ECE 341 - Homework #8

Queueing Theory & Normal Distributions.

Queueing Theory

Assume you are running a fast-food restraunt.

- The time between customers arriving at a restaraunt is an exponential distribution with a mean of 90 seconds.
- The time it takes to serve each customer is an exponential distribution with a mean of 60 seconds.

1) Run a single Monte-Carlo simulation for this restaraunt over the span of one hour.

- Give the formula for each column in you simulation
- What is the longest waiting time for a customer in your simulation?
- What is the largest queue over the span of one hour?

First, generate random arrival and serving times with the above exponential distributions

```
DATA = [];
for i=1:100
   p = rand;
   Tarr = -90*log(1-p);
   p = rand;
   Tser = -60*log(1-p);
   DATA = [DATA ; [i, Tarr, Tser]];
end
round(DATA)

      #  arrive serve
      1   152   142
      2    12   147
      3    90     6
      4    29    47
      5   284   201
      6    15   212
      7   284    40
      8   145     9
```

Equations:

E: E11 = E10+C11

F: F11=MAX(G10,E11)

G: G11=F11+D11

H: H11=F11-E11

I: I11 = (E11<G10)*1+(E11<G9)*1+(E11<G8)*1+(E11<G7)*1+(E11<G6)*1+(E11<G5)*1

Customer	Arrival	Serve	t(Arr)	t(Serve)	t(done)	t(Wait)	Queue
0	0	0	0	0	0	0	0
1	152	142	152	152	294	0	0
2	12	147	164	294	441	130	1
3	90	6	254	441	447	187	2
4	29	47	283	447	494	164	3
5	284	201	567	567	768	0	0
6	15	212	582	768	980	186	1
7	284	40	866	980	1,020	114	1
8	145	9	1,011	1,020	1,029	9	1
9	49	148	1,060	1,060	1,208	0	0
10	141	192	1,201	1,208	1,400	7	1
11	96	2	1,297	1,400	1,402	103	1
12	170	163	1,467	1,467	1,630	0	0
13	102	85	1,569	1,630	1,715	61	1
14	122	30	1,691	1,715	1,745	24	1
15	96	11	1,787	1,787	1,798	0	0
16	110	2	1,897	1,897	1,899	0	0
17	29	3	1,926	1,926	1,929	0	0
18	9	104	1,935	1,935	2,039	0	0
19	107	23	2,042	2,042	2,065	0	0
20	270	2	2,312	2,312	2,314	0	0
21	52	29	2,364	2,364	2,393	0	0
22	131	95	2,495	2,495	2,590	0	0
23	19	40	2,514	2,590	2,630	76	1
24	53	62	2,567	2,630	2,692	63	2
25	111	84	2,678	2,692	2,776	14	1
26	29	68	2,707	2,776	2,844	69	1
27	96	11	2,803	2,844	2,855	41	1
28	11	41	2,814	2,855	2,896	41	2
29	289	25	3,103	3,103	3,128	0	0
30	79	15	3,182	3,182	3,197	0	0
31	125	18	3,307	3,307	3,325	0	0
32	63	72	3,370	3,370	3,442	0	0
33	199	192	3,569	3,569	3,761	0	0

Normal Distribution

The mean and standard deviation for a 6 and 8-sided die are

$$\mu_{d6} = 3.50 \qquad \mu_{d8} = 4.50$$

$$\sigma_{d6} = 1.7078 \qquad \sigma_{d8} = 2.291$$

$$\sigma_{d6}^2 = 2.9166 \qquad \sigma_{d8}^2 = 5.2487$$

2) Let Y be the sum of rolling five 6-sided dice (5d6) plus five 8-sided dice (5d8).

Y = 5d6 + 5d8

a) What is the mean and standard deviation of Y?

$$\mu = 5 \cdot 3.50 + 5 \cdot 4.50$$

$$\mu = 40.0$$

$$\sigma^2 = 5 \cdot 2.9166 + 5 \cdot 5.2487$$

$$\sigma^2 = 40.8263$$

$$\sigma = 6.3895$$

b) Using a normal approximation, what is the 90% confidence interval for Y?

From Stattrek, 5% tails corresponds to a z-score of 1.645

$$\mu - 1.645\sigma < Y < \mu + 1.645\sigma \qquad p = 90\%$$

$$29.489 < Y < 50.511$$

c) Using a normal approximation, what is the probability that the sum the dice will be more than 49.5?

$$z = \left(\frac{49.5-40}{6.3895}\right) = 1.4868$$

From StatTrek, this corresponds to a probability of 0.069

There is a 6.92% chance of rolling 49.5 or more

3) Check your answer using a Monte-Carlo simulation in Matlab with 100,000 rolls:

```
N = 0;
for i=1:1e5
   Y = sum( ceil( 6*rand(5,1) ) ) + sum( ceil( 8*rand(5,1) ) );
   if(Y > 49.5)
      N = N + 1;
      end
   end
N / 1e5


ans =     0.0689        ( vs. 6.92% )
```

which is close to the compted score using a normal approximation

4) Fargo's high temperature in the month of July has been measured by Hector Airport since 1942.

- Determine the mean and standard deviation for the high in July
- Assuming a normal distribution, determine the probability that the high in July will exceed 100F this year

(note: data set is linked on Bison Academy)

```
>> July = DATA(:,8);
>> x = mean(July)

x =   94.63

>> s = std(July)

s =   4.00
```

The z-score for 100F is

$$z = \left(\frac{100-94.63}{4.00}\right) = 1.3425$$

from StatTrek, a z-score of -1.3425 corresponds to a probability of 9.0%

There is a 9.0% chance that it will break 100F this coming July

11 : 1 odds against