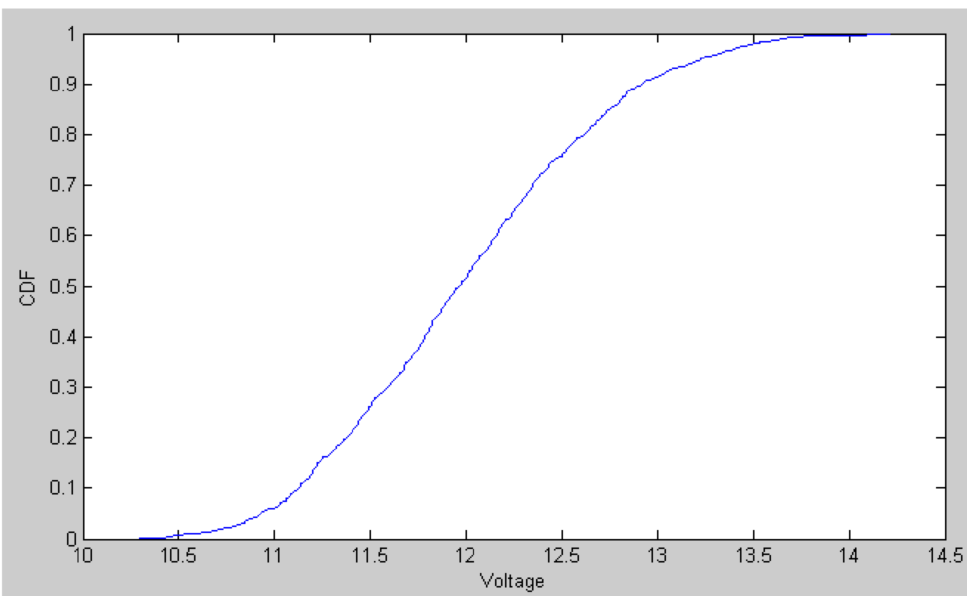
ECE 341 - Homework #9

Weibull Distribution, Central Limit Theorem. Due Wednesday, June 1st

Weibull Distribution

1) Determine and plot the cdf for the voltage, Y, in homework set #7

```
y = [];

for i=1:1000
   R1 = 1000 * (1 + (2*rand-1)*0.05);
   R2 = 2000 * (1 + (2*rand-1)*0.05);
   R3 = 1000 * (1 + (2*rand-1)*0.05);
   R4 = 3000 * (1 + (2*rand-1)*0.05);
   Y = 2*R2*R4 / (R1*R3);
   y = [y ; Y];
end

p = [1:1000]' / 1000;
y = sort(y);

save HW9 y

plot(sort(y),p)
xlabel('Voltage');
ylabel('CDF');
```



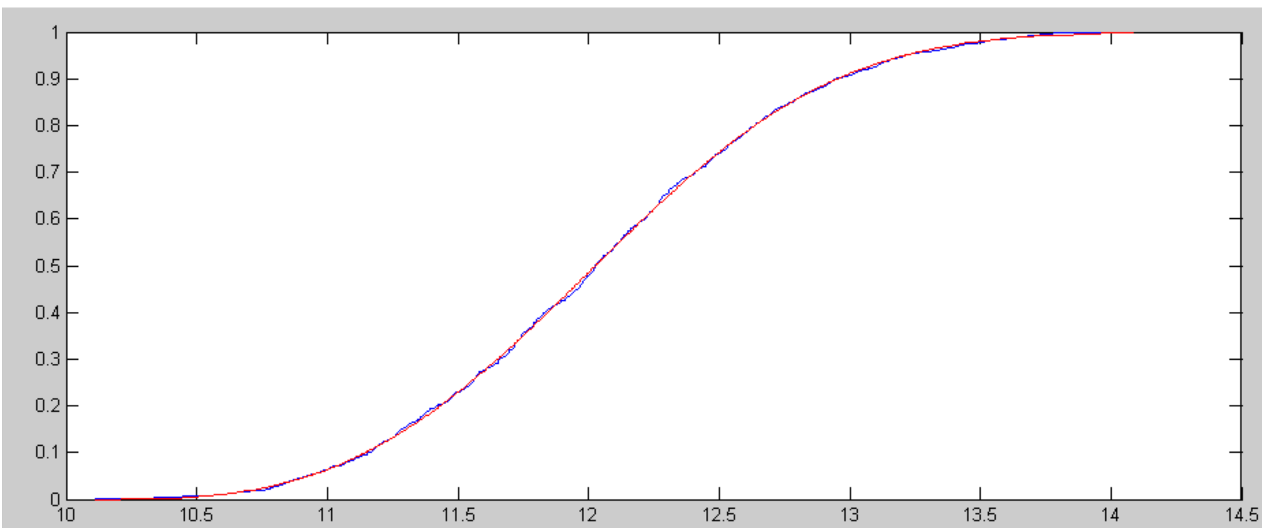
```
>> [Z,e] = fminsearch('Prob1', [2,2,10])

          k        lambda     X0
Z =     3.0199    2.1435   10.1286

e =     0.0194
```

meaning

$$F_x(\lambda, k) = \left(1 - \exp\left(-\left(\frac{x-x_0}{\lambda}\right)^k\right)\right) u(x - x_0)$$

$$F_x(\lambda, k) = \left(1 - \exp\left(-\left(\frac{x-10.1286}{2.1435}\right)^{3.0199}\right)\right) u(x - 10)$$

```
>>
```



Weibull approximation (red) and circuit CDF (blue)

- Note: A Weibull distribution is able to closely approximate a lot of probability funcitons

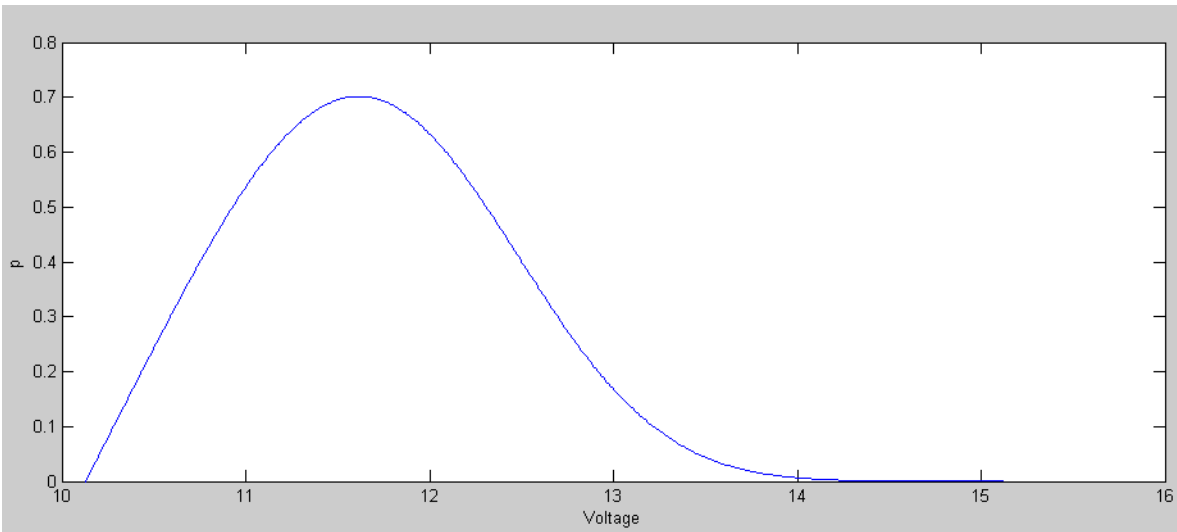
Optimization Code:

```
function y = Prob1(z)
% y = cost(z)
% Weibull distribution curve fit

   k = z(1);
   L = z(2);
   X0 = z(3);

% data to curve fit
   load HW9;

   Vy = sort(y);
   p = [1:length(DATA)]' / length(DATA);

   x = Vy - X0;
   x = max(0,x);

% p(Vy) = target

   W = 1 - exp( -( (x/L) .^ k ) );

   e = p - W;

   plot(Vy,p,'b',Vy,W,'r');
   pause(0.01);

   y = sum(e.^2);

end
```

2) Determine and plot the pdf for the voltage, V4, in homework set #7 using a Weibull approximation for the cdf meaning the pdf is

$$f(x:\lambda,k)=\frac{k}{\lambda}\left(\frac{x-x_0}{\lambda}\right)\exp\left(-\left(\frac{x-x_0}{\lambda}\right)^k\right)u(x-x_0)$$

```
k = Z(1);
L = Z(2);
X0 = Z(3);
x = [0:0.01:5]';
f = k/L * (x/L) .* exp(-( (x/L).^k));
plot(x+X0,f)
xlabel('Voltage');
ylabel('p');
```



Central Limit Theorem

3) Let X be the sum of seven 6-sided dice (7d6 or a level-7 fireball for D&D fans).

- Determine the mean and standard deviation of X
- Determine the probability of doing 34.5 or more damage using a normal approximation

For a single die

$$\mu_1 = \tfrac{1}{n}\sum x_i = 3.5$$

$$\sigma_1^2 = \tfrac{1}{n}\sum (x_i - \mu)^2 = 2.9167$$

For seven dice

$$\mu_7 = 7\mu_1 = 24.5$$

$$\sigma_7^2 = 7\sigma_1^2 = 20.4167$$

$$\sigma_7 = 4.5184$$

Using a normal approximation, the probability of doing more than 34.5 damage is

$$z = \left(\tfrac{34.5-24.5}{4.5184}\right) = 2.213$$

From StatTrek, this corresponds to a probability of 1.3%

4) Using Matlab, determine the actual odds of rolling 35 or more using a Monte Carlo simulation with 1 million level-7 fireballs.

- Roll 7d6 a million times and count how many times you did 35 or more damage

Code:

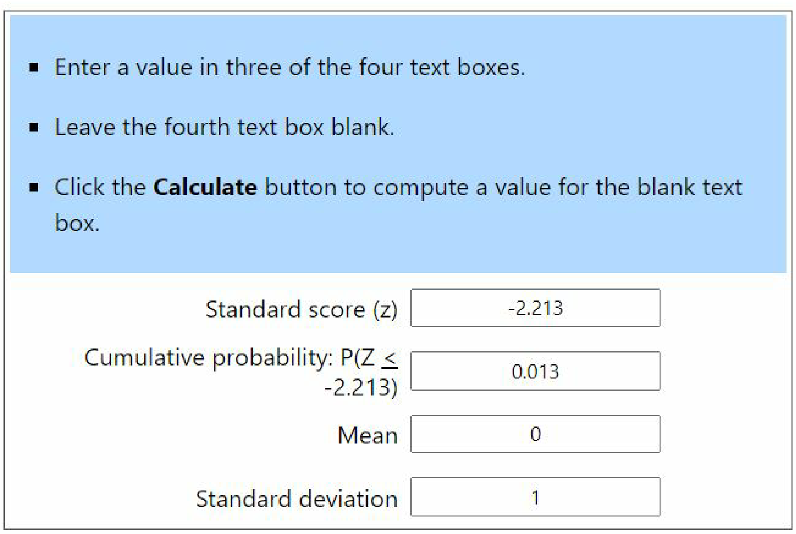
```
% Homework #9 problem 4
% Level 7 fireball

N = 0;

for i=1:1e6
    Dice = sum( ceil(6*rand(7,1)));
    if(Dice > 34.5) N = N + 1; end
end

N
```

Result:

```
N =      12040
N =      12055
N =      12164
N =      12141

Elapsed time is 8.592316 seconds.
```

p = 1.2099% (approximately - using the average)

- StatTrek = 1.3%



Let {a, b, c, d, d, e} each be uniformly distributed over the range of (0, 1).

Let X be the sum: a + b + c + d + e

5) Determine

- The mean and standard deviation of X
- The probability that X is more than 3.50 using a normal approximation

For a uniform distribution over the range of (0,1)

$$\mu = \frac{1}{2}$$

$$\sigma^2 = \frac{1}{12}$$

For the sum of five uniform distributions

$$\mu = \frac{5}{2}$$

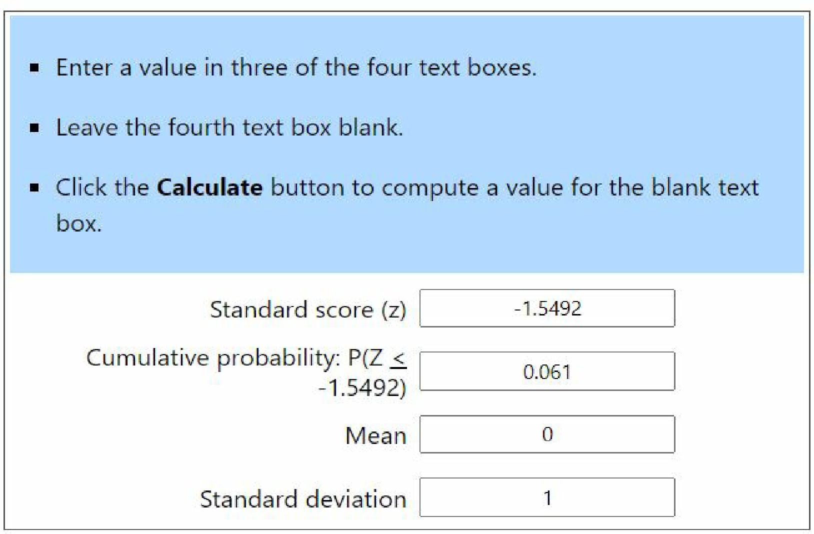
$$\sigma^2 = \frac{5}{12}$$

$$\sigma = 0.6455$$

The probability of the sum being more than 3.50 is thus

$$z = \left(\frac{3.5 - 2.5}{0.6455}\right) = 1.5492$$

From StatTrek, this corresponds to a probability of 6.1%



6) Determine the probability that X is more than 4.00 using a Monte-Carlo simulation with 1 million die rolls.

Code

```
% Homework #9 problem 6
% Sum of five uniform distrubutions

N = 0;

for i=1:1e6
    Dice = sum( rand(5,1) );
    if(Dice > 3.5) N = N + 1; end
end

N
```

Result: (three different runs)

```
N =       61740
N =       62133
N =       62340
```

From a Monte-Carlo simulation, the probability is 6.207% (average of these results)

- vs. 6.1% using a Normal approximation