ECE 341 - Homework #14

Chi-Squared Tests. Due Thursday, June 11th

Please make the subject "ECE 341 HW#13" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

Fair Die

The following Matlab code generates a random number from 1..6 (6-sided die).

```
RESULT = zeros(1,6);

for n=1:120
    d6 = ceil( 6*rand );
    RESULT(d6) = RESULT(d6) + 1;
end

RESULT
```

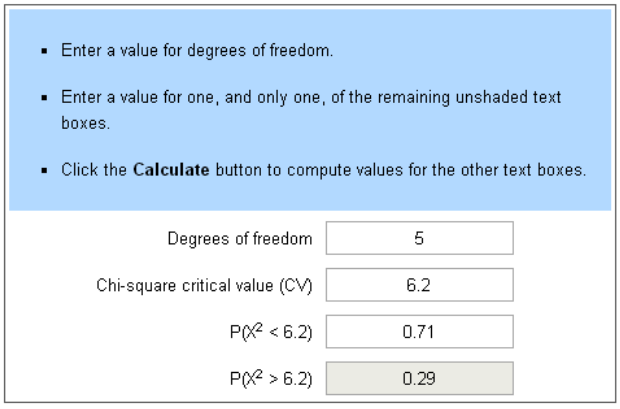
1) Use a chi-squared test to determine if it is a fair die (all numbers equal probability) with 120 rolls.

```
RESULT =    18    27    25    19    14    17
```

Roll	p	np	N	chi squared
1	1/6	20	18	0.2
2	1/6	20	27	2.45
3	1/6	20	25	1.25
4	1/6	20	19	0.05
5	1/6	20	14	1.8
6	1/6	20	17	0.45
			Total	**6.2**

From StatTrek, a chi-squared score of 6.2 with 5 degrees of freedom corresponds to a probability of 0.71

There is a 71% chance that this is not a fair die



2) Use a chi-squared test to determine if it is a fair die (all numbers equal probability) with 120,000 rolls.

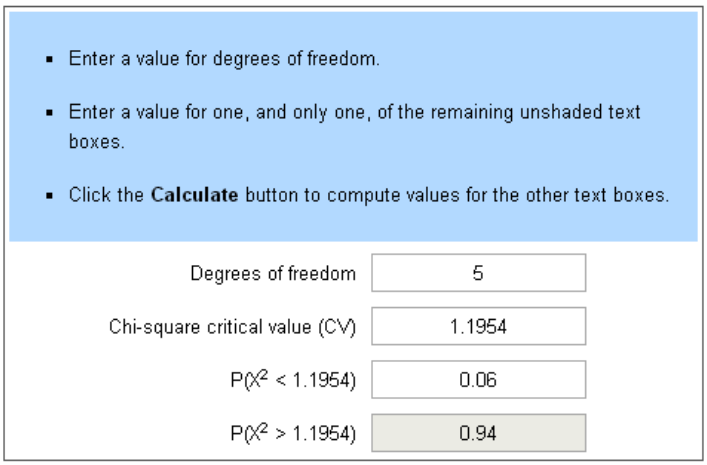
```
RESULT =    19985       20056       19871       20013       20016       20059
```

Setting up a chi-squared table

Roll	p	np	N	chi squared
1	1/6	20,000	19,985	0.0113
2	1/6	20,000	20,056	0.1568
3	1/6	20,000	19,871	0.8321
4	1/6	20,000	20,013	0.0084
5	1/6	20,000	20,016	0.0128
6	1/6	20,000	20,059	0.1741
			Total	**1.1954**

From StatTrek, a chi-squared score of 1.19 corresponds to a probability of 0.06

There is a 6% chance that this is a loaded die



Loaded Die

The following Matlab code generates a random number from 1..6 (6-sided die).

```
RESULT = zeros(1,6);

for n=1:120
    if(rand < 0.04)
        d6 = 6;
    else
        d6 = ceil( 6*rand );
    end

    RESULT(d6) = RESULT(d6) + 1;
end

RESULT
```

3) Use a chi-squared test to determine if it is a fair die (all numbers equal probability) with 120 rolls.

```
RESULT =        29    20    16     9    24    22
```

Roll	p	np	N	chi squared
1	1/6	20	29	4.05
2	1/6	20	20	0
3	1/6	20	16	0.8
4	1/6	20	9	6.05
5	1/6	20	24	0.8
6	1/6	20	22	0.2
			Total	**11.9**

From StatTrek, this corresponds to a probability of 0.96

There is a 96% chance that this is a loaded die

4) Use a chi-squared test to determine if it is a fair die (all numbers equal probability) with 120,000 rolls.

```
RESULT =      19214        19171        19237        19317        19165        23896
```

Roll	p	np	N	chi squared
1	1/6	20,000	19,214	30.8898
2	1/6	20,000	19,171	34.3621
3	1/6	20,000	19,237	29.1085
4	1/6	20,000	19,317	23.3245
5	1/6	20,000	19,165	34.8613
6	1/6	20,000	23,896	758.9408
			Total	911.4868

From StatTrek, is is at least 99.995% certain that the die is loaded.

Am I psychic?

5) Take a deck of playing cards. Predict the suit of the top card then reveal it. If correct, place it in one pile. If incorrect, place it in another pile.

5a) How many times were you correct? Incorrect?

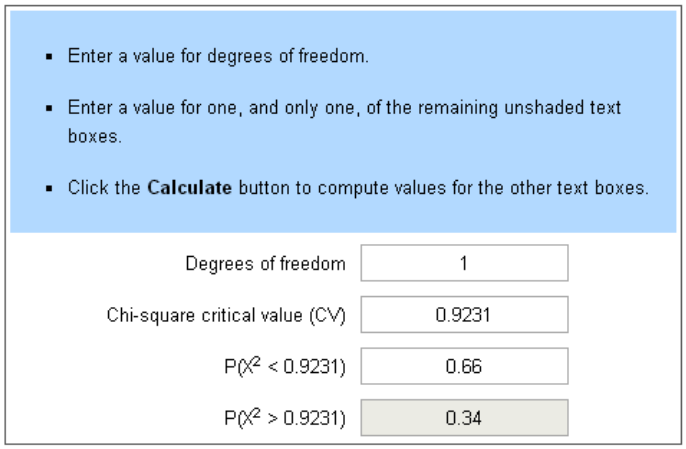
- 10 correct
- 42 incorrect

5b) From your results, determine the odds that you are guessing (25% chance of being correct) using a chi-squared test.

Roll	p	np	N	chi squared
correct	1/4	13	10	0.6923
incorrect	3/4	39	42	0.2308
			Total	**0.9231**

From StatTrek, a chi-squared score of 0.9231 corresponds to a probability of 0.66

There is a 66% chance that I'm not just guessing (i.e. I'm somewhat psychic)



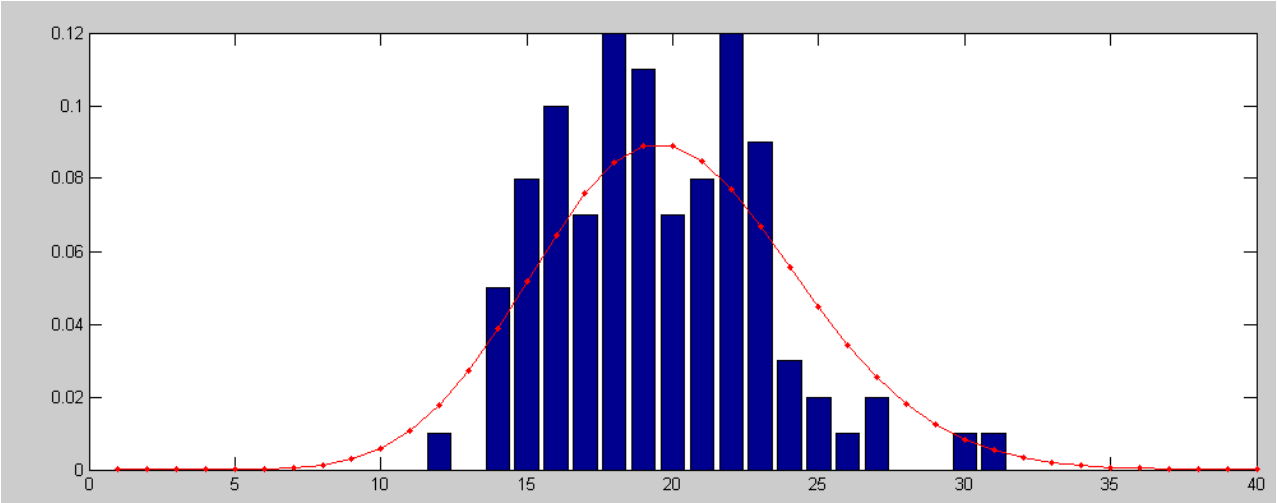
Poisson approximation for a binomial distribution.

5) Let X be the number of 1's and 2's you get when you roll 60 dice. The Poisson approximation for the pdf is

$$\lambda = np = 20$$

$$\binom{60}{x}\left(\frac{2}{6}\right)^{x}\left(\frac{4}{6}\right)^{60-x} \approx \left(\frac{1}{x!}\right) 20^{x}\, e^{-20}$$

- Use Matlab to count the number of 1's and 2's you get when you roll 60 dice
- Repeat 100 times
- Check whether the result is consistent with a Poisson distribution with $\lambda = np = 20$ using a Chi-squred test



Experimental Results (bar graph) and Poisson approximation

bin	p	np	N	chi squared
1-10	0.0108	1.08	0	1.08
11-15	0.1457	14.57	14	0.0223
16-20	0.4026	40.26	47	1.1284
21-25	0.3287	32.87	34	0.0388
26-30	0.0987	9.87	4	3.4911
31-99	0.0135	1.35	1	0.0907
			Total	**5.8513**

From StatTrek, a chi-squared score of 5.85 corresponds to a probability of 0.68

There is a 68% chance that the data is inconsistent with a Poisson distribution