

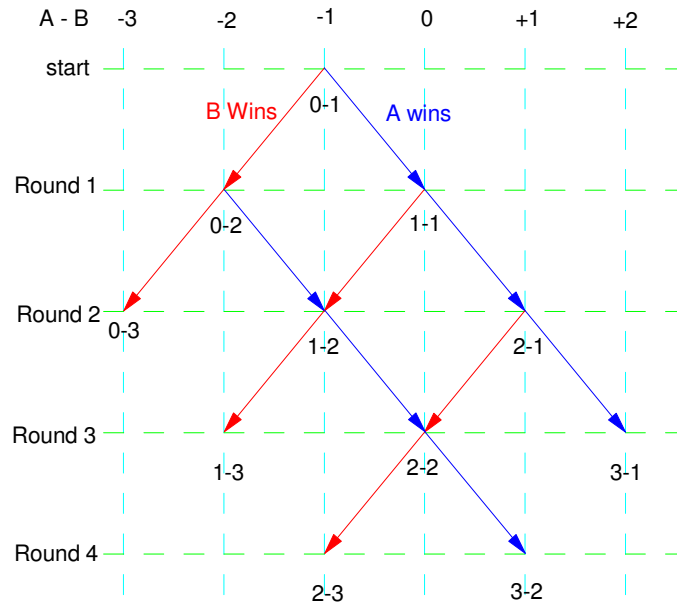
ECE 341 - Homework #1

Tree Diagrams and Enumeration.

1) Two teams, A and B, are playing a best of 5 game series.

- The series is over once one team wins 3 games.
- B starts with +1 point (odds)

Draw the tree diagram for all possible outcomes of the series.



2) List all possible combinations of rolling a 4-sided die (d4) and a 6-sided die (d6) (enumeration).

(d4, d6) min(d4, d6)		6-sided die					
		1	2	3	4	5	6
4-sided die	1	(1,1) 1	(1,2) 1	(1,3) 1	(1,4) 1	(1,5) 1	(1,6) 1
	2	(2,1) 1	(2,2) 2	(2,3) 2	(2,4) 2	(2,5) 2	(2,6) 2
	3	(3,1) 1	(3,2) 2	(3,3) 3	(3,4) 3	(3,5) 3	(3,6) 3
	4	(4,1) 1	(4,2) 2	(4,3) 3	(4,4) 4	(4,5) 4	(4,6) 4

Also determine the probability $X \in \{1..6\}$ where X is the smallest of the two numbers.

$X = 1$: 9 chances out of 24 outcomes ($p = 9/24$)

$X = 2$: 7 chances out of 24 outcomes ($p = 7/24$)

$X = 3$: 5 chances out of 24 outcomes ($p = 5/24$)

$X = 4$: 3 chances out of 24 outcomes ($p = 3/24$)

$X = 5$: 0 chances out of 24 outcomes ($p = 0/24$)

$X = 6$: 0 chances out of 24 outcomes ($p = 0/24$)

Two players, A and B, are playing a game of dice.

- Player A rolls a d4 and a d6 and takes the smallest of the two numbers (i.e. problem #2)
- Player B rolls a 6-sided die and subtracts one ($d6 - 1$).

Player A wins on ties.

3) What is the conditional probability

- Player A wins given B's score is 3 (B rolled a 4)

The probability that A wins given that B scores 3 points is

$$\begin{aligned} p(\text{A wins}) &= p(\text{A scores 3 points}) + p(\text{A scores 4 points}) \\ &= (5/24) + (3/24) \\ &= 8/24 \end{aligned}$$

Player A has a $8/24$ chance of winning knowing that B scored 3 points

4) What is the probability that player A will win any given game?

$$\begin{aligned} p(\text{A wins}) &= p(\text{A wins} \mid \text{B scores 0 points}) * p(\text{B scores 0 points}) + \\ &\quad p(\text{A wins} \mid \text{B scores 1 point}) * p(\text{B scores 1 point}) + \\ &\quad p(\text{A wins} \mid \text{B scores 2 points}) * p(\text{B scores 2 points}) + \\ &\quad p(\text{A wins} \mid \text{B scores 3 points}) * p(\text{B scores 3 points}) + \\ &\quad p(\text{A wins} \mid \text{B scores 4 points}) * p(\text{B scores 4 points}) + \\ &\quad p(\text{A wins} \mid \text{B scores 5 points}) * p(\text{B scores 5 points}) \end{aligned}$$

$$p(\text{A wins} \mid \text{B scores 0 points}) = 1.000$$

$$p(\text{A wins} \mid \text{B scores 1 points}) = 1.000$$

$$\begin{aligned} p(\text{A wins} \mid \text{B scores 2 points}) &= p(\text{A}=2) + p(\text{A}=3) + p(\text{A}=4) \\ &= (7/24) + (5/24) + (3/24) = 15/24 \end{aligned}$$

$$\begin{aligned} p(\text{A wins} \mid \text{B scores 3 points}) &= p(\text{A}=3) + p(\text{A}=4) \\ &= (5/24) + (3/24) = 8/24 \end{aligned}$$

$$\begin{aligned} p(\text{A wins} \mid \text{B scores 4 points}) &= p(\text{A}=4) \\ &= (3/24) \end{aligned}$$

$$p(\text{A wins} \mid \text{B scores 5 points}) = 0$$

$$p(\text{A wins}) = (1.00)(1/6) + (1.00)(1/6) + (15/24)(1/6) + (8/24)(1/6) + (3/24)(1/6) + (0)(1/6) + (0)(1/6)$$

$$p(\text{A wins}) = 0.513889$$

A has a 51.38889% chance of winning any given game

Enumeration & Farkle

Write a Matlab program to go through every combination of 6d6 and determine...

5) The odds of rolling 5-of-a-kind

dice = xxxxx a a not equal to x

6) The odds of rolling two tripples

dice = xxx yyy x and y different

 five of a kind two tripples
p = 0.003858024691358 0.006430041152263

Elapsed time is 2.485368 seconds

Code:

```
% Lecture #1: Enumeration
% Roll six dice
% Count how many times you get 4 of a kind
% xxxx a b

tic
Pair5 = 0;
Pair33 = 0;

for d1 = 1:6
    for d2 = 1:6
        for d3 = 1:6
            for d4 = 1:6
                for d5 = 1:6
                    for d6 = 1:6
                        Dice = [d1,d2,d3,d4,d5,d6];
                        % check for pairs
                        N = zeros(1,6);
                        for i=1:6
                            N = sum(Dice == i);
                        end
                        [N,b] = sort(N, 'descend');
                        if ( N(1) == 5)
                            Pair5 = Pair5 + 1;
                        end
                        if( (N(1) == 3)*(N(2) == 3))
                            Pair33 = Pair33 + 1;
                        end
                    end
                end
            end
        end
    end
end

% probability:
disp('Odds')
disp([ Pair5, Pair33] / (6^6) )
toc
```

Enumeration in 4-card Poker

7) In 4-card poker, you're dealt just 4 cards. Determine using enumeration the odds of being dealt 2-pair

```
hand = xx yy
```

8) Determine using enumeration the odds of being dealt one-pair

```
hand = xx y z
```

```
n =          2-pair          pair
      2808          82368
```

```
p = 0.010372148859544 0.304249699879952
```

```
Elapsed time is 30.628046 seconds.
```

Code

```
% 4-Card Stud
tic
Pair22 = 0;
Pair2 = 0;

for c1=1:52
    for c2 = c1+1:52
        clc
        disp([c1,c2])
        for c3 = c2+1:52
            for c4 = c3+1:52
                Hand = [c1,c2,c3,c4];
                Value = mod(Hand,13) + 1;

                N = zeros(1,13);
                for n=1:13
                    N(n) = sum(Value == n);
                end

                [N,a] = sort(N, 'descend');
                if ((N(1) == 2)*(N(2) == 2)) Pair22 = Pair22 + 1; end
                if ((N(1) == 2)*(N(2) == 1)) Pair2 = Pair2 + 1; end
            end
        end
    end
end

disp([Pair22, Pair2]/270725)
toc
```