ECE 341 - Test #2

Continuous Probability

Open-Book, Open Notes. Calculators, Matlab, Tarot cards, Internet allowed. Just not other people.

Please sign if possible (i.e. you did not get help from someone else).

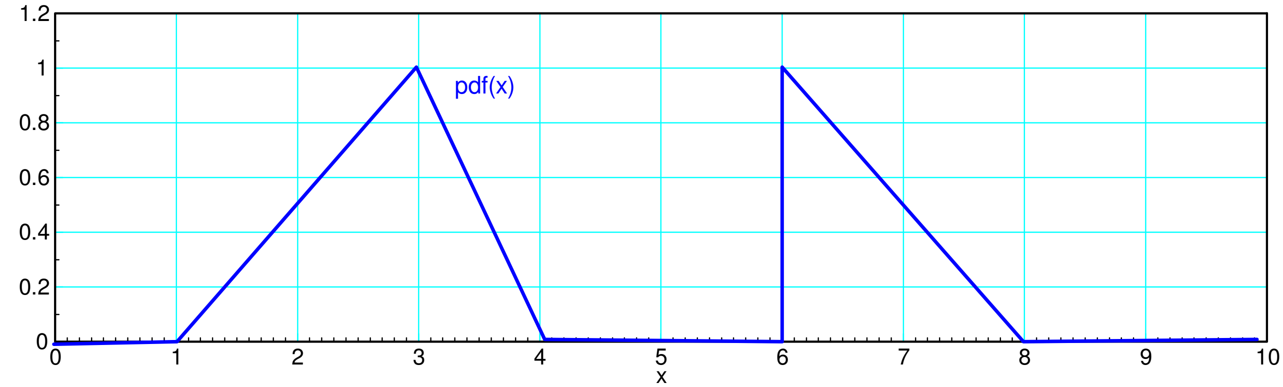
No aid given, received, or observed: ___________________________________

Due Sunday, June 7th, 8am

Please make the subject "ECE 341 Test2" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

1) Continuous PDF

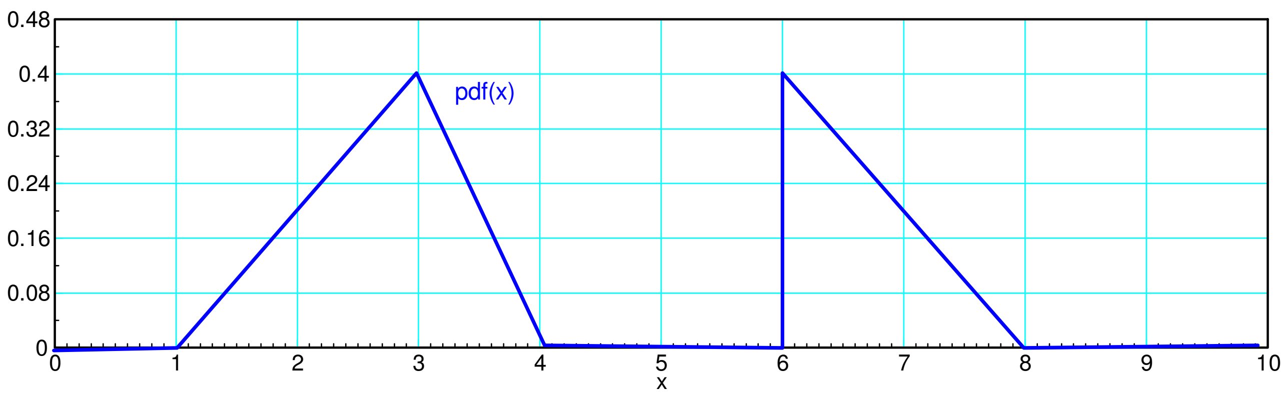
For the following probability density function



a) Determine the scalar to multiply this curve so that it is a valid pdf (i.e. the total area = 1.0000)

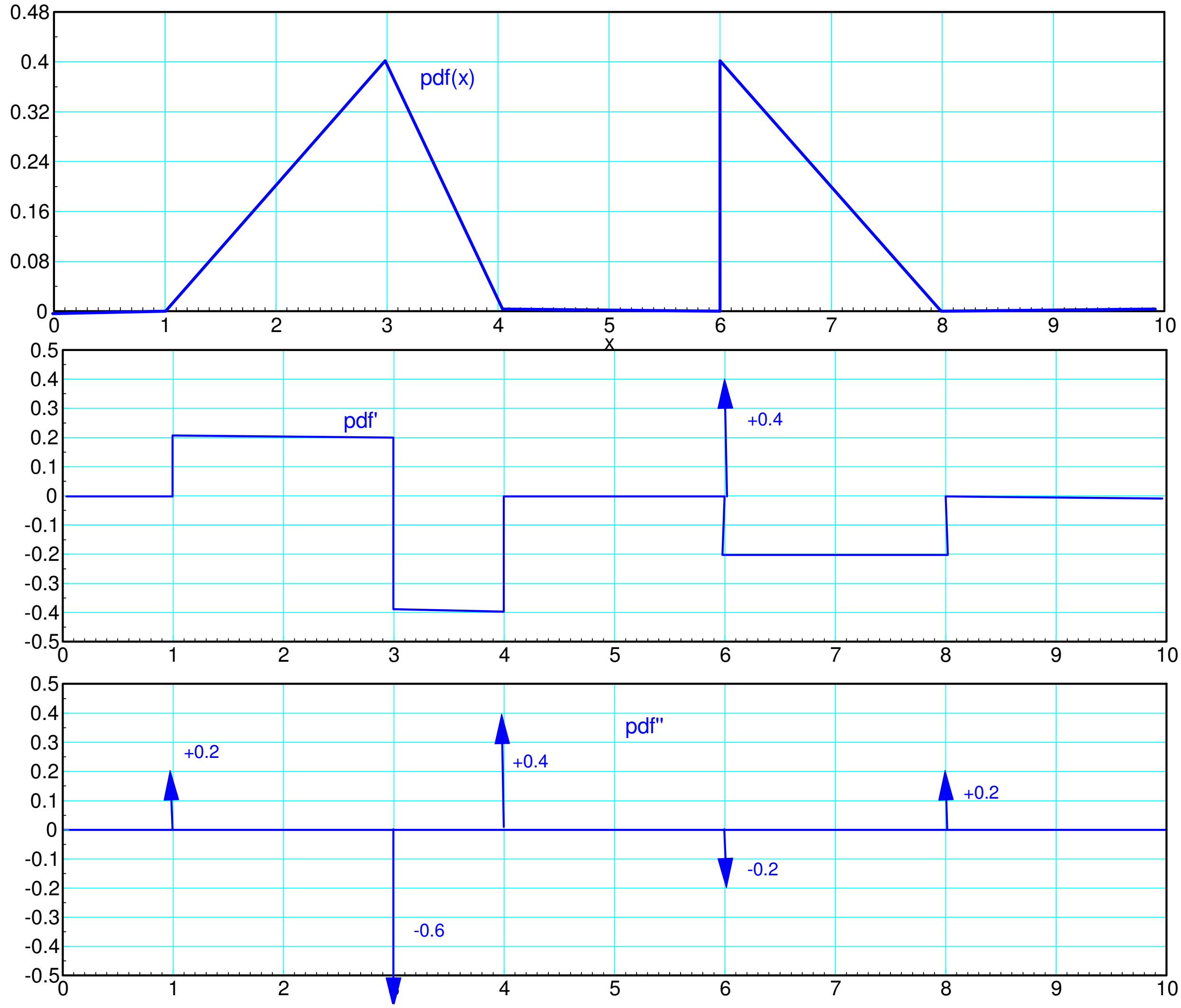
As drawn, the total area = 2.5

Scalar = 1/2.5 = 0.4



b) Determine the moment generating function (i.e. LaPlace transform)

$$\psi(s) = \left(\frac{1}{s}\right)(0.4e^{-6s}) + \left(\frac{1}{s^2}\right)(0.2e^{-s} - 0.6e^{-3s} + 0.4e^{-4s} - 0.2e^{-6s} + 0.2e^{-8s})$$

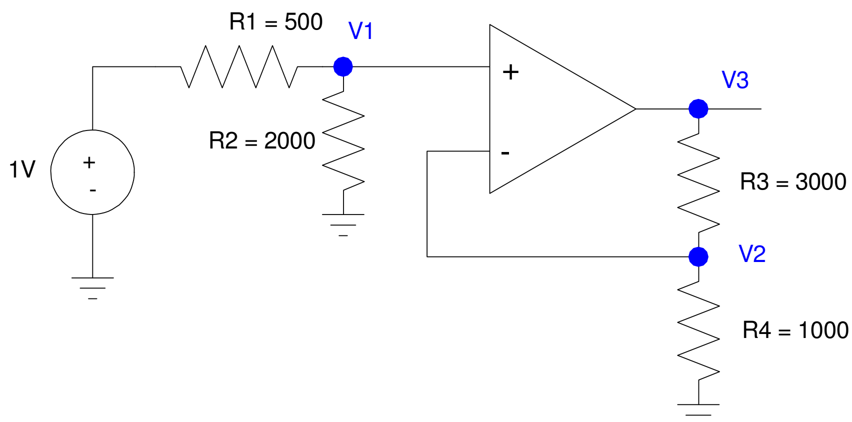


2) Uniform PDF

Assume each resistor has 5% tolerance with a uniform distribution:

$$R = (1 + 0.05x)R_0$$

where x is uniform(-1, 1).



a) Write the voltage node equations for this circuit in terms of {R1, R2, R3, R4}

$$\left(\frac{V_1-1}{R_1}\right) + \left(\frac{V_1}{R_2}\right) = 0$$

$$V_1 = V_2$$

$$\left(\frac{V_2-V_3}{R_3}\right) + \left(\frac{V_2}{R_4}\right) = 0$$

b) Run a Monte Carlo simulation to solve for V3 with 100 sets of R's (give a copy of your Matlab code)

```
V3 = [];
for i=1:1000

   R1 = 500*(1 + 0.05*(rand*2-1));
   R2 = 2000*(1 + 0.05*(rand*2-1));
   R3 = 3000*(1 + 0.05*(rand*2-1));
   R4 = 1000*(1 + 0.05*(rand*2-1));

   A = [1/R1+1/R2,0,0 ; 1,-1,0 ; 0,1/R3+1/R4,-1/R3];
   B = [1/R1;0;0];
   V = inv(A)*B

   V3 = [V3;V(3)];

end
```

c) Determine the mean and standard deviation of V3

```
mean(V3)
     mean  =    3.2019

srd(V3)
     stdev =    0.1011
```

3) Geometric & Gamma PDF

Let A, B, and C be continuous exponential distributions:

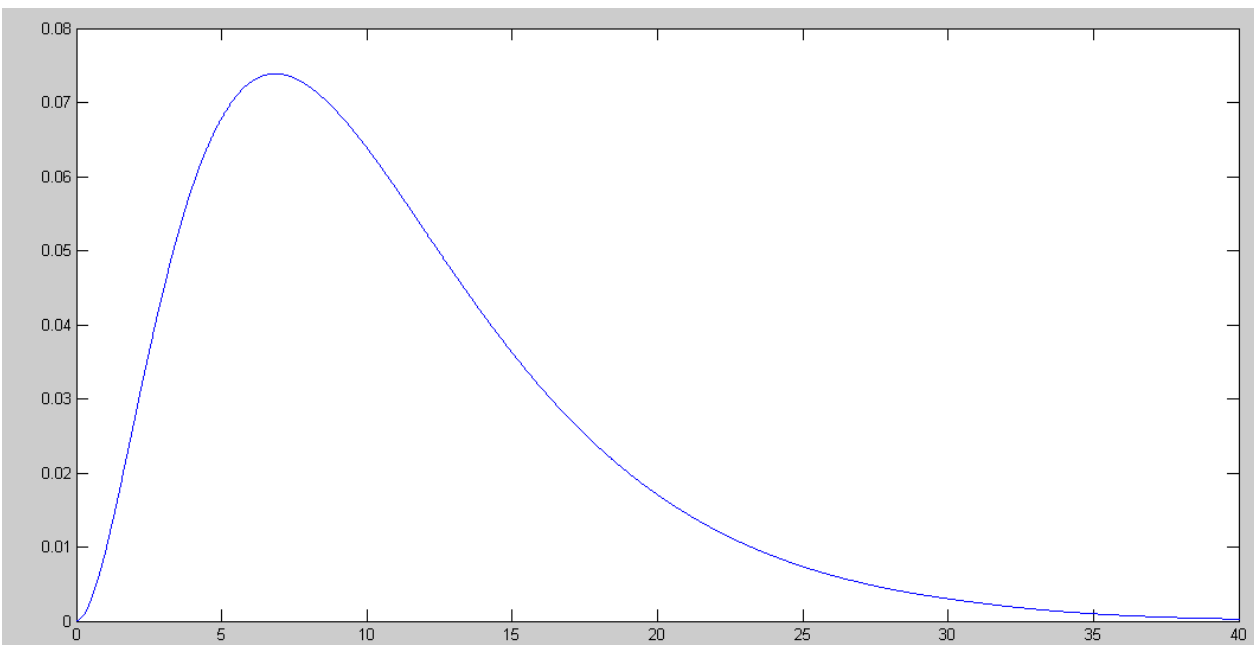
- A has a mean of 2
- B has a mean of 4, and
- C has a mean of 5

a) Determine the pdf of A + B + C using convolution

```
dt = 0.01;
t = [0:dt:30]';
A = 1/2 * exp(-t/2);
B = 1/4 * exp(-t/4);
C = 1/5 * exp(-t/5);

AB = conv(A,B) * dt;
ABC = conv(AB,C) * dt;

t1 = [0:length(ABC)-1]' * dt;
plot(t1,ABC)
```



b) Determine the pdf of Y = A + B + C using moment generating functions (LaPlace transforms)

$$A(s) = \left(\frac{0.5}{s+0.5}\right)$$

$$B(s) = \left(\frac{0.25}{s+0.25}\right)$$

$$C(s) = \left(\frac{0.2}{s+0.2}\right)$$

Then

$$Y(s) = \left(\frac{0.5}{s+0.5}\right)\left(\frac{0.25}{s+0.25}\right)\left(\frac{0.2}{s+0.2}\right)$$

Use partial fractions

$$Y(s) = \left(\frac{0.3333}{s+0.5}\right) + \left(\frac{-2}{s+0.25}\right) + \left(\frac{1.6667}{s+0.2}\right)$$

Take the inverse LaPlace transform

$$y(t) = (0.3333\ e^{-0.5t} - 2\ e^{-0.25t} + 1.667\ e^{-0.2t})u(t)$$

4) Central Limit Theorem

Let A, B, and C be continuous uniform distributions

- A = uniform over the interval of (1, 4)
- B = uniform over the interval of (1, 5)
- C = uniform over the interval of (1, 6)
- Y = A + B + C

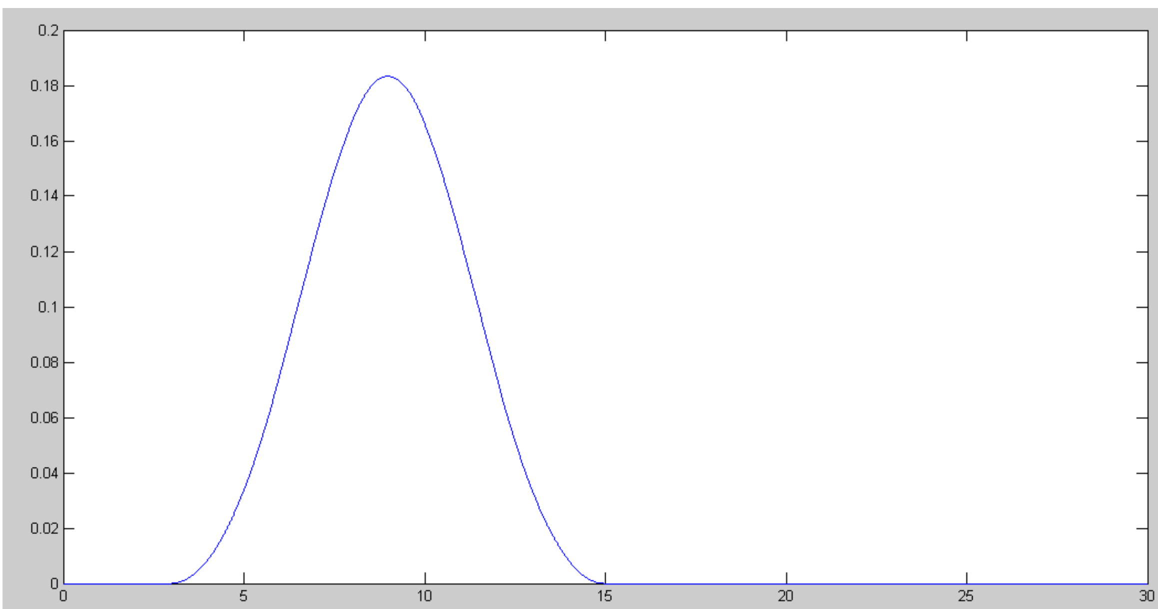
a) Using convolution, determine the pdf for Y

- Give your Matlab code and resulting plot of the pdf

```
dt = 0.01;
t = [0:dt:10]';
A = 1/3 * (t>=1) .* (t<4);
B = 1/4 * (t>=1) .* (t<5);
C = 1/5 * (t>=1) .* (t<6);

AB = conv(A,B) * dt;
Y = conv(AB,C) * dt;

t = [0:length(Y)-1]' * dt;
plot(t,Y)
```



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b) Determine the probability that Y > 12

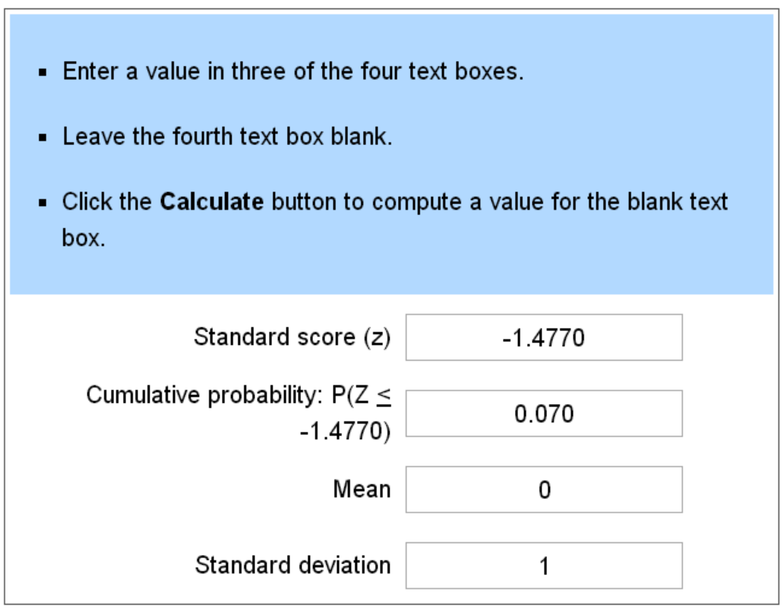
```
length(Y)

ans =       3001

sum(Y(1201:3001))*dt

ans =    0.0743
```

c) Use a normal approximation to Y to determine the z-score corresponding to Y=12 and the probability that Y > 12

```
x = sum(t .* Y) * dt

x =    8.9850

s2 = sum( (t - x).^2 .* Y) * dt

s2 =    4.1666

s = sqrt(s2)

s =    2.0412

z = (12 - x) / s

z =    1.4770
```

From StatTrek, a z-score of 1.4770 corresponds to a probability of 0.070

p = 7.0%



5) Testing with Normal PDF

Let A and B NPN transistors. The data sheets specify the minimum and maximum current gain

		min hfe (0.5%)	max hfe (99.5%)	mean	standard deviation
A	ZTX857	100	300	**200**	**38.82**
B	ZTX690B	150	500	**325**	**67.93**

Assume both transistors have a normal distribution and the min/max corresponds to the 99% confidence interval (each tail is 0.005 or 0.5%).

5a) What is the mean and standard deviation for both transistors with this assumption?

The mean is the midpoint between the endpoints

From StatTrek, a probability of 0.005 (0.5%) corresponds to a z-score of 2.576

100 = 200 - 2.576 s

s = 38.82

150 = 325 - 2.576 s

s = 67.93

5b) If you pick one transistor at random for each type, what is the probability that the ZTX690B transistor will have the higher gain (hfe)?

Let W = B - A

$$\mu_w = \mu_b - \mu_a = 125$$

$$\sigma_w^2 = \sigma_b^2 + \sigma_a^2 = 6122$$

$$\sigma_w = 78.24$$

The z-score to zero is

$$z = \left(\frac{\mu_w - 0}{\sigma_w}\right) = \left(\frac{125}{78.24}\right) = 1.5976$$

From StatTrek, this z-score corresponds to a probability of 0.945

Transistor B has a 94.5% chance of having a larger gain than transistor A