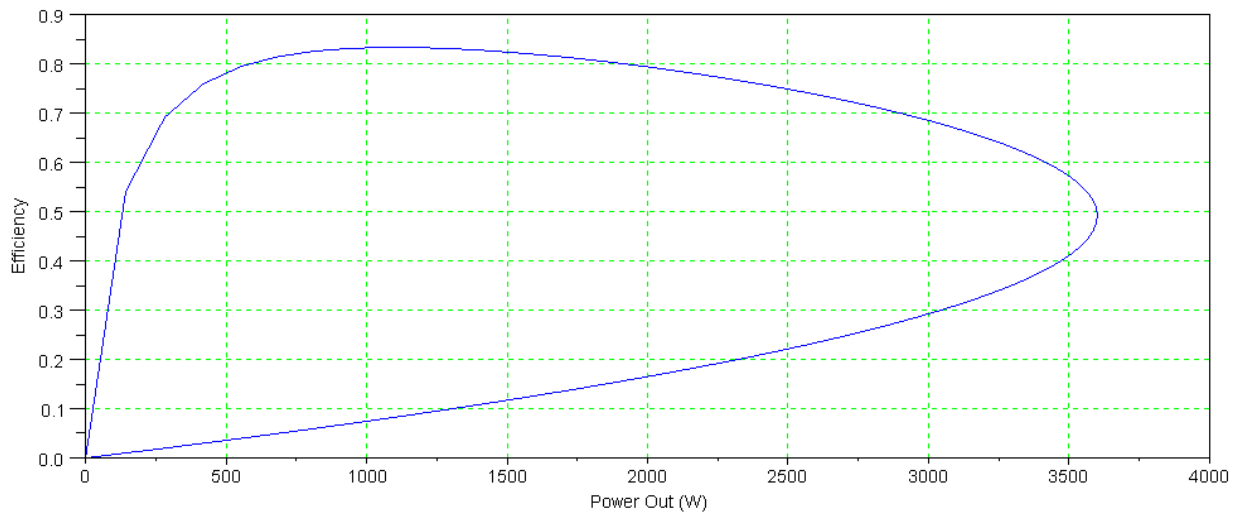
Output power (blue), Field Copper Losses (dark green), Rotor Copper Losses (red)

```
-->Eff = Pout / Pin;

-->plot(w,Eff)
-->xlabel('Speed');
-->ylabel('Efficiency')
-->xlabel('Speed (rad/sec)');
-->xgrid(3)
```



Efficiency vs. Speed



Efficiency vs. Output Power