ECE 341 - Homework #5

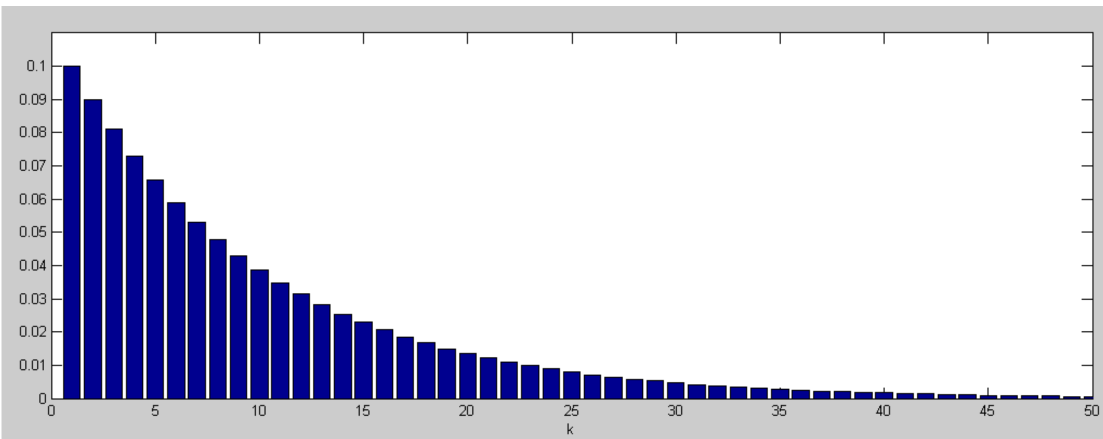
Geometric, Pascal Distributions

Let

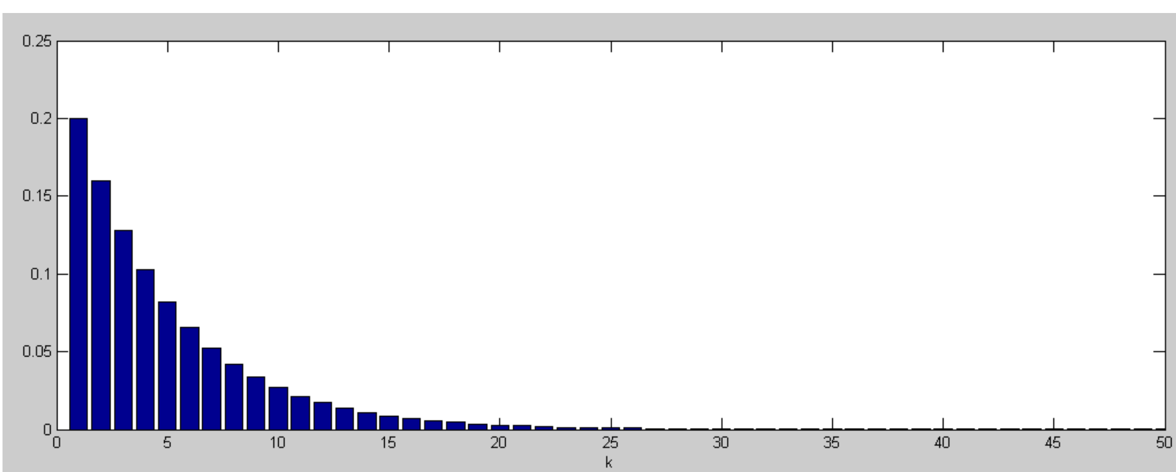
- A be the number of times you roll a 10-sided die until you roll a 1 (p = 1/10)
- B be the number of times you roll a 10-sided die until you get a 1, or 2 (p = 1/5)

1) Determine the pdf of A+B using convolution

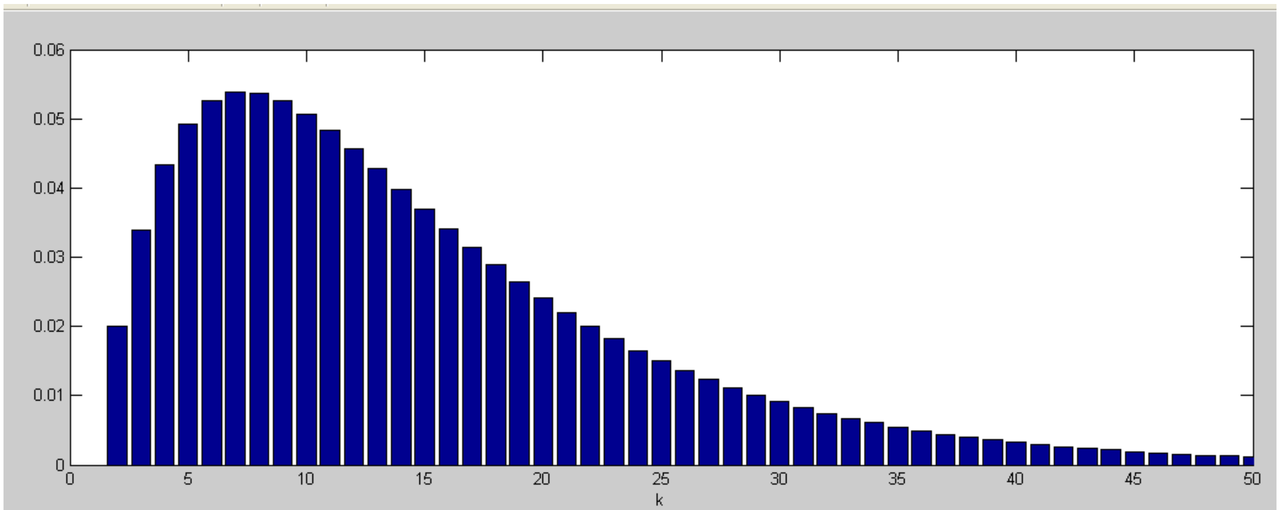
```
>> k = [0:50]';
>> p = 1/10;
>> A = (1-p).^(k-1) .* p;
>> A(1) = 0;
>> bar(k,A)
>> xlabel('k');
```




```
>> p = 1/5;
>> B = (1-p).^(k-1) .* p;
>> B(1) = 0;
>> bar(k,B)
>> xlabel('k');
```



```
>> C = conv(A,B);
>> C = C(1:51);
>> bar(k,C)
>> xlim([0,50]);
>> xlabel('k');
```




```
>> [k,C]

      k          p(k)
           0          0
      1.0000          0
      2.0000     0.0200
      3.0000     0.0340
      4.0000     0.0434
      5.0000     0.0493
      6.0000     0.0526
      7.0000     0.0539
      8.0000     0.0537
      9.0000     0.0525
     10.0000     0.0506
```

2) Determine the pdf of A+B using z-transforms

The z-transform of A is

$$A = \left(\frac{0.1}{z-0.9}\right)$$

The z-transform of B is

$$B = \left(\frac{0.2}{z-0.8}\right)$$

The z-transform of C is then

$$C = \left(\frac{0.1}{z-0.9}\right)\left(\frac{0.2}{z-0.8}\right)$$

Taking the inverse-z transform

$$z^2 C = \left(\frac{0.02z}{(z-0.9)(z-0.8)}\right) z$$

using partial fractions

$$z^2 C = \left(\left(\frac{0.18}{z-0.9}\right) - \left(\frac{0.16}{z-0.8}\right)\right) z$$

$$z^2 C = \left(\left(\frac{0.18z}{z-0.9}\right) - \left(\frac{0.16z}{z-0.8}\right)\right)$$

Take the inverse-z transform

$$z^2 c(k) = \left(0.18(0.9)^k - 0.16(0.8)^k\right) u(k)$$

Divide by z2 (delay 2 samples)

$$c(k) = \left(0.18(0.9)^{k-2} - 0.16(0.8)^{k-2}\right) u(k-2)$$

Checking at k=5 (convolution gave 0.0493)

$$c(5) = \left(0.18(0.9)^3 - 0.16(0.8)^3\right)$$

$$c(5) = 0.0493$$

that checks

Let

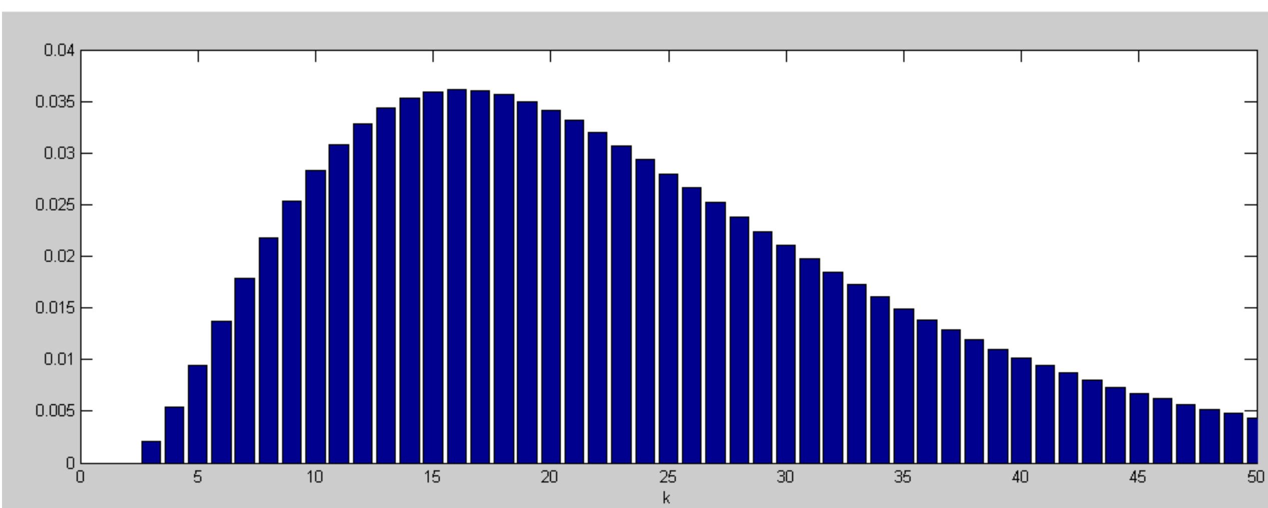
- A be the number of times you roll a 6-sided die until you roll a 1 (p = 1/6)
- B be the number of times you roll a 8-sided die until you get a 1 (p = 1/8)
- C be the number of times you roll a 10-sided die until you get a 1 (p = 1/10)
-

3) Determine the pdf of A+B+C using convolution

```
k = [0:100]';
p = 1/6;
A = (1-p).^(k-1) * p;
A(1) = 0;
p = 1/8;
B = (1-p).^(k-1) * p;
B(1) = 0;
p = 1/10;
C = (1-p).^(k-1) * p;
C(1) = 0;
AB = conv(A,B);
ABC = conv(AB,C);
ABC = ABC(1:101);
bar(k,ABC)
xlim([0,50]);
xlabel('k');

[k(1:11),ABC(1:11)]
```

k	p(k)
0	0
1.0000	0
2.0000	0
3.0000	0.0021
4.0000	0.0054
5.0000	0.0095
6.0000	0.0137
7.0000	0.0179
8.0000	0.0218
9.0000	0.0253
10.0000	0.0283



4) Determine the pdf of A+B+C using z-transforms

The z-transform of A, B, C are

$$A = \left(\frac{1/6}{z-5/6}\right)$$

$$B = \left(\frac{1/8}{z-7/8}\right)$$

$$C = \left(\frac{1/10}{z-9/10}\right)$$

The sum is then

$$D = \left(\frac{1/6}{z-5/6}\right)\left(\frac{1/8}{z-7/8}\right)\left(\frac{1/10}{z-9/10}\right)$$

Doing a partial fraction expansion

trick: multiply by z^3 so that the numberator and denominator are of the same order.

$$z^3 D = \left(z^2\left(\frac{1/6}{z-5/6}\right)\left(\frac{1/8}{z-7/8}\right)\left(\frac{1/10}{z-9/10}\right)\right)z$$

$$z^3 D = \left(\left(\frac{0.5208}{z-5/6}\right) + \left(\frac{-1.5312}{z-7/8}\right) + \left(\frac{1.0125}{z-9/10}\right)\right)z$$

$$z^3 D = \left(\left(\frac{0.5208z}{z-5/6}\right) + \left(\frac{-1.5312z}{z-7/8}\right) + \left(\frac{1.0125z}{z-9/10}\right)\right)$$

Take the inverse-z trasnform

$$z^3 d(k) = \left(0.5208\left(\frac{5}{6}\right)^k - 1.5312\left(\frac{7}{8}\right)^k + 1.0125\left(\frac{9}{10}\right)^k\right)u(k)$$

Divide by z^3 (delay 3 samples)

$$d(k) = \left(0.5208\left(\frac{5}{6}\right)^{k-3} - 1.5312\left(\frac{7}{8}\right)^{k-3} + 1.0125\left(\frac{9}{10}\right)^{k-3}\right)u(k-3)$$

Checking: at k=8, d(k) = 0.0218 from convolution

$$d(8) = \left(0.5208\left(\frac{5}{6}\right)^5 - 1.5312\left(\frac{7}{8}\right)^5 + 1.0125\left(\frac{9}{10}\right)^5\right)$$

$$d(8) = 0.0218$$