ECE 341 - Homework #2

Card Games. Due Thursday, May 20th

Please make the subject "ECE 341 HW#2" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

The card game *bridge* uses a 52-card deck. Each person is dealt 13 cards for their hand.

1) How many different hands are possible? (order doesn't matter)

$$N = \binom{52}{13} = \left(\frac{52!}{13!\cdot 39!}\right) = 635,013,559,600$$

2) What is the probability of having 8 cards of one suit in your hand?

(4 suits, pick one)(13 cards in that suit, pick 8)(39 other cards, pick 5)

$$M = \binom{4}{1}\binom{13}{8}\binom{39}{5} = 2,963,997,036$$

The probability of having 8 cards of the same suit are

$$p = \left(\frac{2,963,997,036}{635,013,559,600}\right) = 0.0008768$$

The odds are 1140.45 : 1 against having 8 cards of one suit

3) What is the probability of having honors? (you have four or five of the top 5 cards of any suit)

4 of top 5: (100 points)

(4 suits, pick 1)(5 top cards, pick 4)(of the remaining 47 cards, pick 9)

$$M_4 = \binom{4}{1}\binom{5}{4}\binom{47}{9} = 27,252,982,900$$

5 of top 5: (150 points)

(4 suits, pick 1)(5 top cards, pick 5)(of the remaining 47 cards, pick 8)

$$M_5 = \binom{4}{1}\binom{5}{5}\binom{47}{8} = 1,257,829,980$$

The odds of having honors is

$$p = \left(\frac{M_4+M_5}{\binom{52}{13}}\right) = 0.044898$$

The odds are 22.27 : 1 against

In 4-card poker, you are dealt just four cards

4) Compute the odds of a flush in 4-card poker.
   - 4 cards of the same suit

The total number of hands in 4-card poker are

$$N = \binom{52}{4} = 270,725$$

The number of hands that are a flush are

(4 suits, choose 1) * (13 cards, pick 4)

$$M = \binom{4}{1}\binom{13}{4} = 2,860$$

The odds of drawing a flush in 4-card poker are

$$p = \left(\frac{2,860}{270,725}\right) = 0.01056$$

There is a 1.056% chance of drawing a flush

The odds against drawing a flush are 94.65 : 1 against

5) Compute the odds of 3 of a kind in 4-card poker.
   - xxx y

The number of hands that are 3 of a kind are...

(13 cards, pick 1 for x) (4 cards of that value, pick 3)(48 other cards, pick 1)

$$M = \binom{13}{1}\binom{4}{3}\binom{48}{1} = 2,496$$

The probability of drawing 3 of a kind is

$$p = \left(\frac{2,496}{270,725}\right) = 0.00922$$

There is a 0.922% chance of drawing 3 of a kind

The odds are 108.46 : 1 against drawing a three of a kind

6) Determine the odds of drawing

- A flush and
- 3 of a kind

using Matlab and a Monte-Carlo simulation for 4-card poker

Code:

```
% ECE 341 Homework #2
% 4-Card Stud
% Flush and 3 of a kind
tic
Pair3 = 0;
Flush = 0;

for i0 = 1:1e5

   X = rand(1,52);
   [a,Deck] = sort(X);
   Hand = Deck(1:4);
   Value = mod(Hand,13) + 1;
   Suit = floor(Hand/13) + 1;

   N = zeros(1,13);
   for n=1:13
       N(n) = sum(Value == n);
   end

   [N,a] = sort(N, 'descend');

   S = zeros(1,4);
   for n=1:4
      S(n) = sum(Suit == n);
   end

   [S,a] = sort(S, 'descend');


   if (N(1) == 3) Pair3 = Pair3 + 1; end
   if (S(1) == 4) Flush = Flush + 1; end

end

[Pair3, Flush]
toc
```

Result: 100,000 hands of 4-card poker

```
ans =   901   958

Elapsed time is 13.670885 seconds.
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3 of a kind:: 0.901% chance

- 901 flushes in 100,000 hands

Flush: 0.958% chance

- 958 3-of-a-kind in 100,000 hands

Conditional Probabilities & 4-card poker

7) Compute the probability of getting a flush if there is a single draw step

- If you are dealt a flush, you draw zero cards
- If you are dealt 3 cards of a suit, you keep those cards and draw one
- If you are dealt 2 cards of a suit, you keep those cards and draw two more
- Otherwise, draw 4 new cards

Only flushes count for this problem.

p(Flush) = p(flush | drew 4) * p(drew 4) (case A)

+ p(flush | drew 3) * p(drew 3) (case B)

+ p(flush | drew 2) * p(drew 2) (case C)

+ p(flush | drew 1) * p(drew 1) (case D)

Case A:

p(flush | drew 4) = 1.000

p(drew 4) = 0.01056 from problem #4

Case B:

$$p(\text{flush} \mid \text{drew } 3) = \left(\frac{\text{(10 remaining cards of that suit, choose 1)}}{\text{(48 cards choose 1)}} \right)$$

$$= \frac{\binom{10}{1}}{\binom{48}{1}} = \left(\frac{10}{48} \right) = 0.20833$$

$$p(\text{draw } 3) = \left(\frac{\text{(4 suits choose 1)(13 cards of that suit, choose 3)(42 other cards choose 1)}}{\text{(52 cards choose 4)}} \right)$$

$$= \left(\frac{\binom{4}{1}\binom{13}{3}\binom{42}{1}}{\binom{52}{4}} \right) = \left(\frac{48{,}048}{270{,}725} \right) = 0.17748$$

p(flush | drew 3) * p(drew 3) = 0.20833 * 0.17748 = 0.03697

Case C:

$$\text{p(flush | drew 2)} = \left(\frac{\text{(11 cards of the suit choose 2)}}{\text{(48 cards choose 2)}}\right) = \left(\frac{\binom{11}{2}}{\binom{48}{2}}\right) = \left(\frac{55}{1128}\right) = 0.04876$$

The number of ways to draw 2 cards of the same suit is

xx yy + xx y z

xx yy = (4 suits choose 2)(13 cards choose 2)(13 cards choose 2)

$$= \binom{4}{2}\binom{13}{2}\binom{13}{2} = 36,504$$

xx y z = (4 suits choose 1 for x)(13 cards choose 2)(3 remaining suits choose 2)(13 cards choose 1 for y)(13 cards choose 1 for z)

$$= \binom{4}{1}\binom{13}{2}\binom{3}{2}\binom{13}{1}\binom{13}{1} = 158,184$$

The total number of ways to draw 2 of a suit is then

$$M = 36,504 + 158,184 = 194,688$$

The odds are then

$$p = \left(\frac{194.688}{270,725}\right) = 0.71914$$

p(flush | drew2) * (drew 2) = 0.04876 * 0.71914 = 0.03507

Case D:

$$\text{p(flush | drew 1)} = \left(\frac{\text{(4 suits pick 1)(12 cards pick 4)}}{\text{(48 cards pick 4)}}\right)$$

$$= \left(\frac{\binom{4}{1}\binom{12}{4}}{\binom{48}{4}}\right) = \left(\frac{1,980}{195,580}\right) = 0.01018$$

$$\text{p(drew 1)} = \left(\frac{\text{(13 clubs pick 1)(13 diamonds pick 1)(13 hearts pick 1)(13 spades pick 1)}}{\text{(52 choose 4)}}\right)$$

$$= \left(\frac{\binom{13}{1}\binom{13}{1}\binom{13}{1}\binom{13}{1}}{\binom{52}{4}}\right) = \left(\frac{28,561}{270,725}\right) = 0.10550$$

p(flush | drew 1) = 0.01018 * 0.10550 = 0.00107

The total odds of getting a flush are

p(flush) = Case A + Case B + Case C + Case D

= 0.01056 + 0.03697 + 0.03507 + 0.00107

= 0.08367

Or the odds are 11.95 : 1 odds against

The odds go up from 104 : 1 odds against to 11.95 : 1 odds agains if you're allowed to draw cards

8) Check your answer using a Monte Carlo simulation

With 1,000,000 hands, the number of flushes are

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Flush =      76288

Elapsed time is 77.767361 seconds.
>>
```

giving a 7.6288% chance of a flush (vs. 8.367% computed)

Code

```
% ECE 341 Homework #2
% 4-Card draw looking for a flush

tic
Flush = 0;

clc

for i0 = 1:1e6

   X = rand(1,52);
   [a,Deck] = sort(X);
   Hand = Deck(1:4);
   Value = mod(Hand,13) + 1;
   Suit = floor(Hand/13) + 1;

   N = Suit / 10;
   for i=1:4
       for j=1:4
           if(Suit(i) == Suit(j))
               N(i) = N(i) + 1;
           end
       end
   end

   [a,b] = sort(N, 'descend');
   Hand = Hand(b);
   Value = Value(b);
   Suit = Suit(b);

   N = N(b);
   N = floor(N);

   % draw cards

   if(N(1) == 4) end
   if(N(1) == 3) Hand(4) = Deck(5); end
   if(N(1) == 2) Hand(3:4) = Deck(5:6); end
   if(N(1) == 1) Hand(1:4) = Deck(5:8);  end

   Value = mod(Hand,13) + 1;
   Suit = floor(Hand/13) + 1;

   N = Suit / 10;
   for i=1:4
       for j=1:4
           if(Suit(i) == Suit(j))
               N(i) = N(i) + 1;
           end
       end
   end
   N = floor(N);

   if (N(1) == 4) Flush = Flush + 1; end

end

[Flush]
toc
```