ECE 341 - Homework #13

t-Tests. Due Wednesday, June 10th

Please make the subject "ECE 341 HW#13" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

Test of a Single Population: 4-card poker

1) Calculate the odds of getting 3-of-a-kind in 4-card stud poker.

- You are dealt (xxx y) where x and y are different values (Ace through King)
  (13 choose 1 for x)(4 x's choose 3)(48 other cards choose 1 for y)

$$M = \binom{13}{1}\binom{4}{3}\binom{48}{1} = 2496$$

$$N = \binom{52}{4} = 270,725$$

$$p = \left(\frac{2496}{270,725}\right) = 0.00922$$

2) Run a Monte Carlo simulation to determine the odds of getting 3-of-a-kind in 4-card stud poker:

- Each simulation deals 10,000 hands of poker
- Run the simulation 5 times

```
Pair3 =   104
Pair3 =    86
Pair3 =    86
Pair3 =    89
Pair3 =    91
```

3) From the results of problem #2, use a t-Test to determine the 90% confidence interval for the odds of getting 3-of-a-kind in 4-card stud poker.

Option 1: Infividual. What is the 90% confidence interval for the *next* time I run this Monte-Carlo simulation?

```
>> DATA = [104,86,86,89,91]';

x = mean(DATA)

x =   91.2000

>> s = std(DATA)

s =    7.4632

>> x + 2.132*s

ans =  107.1116

>> x - 2.132*s

ans =   75.2884
```

I'm 90% certain that the number of 3-of-a-kind hands will be in the range of (75.2 ... 107.1) hands out of 100,000

Option 2: Population. What is the 90% confidence interval for the *actual (population's)* mean?

In this case, the more data you have, the more you know: divide the standard deviation by the square root of the sample size.

```
>> x + 2.132*s/sqrt(5)

ans =   98.3159

>> x - 2.132*s/sqrt(5)

ans =   84.0841
```

I'm 90% certain the the population's average (i.e. the actual probability of drawing 3-of-a-kind) is in the range of (84.08 ... 98.32) for 100,000 hands.

From problem #1, the actual average is 92.2 hands out of 100,000 - which is in this range.

Test of a Single Population: Reflex Time

4) Go to the web site

-

Play the game and record your reaction times (5 measurements)

Reaction Times:

{0.329, 0.260, 0.257, 0.377, 0.274}

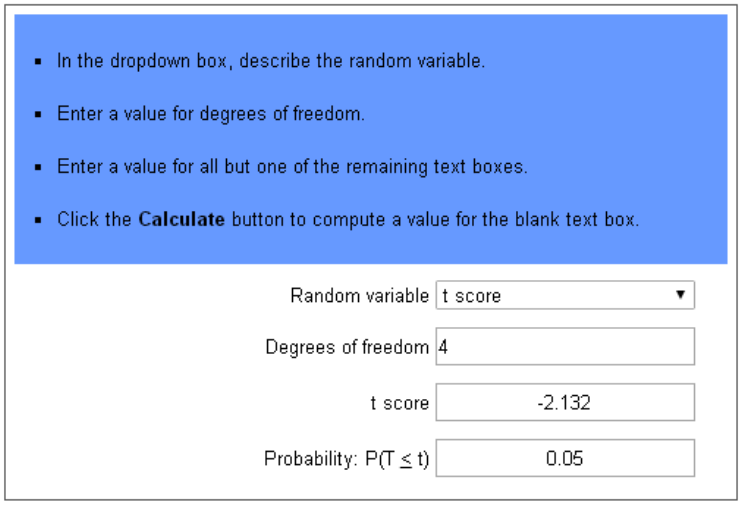
5) From the data from problem #4, determine the 90% confidence interval for your reaction time.

```
>> Data = [0.329, 0.260, 0.257, 0.377, 0.274];
>> x = mean(Data)

x =    0.2994

>> s = std(Data)

s =    0.0522
```

From StatTrek the t-score for 5% tails and 4 degrees of freedom (5 data points) is 2.132

```
>> x - 2.132*s

ans =    0.1881

>> x + 2.132*s

ans =    0.4107
```

I'm 90% certain that my reaction time will fall in the range of (0.1881, 0.4107) seconds



6) From the data from problem #4, determine the odds that your next reaction time will be less than 200ms.

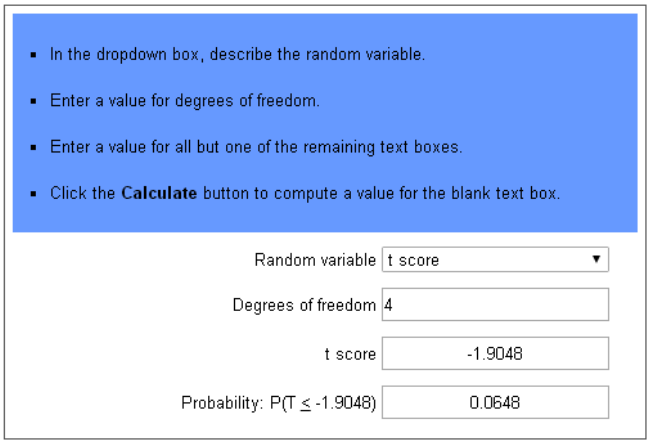
The z-score is

$$t = \left(\frac{\bar{x} - 200ms}{s}\right) = \left(\frac{0.2994 - 0.2}{0.0522}\right)$$

```
>> t = (x - 0.2)/s

   t =    1.9048
```

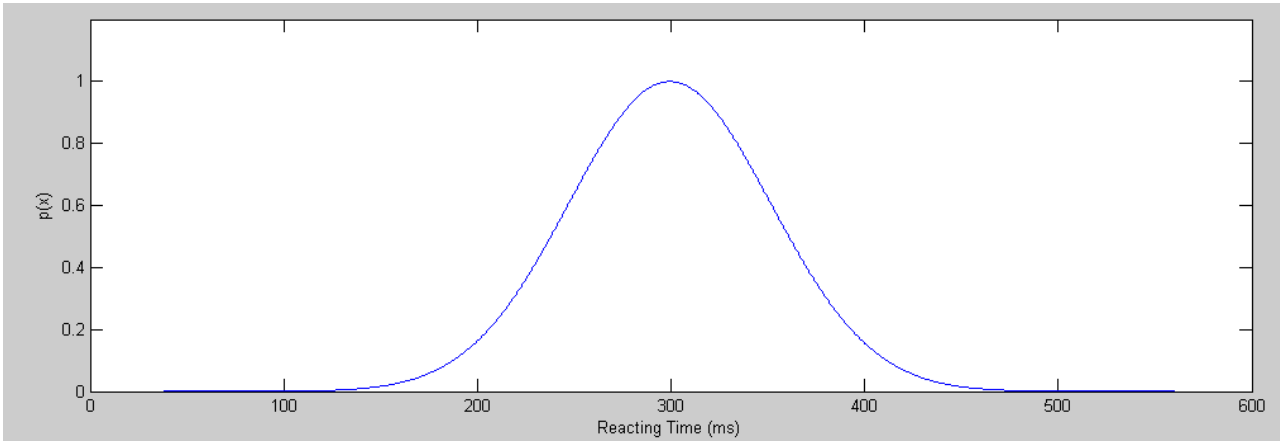
From StatTrek, this corresponds to a probability of 6.48%

There is a 6.48% chance my next reaction time will be less than 200ms



Just for fun, the pdf for my reaction time is approximately:

```
s1 = [-5:0.01:5]';
p = exp( - (s1 .^ 2)/2 );
plot(1000*(s1*s + x), p);
xlabel('Reacting Time (ms)');
ylabel('p(x)');
```

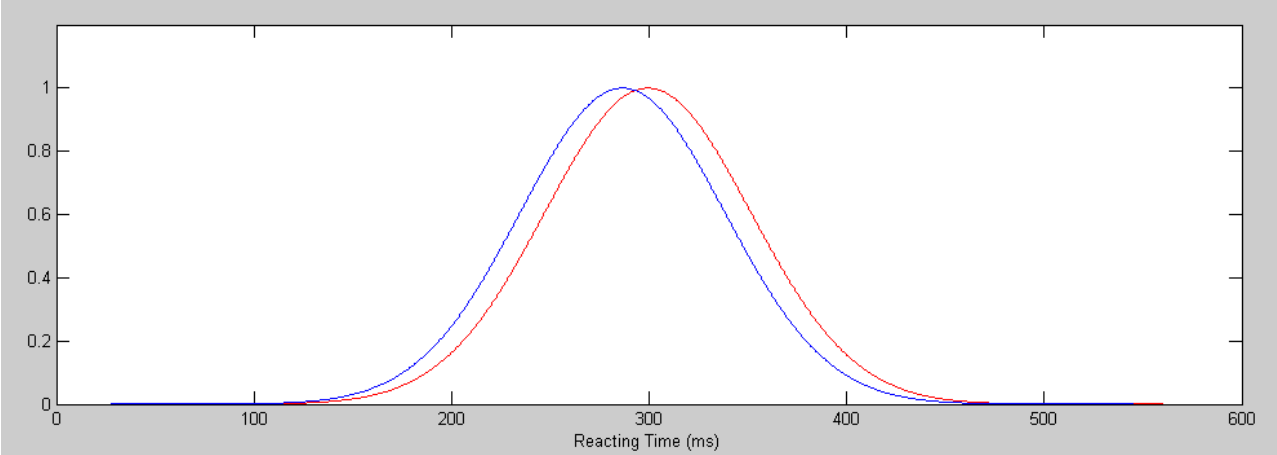


Comparison of Means Test:

The reaction time of Glower was

B = { 0.378, 0.258, 0.267, 0.276, 0.254 }

```
>> A = [.329, 0.260, 0.257, 0.377, 0.274];
>> B = [0.378, 0.258, 0.267, 0.276, 0.254];
>>
>> xa = mean(A)

xa =    0.2994

>> sa = std(A)

sa =    0.0522

>> xb = mean(B)

xb =    0.2866

>> sb = std(B)

sb =    0.0518

>> plot(1000*(s1*sa + xa), p, 'r', 1000*(s1*sb + xb), p, 'b');
>> xlabel('Reacting Time (ms)');
>> ylim([0,1.2])
```



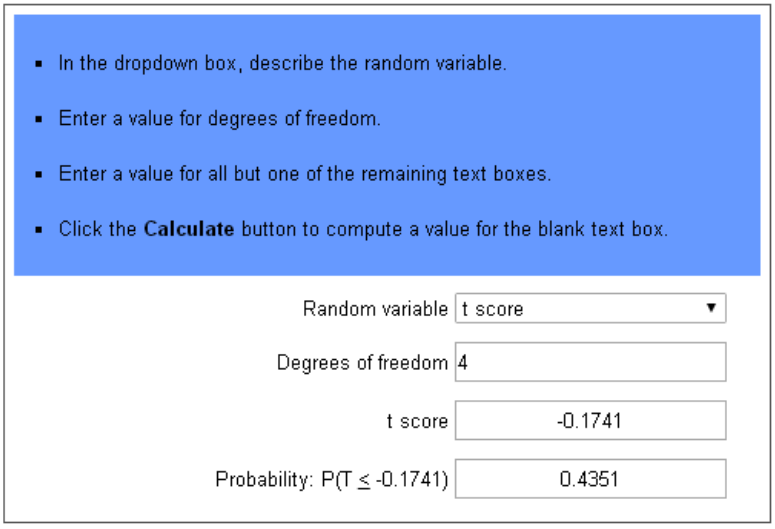
pdf for A (red - positive) and B (blue - negative)

7) Determine the probability that your reaction time will be less than Glower's if there was one-more test

```
>> xw = xa - xb

xw =    0.0128

>> sw = sqrt(sa^2 + sb^2)

sw =    0.0735

>> t = (xw - 0)/sw

t =    0.1741
```

From StatTrek, a t-score of 0.1741 corresponds to a probability of 0.4351

There is a 43.51% chance that A's reaction time will be less than B's reaction time (individual)



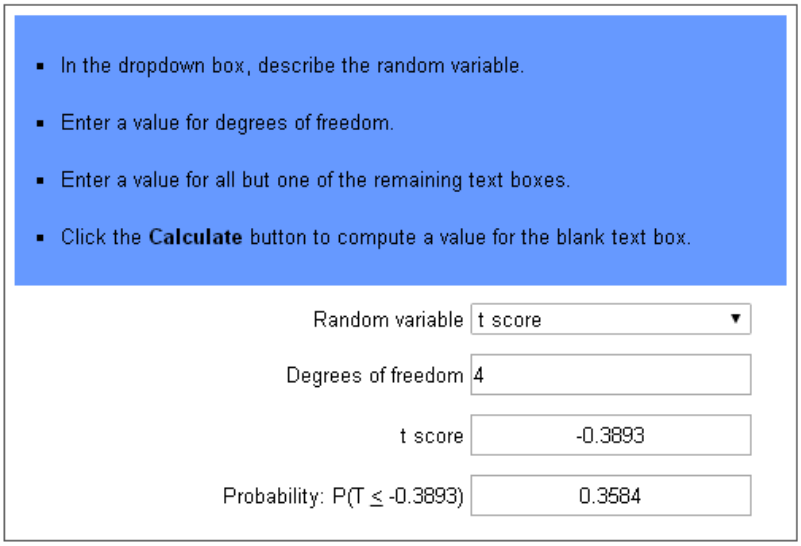
8) Determine the probability that your reaction time is less than Glower's (population's mean)

This is a population question (which population has the lower mean)

- meaning you divide the variance by the sample size

```
>> xw = xa - xb

xw =    0.0128

>> sw = sqrt(sa^2 / 5 + sb^2 / 5)

sw =    0.0329

>> t = (xw - 0)/sw

t =    0.3893
```

There isa 38.93% chance that A's reaction time is less than B's (population)



Type-1 / Type-2 Errors

9) Suppose you want to determine who took the test based upon their reaction time.

- Pick a thereshold such as 300ms
- If the person's reaction time is less / more than 300ms, you declare the person taking the test was yourself or Glower

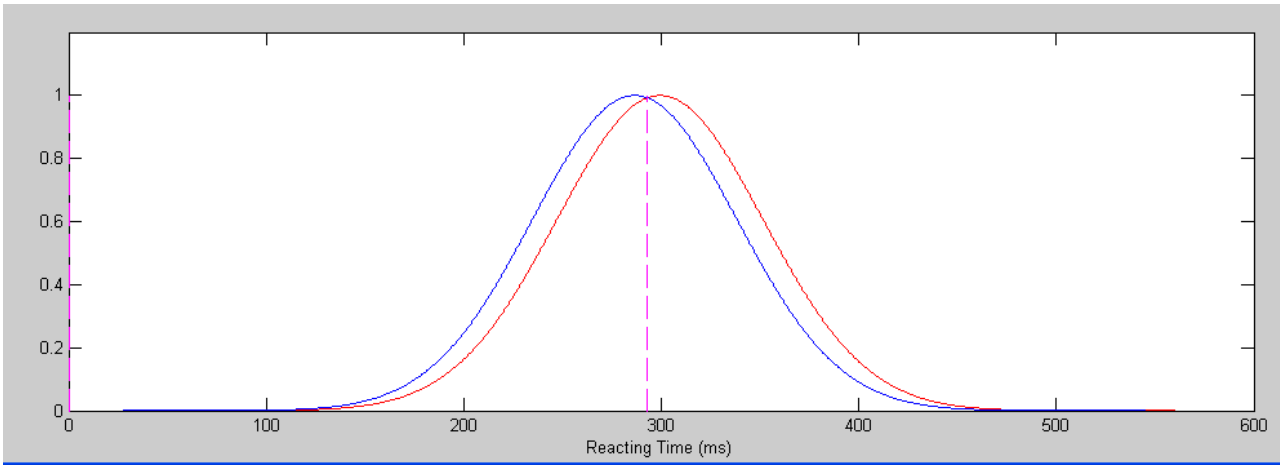
9a) What theshold do you pick and why?

Pick the midpoint between the two means:

```
>> T = (xa + xb) / 2

T =    0.2930
```

Test:

- If the reaction time is more than 293ms, A took the test (red pdf)
- If the reaction time is less than 293ms, B took the test (blue pdf)



9b) With this threshold, what is the probability of a false positive

- Glower took the test but your test results say you took it
- The area of B to the right of the threshold

The t-score is

```
>> t = (xb - 0.293)/sb

t =   -0.1236
```

This corresponds to a probability of 0.4538

There is a 45.38% chance of a false positive.

If the results are positive (conclusion is A took the test), 45% of the time you're wrong. There is a *lot* of overlap between the two pdf's

9c) What is the probability of a false negative?

- You took the test bur your test results say Glower took it
- The area of the red curve left of T

```
>> t = (xa - 0.293)/sa

t =    0.1226
```

From StatTrek, a t-score of -0.1226 corresponds to a probability of 0.4542

There is a 45.42% chance of a false negative

If the results are negative (conclusion is B took the test), 45.42% of the time you'll be wrong.

10) Take the test one more time. Who does your test from problem #9 say took the test?

Reaction time was 0.24 seconds

- This is less than 293ms
- I conclude that B took the test
- It was actually A taking the test
- The test result is incorrect (false negative)