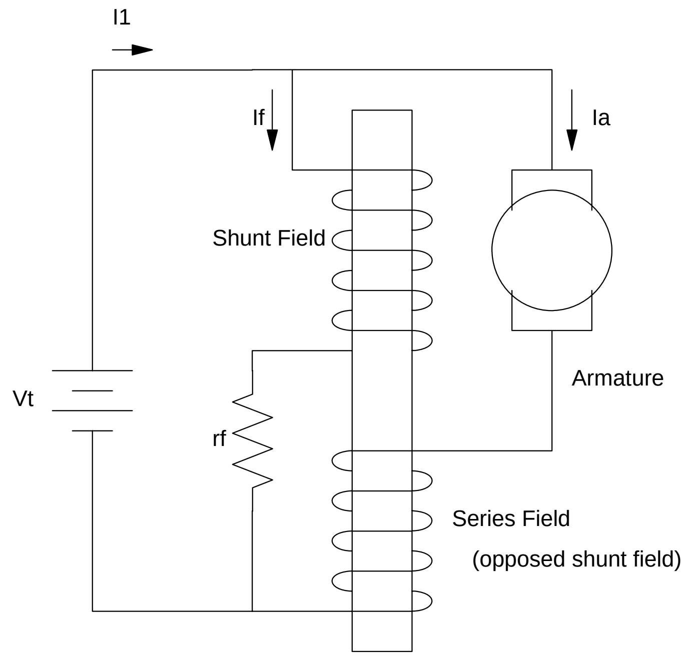
DC Series-Shunt Windings

Series-shunt motors are constant-speed motors (almost). They're designed to keep the speed nearly constant as you increase the load. This implies that they need to draw more power as the load increases with positive feedback.

As you load a DC motor, it slows down. If you compensate by adjusting the field strength, changing Kt, you can keep the speed constant.

One way to measure the load is to monitor the armature current, Ia. As the load increases, Ia increases.



Assume you add a second electromagnet, powerd by Ia. Then:

$$\Phi_P = \frac{N_f I_f}{R_{rel:f}} \pm \frac{N_{f2} I_a}{R_{rel:a}}$$

(The $\pm$ comes from being able to have the second coil add or subtract from the field coil's magnetic field).

$$E_a = \frac{2\omega N_a \Phi_P}{\pi} = \left(\frac{2N_a}{\pi}\right)\left(\frac{N_f I_f}{R_{rel:f}} + \frac{N_a I_a}{R_{rel:a}}\right)\omega = K_t \omega$$

$$K_t = \left(\frac{2N_a}{\pi}\right)\left(\frac{N_f I_f}{R_{rel:1}}\right) \pm \left(\frac{2N_a}{\pi}\right)\left(\frac{N_{f2}}{R_{rel:2}}\right) I_a$$

$$K_t = K_{t0} + K_{t1} I_a$$

The speed vs. load relationship is then:

$$V_t = I_a R_x + E_a$$

$$T = K_t I_a$$

$$E_a = K_t \omega$$

$$K_t = K_{t0} + K_{t1} I_a$$

Substituting

$$T = (K_{t0} + K_{t1} I_a) I_a \qquad \text{determines Ia}$$

$$V_t = I_a R_x + (K_{t0} + K_{t1} I_a)\omega \qquad \text{determines speed}$$

Example: Find the no-load speed with Vt = 120VDC, Rf = 120 Ohms, Rx = 1 Ohm, Nf = 20, Na = 20, and a reluctance of 800. Kt1 = 0 and T = 10Nm.

$$K_{t0} = 0.3183 \frac{\text{Volt}}{rad/\sec}$$

$$\Phi_P = 0.025Wb$$

Solving for the field current:

$$T = 0Nm = (0.3183 + 0 \cdot I_a) I_a$$

$$I_a = 0A$$

Solving for speed:

$$V_t = I_a R_x + (K_{t0} + K_{t1} I_a)\omega$$

$$(120) = (0A)(1\Omega) + \left(0.3183 \frac{V}{\text{rad/sec}} + 0\right)\omega$$

$$\omega = 377 \frac{rad}{\sec}$$

Change the load to 10Nm with Kt1 = 0:

$$T = 10Nm = (0.3183 + 0 \cdot I_a) I_a$$

$$I_a = 31.4169A$$

The speed is then

$$(120) = (31.4169A)(1\Omega) + \left(0.3183 \frac{V}{\text{rad/sec}} + 0\right)\omega$$

$$\omega = 278.3 \frac{rad}{\sec}$$

$$\text{Speed Regulation} = \left(\frac{377-278}{377}\right) = 26.3\%$$

The motor slows down as you increase the load.

Now, let Kt1 = -0.0025

$$T = 10Nm = (0.3183 + -0.0025 I_a) I_a$$

$$I_a = 56.4077A$$

Solving for speed:

$$120V = (56.4077A)(1\Omega) + (0.3183 - 0.1410)\omega$$

$$\omega = 358.7 \text{ rad/sec}$$

$$\text{Speed Regulation} = \left(\frac{377-358}{377}\right) = 4.8\%$$

The series winding weakened the field, reducing the torque constant by 0.0862 (27%). This resulted in less back-EMF and greater speed. The net result is

- The greater the load, the greater the current draw
- This positive feedback keeps the speed nearly constant.

This is somewhat dangerous, however.

- If you add too little positive feedback, the speed drops as the load increases.
- If you increase the positive feedback, the speed gets closer to being constant.
- If you add just a little too much positive feedback, the motor accelerates, drawing more power, accelerating more, drawing more power, etc. Either the circuit breaker kicks in or the motor flies apart.

Torque vs. Speed Curves

Plot the torque vs. speed curves for

$$K_t = (0.3183 - \alpha I_a)\ \frac{Nm}{A}$$

as α varies

No-Load Speed: $I_a = 0$ so α doesn't matter.

$$\omega_{nl} = \frac{V_t}{K_t} = \frac{120V}{0.3183} = 377\frac{rad}{\sec}$$

T = 10Nm Speed: This does depend on α

a) $\alpha = 0$

$$T = K_t I_a$$

$$I_a = 31.41A$$

$$E_a = V_t - I_a R_x = 88.58V$$

$$E_a = K_t\omega$$

$$\omega = 278\frac{rad}{\sec}$$

(73.8% of no-load speed)

b) $\alpha = 0.001$

Here you have to iterate until the equations balance

$$K_t = 0.3183 - 0.001 I_a$$

Guess

$$I_a = 0$$

$$K_t = 0.3183$$

$$I_a = \frac{T}{K_t} = \frac{10Nm}{0.3183\frac{Nm}{A}} = 31.41A$$

Guess

$$I_a = 31.41A$$

$$K_t = 0.3183 - 0.001 I_a = 0.2869$$

$$I_a = \frac{T}{K_t} = \frac{10Nm}{0.2869\frac{Nm}{A}} = 34.85A$$

(time passes)

$$I_a = 35.34A$$

$$K_t = 0.3183 - 0.001 I_a = 0.2830$$

$$I_a = \frac{T}{K_t} = \frac{10Nm}{0.2830\frac{Nm}{A}} = 35.34A$$

Ia balances. Then

$$E_f = V_t - I_a R_x = 84.6592V$$

$$\omega = \frac{E_f}{K_t} = 299.19$$

Repeating for different values of alpha with T = 10Nm

(No-Load Speed = 377 rad/sec)

	alpha = 0	alpha = 0.001	alpha = 0.002	alpha = 0.0025
Ia	31.4169 A	35.3408 A	43.075 A	56.28A
Kt	0.3183	0.2830	0.2321	0.1777
Ef	88.58 V	84.65 V	76.92 V	63.71V
speed	278	299	331	358
% No-Load Speed	73.7%	79.3%	87.8%	94.9%

Operating Modes for DC Motors:

Constant Torque

$$T = K_t I_a$$

If Kt is fixed (i.e. a shunt-type motor) and Ia is fixed, the torque is fixed.

Use a constant current source if you want constant torque. The voltage you need to apply then varies with speed:

$$V_t = I_a R_a + E_f$$

$$V_t = I_a R_a + K_t \omega$$

Constant Speed:

The voltage you need to apply to maintain constant speed is proportional to the load torque

$$T = K_t I_a$$

$$V_t = E_f + I_a R_a$$

$$V_t = K_t \omega + R_a \left(\frac{T}{K_t}\right)$$

$$V_t = K_t \omega + \left(\frac{R_a}{K_t}\right) T$$

Constant Power

$$P = I_a E_f = T\omega$$

$$P = \left(\frac{V_t - E_f}{R_a}\right) E_f = T\omega$$

$$P = \left(\frac{V_t - E_f}{R_a}\right) (K_t \omega)$$

$$P = \left(\frac{K_t}{R_a}\right) (V_t - K_t \omega)\omega$$

$$V_t = \left(\left(\frac{R_a}{\omega K_t}\right) P + K_t \omega^2\right)$$

Efficiency

$$\eta = \frac{P_{out}}{P_{in}} = \frac{I_a E_f}{\left(I_a + I_f\right) V_t} \approx \frac{E_f}{V_t}$$